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Feigenbaum universality in String theory

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Brane-like vertex operators, defining backgrounds with the ghost-matter mixing in NSR superstring theory, play an important role in a world-sheet formulation of D-branes and M theory, being creation operators for extended objects in the second quantized formalism. In this paper we show that dilaton's beta function in ghost-matter mixing backgrounds becomes stochastic. The renormalization group (RG) equations in ghost-matter mixing backgrounds lead to non-Markovian Fokker-Planck equations which solutions describe superstrings in curved space-times with brane-like metrics. We show that Feigenbaum universality constant $\delta = 4,669\ldots$ describing transitions from order to chaos in a huge variety of dynamical systems, appears analytically in these RG equations. We find that the appearance of this constant is related to the scaling of relative space-time curvatures at fixed points of the RG flow. In this picture the fixed points correspond to the period doubling of Feigenbaum iterational schemes.

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Superstring theory is our current hope to put gravity in a Prokrust's bed of quantum mechanics. In spite of all the spectacular progress in the last quarter of the century [1] the full structure and underlying symmetries of the theory have yet to be unveiled. One of the most striking features of String theory is a deep relation between renormalization group (RG) flows on a world sheet and an evolution in a target space. Critical points of these RG flows, described by 2D conformal field theories (CFT), determine equations of motion in a target space. The structure of these equations is determined by the world sheet correlation functions of the appropriate vertex operators in respective CFT [2]. The conformal field theory description of strings in curved backgrounds, such as of strings in the presence of branes, as well as the underlying CFT of strongly coupled strings is much harder a problem to tackle, in particular because the adequate knowledge of quantum degrees of freedom of M-theory and non-perturbative strings is still lacking. Some time ago, we have proposed the formalism [3 – 6] that describes the non-perturbative dynamics of solitons in string and M-theory in terms of a special class of vertex operators, called brane-like states. The crucial distinction of these vertex operators from usual one (such as a photon or a graviton) is that they exist at nonzero ghost pictures only. The simplest example of these vertices in the closed string case is given by (before integration over the world-sheet):

$$\begin{aligned} V_5^{(-3)}(q) &= e^{-3\phi-\bar\phi}\psi_{t_1}...\psi_{t_5}\bar\psi_{t_6}e^{iq_aX_a}(z,\bar z),\\ V_5^{(-2)}(q) &= c\partial\chi e^{\chi-3\phi-\bar\phi}\psi_{t_1}...\psi_{t_5}\bar\psi_{t_6}e^{iq_aX_a}(z,\bar z),\\ V_5^{(+1)}(q) &= e^{\phi-\bar\phi}\psi_{t_1}...\psi_{t_5}\bar\psi_{t_6}e^{iq_aX_a}(z,\bar z)+\\ &+b-c\ \ ghosts,\quad a=0,...,3;\ t_i=4,...9. \end{aligned} \quad (1)$$

It is important that the discrete picture-changing gauge symmetry is broken for such operators and their superconformal ghost dependence cannot be removed by any picture-changing transformation. We shall refer to this property of the brane-like vertices as the ghost-matter mixing. The crucial property of these special vertex operators is that they do not correspond to any perturbative string excitation but describe the nonperturbative dynamics of extended solitonic objects, such as D-branes.

In [6] we have shown that the low-energy effective action of the sigma-model with the brane-like states is given by the DBI action for D-branes. From the world sheet point of view this means that the insertion of vertices with the ghost-matter mixing makes the deform CFT describing strings in flat space-time and it flows to a new fixed point, corresponding to the CFT of strings in a curved background induced by D-branes. In this paper we shall further investigate RG flows in the ghost-matter mixing backgrounds. It appears that properties

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of these RG flows are stunningly different from the usual ones. We found that ghost-matter mixing adds to RG flow operator-valued stochastic terms. Even more intriguing is the emergence of universal constant in the RG equations which with accuracy less than 0.5% is nothing but the logarithm of the famous Feigenbaum constant $\delta = 4.669$ [7]. This coincidence is not accidental but reflects remarkable and new relations between superstrings, chaos, gravity and stochastic processes which is the subject of this Letter.

The crucial property of world sheet conformal beta-functions (e.g. of a dilaton) in ghost-matter mixing backgrounds is the presence of stochastic terms in the RG equations. One specific property of the brane-like states, is that their OPE algebra is picture-dependent. This picture dependence leads to non-deterministic stochastic terms in the dilaton's beta-function. Namely, consider the NSR sigma model in $D = 10$ perturbed by the dilaton and the ghost-matter mixing vertex (1). The generating functional for this model is

$$Z(\varphi,\lambda) = \int DXD\psi D[ghosts] : f(\Gamma) :: f(\bar\Gamma) : \times \\ \times \exp\{-S_{NSR} + \int d^4q\lambda(q)\int d^2zV_5(q,z,\bar z) + \\ + \int d^{10}p\varphi(p)\int d^2wV_\varphi^{(-2)}\varphi(p,w,\bar w)\}. \qquad (2)$$

Here

$$: f(\Gamma) :=: \frac{1}{1-\Gamma} := 1+ : \Gamma : + : \Gamma^2 : + \ldots$$

is the measure function of picture-changing operator, $: \Gamma :=: e^\phi G :$ with $G = G_m + G_{gh}$ being the full matter + ghost world sheet supercurrent. The dilaton vertex operator can be taken at any negative picture. It is convenient to take V_φ at picture -2 (both left and right) as in this case the dilaton vertex operator is given by

$$V_\varphi(p) = \int d^2ze^{-2\phi-2\bar\phi}\partial X^m\partial X_n(\eta_{mn} - k_m\bar k_n - k_n\bar k_m).$$

Let us expand the generating functional (2) up to the third order of λ and the second order of φ, which symbolically can be written as (keeping only relevant terms)

$$Z =< \ldots + \lambda\varphi V_5V_\varphi + \lambda^2V_5V_5 + \\ + \varphi^2V_\varphi V_\varphi + \lambda^3V_5V_5V_5 + \ldots > . \qquad (3)$$

To determine the UV divergences in the partition function (2), relevant to the dilaton's beta-function, one has to point out the relevant singular terms in the OPE algebra of the dilaton and V_5. In the on-shell limit, the relevant terms in the operator algebra are given by:

$$V_5^{(a)}(w_1,\bar w_1;q_1)V_5^{(b)}(w_2,\bar w_2;q_2) \sim \\ \sim \frac{C_{[-a|b]}(q_1,q_2)V_\varphi^{(a+b)}(q_1+q_2)}{|z_1-z_2|^2} + \ldots,\ a,b = +1,-3, \quad (4)$$

where

$$C_{[-3|-3]}(q_1,q_2) \sim (q_1q_2)(1 + (q_1+q_2)^2), \\ C_{[-3|1]}(q_1,q_2) \sim (q_1q_2), \\ C_{[-3|1]}(q_1,q_2) \sim (q_1q_2)(1 - (q_1+q_2)^2). \qquad (5)$$

Next, one has to point out the picture changing rules for the left part of the V_5-operator, in order to specify how it is acted on by $: f(\Gamma) :$ The picture changing transformation rules for the V_5 operators (1) can be written in the form

$$: \Gamma :^n V_5^{(k)}(p) = \alpha_{[k|n+k]}V_5^{(N+k)}(p), \\ \alpha_{[i|j]} = \alpha_{[m|n]} = \alpha_{[n|m]} = 1, \\ \alpha_{[a|j]} = \alpha_{[a|b]} = \alpha_{[s|a]} = 0, \\ \alpha_{[i|m]} = \alpha_{[s|m]} = 1 + p^2, \\ s,t = -\infty, \ldots. - 4;\ i,j = -3,-2; \\ a,b = -1,0;\quad m,n = 1,2,\ldots \qquad (6)$$

In the beta-function calculations, when the vertex operators are taken slightly off-shell, the following identities are useful:

$$\alpha_{[i|m]}C_{[m|n]} = C_{[i|n]};\quad \alpha_{[i|m]}C_{[m|j]} = C_{[i|j]}. \qquad (7)$$

Finally, using the fact that picture changing operators form the polynomial ring:

$$: \Gamma :^{m+n} =: \Gamma^m\Gamma^n : +[Q_{BRST},\ldots] \qquad (8)$$

the action of the $: \Gamma^n :$ operator on the vertex operators inside the functional integral can be expressed as:

$$<: \Gamma^n : (w)V_1(z_1)\ldots.V_(z_N) >= \\ = \sum_{k_1,\ldots k_N=0}^{k_1+\ldots+k_{N-1}=n} N^{-n}\frac{n!}{k_1!\ldots k_{N-1}!(n-k_1-\ldots-k_{N-1})!} \times \\ \times <: \Gamma :^{k_1} V_1(z_1)\ldots : \Gamma :^{k_{N-1}} V_{N-1}(z_{N-1}) \\ : \Gamma :^{n-k_1-\ldots-k_{N-1}} V_N(z_N) > \qquad (9)$$

i.e. the correlator does not depend on w. The factor of N^{-n} in (9) insures the correct normalization of amplitudes in the picture-independent case. Using the relations (6)–(9) we are finally in the position to start evaluating the beta-function. The first contribution of interest to the beta-function comes from the λ^2-term, bilinear in the V_5-operator. At a given picture level n this term leads to the following divergence in the order of λ^2:

$$\frac{1}{2}\int d^2w_1d^2w_2 <:\Gamma^{n+6} : V_5^{(-3)}(w_1)V_5^{(-3)}(w_2)\ldots >= \\ = 2^{-n-7}\sum_{k=0}^{n+6}\frac{(n+6)!}{k!(n+6-k)!}\alpha_{[-3|k-3]}C_{[k-3|n+3-k]} \times \\ \times \alpha_{[-3|n+3-k]}\int d^2\xi < V_\varphi^{(n)}(\xi,\bar\xi)\ldots > \times \log\Lambda, \qquad (10)$$

where $\xi = 1/2(w_1 + w_2), \eta = 1/2(w_1 - w_2)$ and $\log\Lambda = \int_\Lambda \frac{d^2\eta}{|\eta|^2}$ is the log of the world sheet UV cutoff. For the sake of brevity we suppress the momentum dependence of fields, vertices and structure constants here and below. This divergence is removed by renormalizing the dilaton field as

$$\varphi \to \varphi - \sum_{n=0}^{\infty} 2^{-n-7} \sum_{k=0}^{n+6} \frac{(n+6)!}{k!(n+6-k)!} \alpha_{[-3|k-3]} \times \times C_{[k-3|n+3-k]} \alpha_{[-3|n+3-k]} \lambda^2 \log\Lambda. \quad (11)$$

In the absence of picture-dependence the sum over k would have been reduced to $\frac{1}{2}C\lambda^2 \log\Lambda$ for each picture, as it should be in the standard case when ghost-matter mixing is absent.

As a result of the dilaton's RG flow, the $\lambda\varphi$ cross-term is renormalized by λ^3 logarithmic divergence:

$$\varphi \to \varphi - \text{const}\lambda^2 \log\Lambda,$$
$$\lambda\varphi V_5 V_\varphi \to \lambda\varphi V_5 V_\varphi - \text{const}\lambda^3 \log\Lambda V_5 V_\varphi. \quad (12)$$

Using the identities (7) relating α and C after some straightforward transformations we can cast the renormalization of the $\lambda\varphi$-term under the flow (11) as

$$-\lambda^3 \log\Lambda C_{[-3|-3]} \alpha_{[-3|1]} \times \times \int d^2 w_2 V_\varphi^{(-6)}(w_1) V_5^{(n+6)}(w_2) \times \times \sum_{n=0}^{\infty} \sum_{k,l=0}^{k=n+6;l=n+5} \frac{(n+5)!(n+6)!}{k!l!(n+5-l)!(n+6-k)!}, \quad (13)$$

where in the sum over k and l one must have $ak \neq 2, 3, n+3, n+6$; $l \neq n+2, n+3$. This gives the renormalization of the $\lambda\varphi$ cross-term under the RG flow (11) of the dilaton field in the ghost-matter mixing case. The other contribution of the same order of λ^3 to the dilaton beta-function comes from the OPE singularities inside the λ^3-term itself, appearing in the expansion (3) of the partition function. After simple calculations we get:

$$\frac{1}{2}\lambda^3 \log\Lambda \sum_{n=0}^{\infty} 3^{-n-9} \sum_{k,l=0}^{k+l=n+9} \frac{(n+9)!}{k!l!(n+9-k-l)!} \times \times C \int d^2 w_1 \int d^2 w_2 < V_\varphi^{(-6)}(w_1) V_5^{(n+6)}(w_2)... >;$$

$$3C = C_{[k-3|l-3]} \alpha_{[-3|k-3]} \alpha_{[-3|n+6-k-l]} \alpha_{[-3|n+6]} + + C_{[k-3|n+6-k-l]} \alpha_{[-3|k-3]} \alpha_{[-3|l-3]} \alpha_{[-3|n+6]} + + C_{[k-3|n+6-k-l]} \alpha_{[-3|l-3]} \alpha_{[-3|n+6-k-l]} \alpha_{[-3|n+6]}. \quad (14)$$

Again, it is easy to see that in the absence of the ghost-matter mixing ($\alpha = 1$, all C are picture-independent) this contribution would sum up to

$$\frac{1}{2} C\lambda^3 \log\Lambda \int d^2 w_1 \int d^2 w_2 V(w_1, \bar{w_1}) V(w_2, \bar{w_2}) \quad (15)$$

precisely cancelling the divergence of the same λ^3-type, originating from the renormalization of the $\lambda\varphi$ cross term under the flow. In the picture-independent case this insures that the renormalization (11) of the dilaton field under the flow does not bring about any additional singularities from higher order terms, such as the cubic one and the $\lambda\phi$ cross-term. In particular, this guarantees that terms of the type

$$\sim C\lambda^3 \log\Lambda \int_\Lambda d^2 w V_5(w, \bar{w}) \quad (16)$$

never appear in the dilaton or other perturbative close string field beta-functions in the picture-independent case. On the contrary, should the terms of this type appear in the beta-function, that would imply that the RG equations become stochastic, since from the point of view of the space-time fields, world sheet operator $\int_\Lambda d^2 w V_5(w, \bar{w})$ is a stochastic random variable, with the cutoff parameter Λ playing the role of the stochastic time. In this case, the RG equations have the form of non-Markovian Langevin equations where the memory of the noise is determined by the world sheet correlations of the V_5-operators. This exactly is what happens in the ghost-matter mixing backgrounds, due to the OPE picture dependence. To get the total flow on the λ^3 level, one has to subtract the sum (14) from (12) using the identities (7). We obtain

$$\varphi \to \varphi + \sigma C_{[-3|-3]} \alpha_{[-3|1]} \lambda^3 \log\Lambda \int d^2 w V_5^{(-3)}(w, \bar{w}) \quad (17)$$

where

$$\sigma = 1 - \sum_{n=0}^{\infty} [(n+4)^2 (n+5)^3 (n+6) 2^{-2n-12} + + \frac{(n+8)(n+9)(n+13)}{96} (\frac{2}{3})^{n+9} - ((n+4)^2 (n+5) + + (1/2)(n+5)^2 (n+6)) 2^{-n-6}] = 1.534. \quad (18)$$

Therefore the resulting beta-function equations for the dilaton in the ghost-matter mixing background gives (with the momentum dependence restored):

$$\frac{d\varphi(p)}{d\log\Lambda} = -\frac{\delta S_\varphi}{\delta\varphi(p)} + \sigma C(p) \int d^4 q \lambda(q) \eta,$$
$$C(p) = \int d^4 k C_{[-3|-3]}(p, k) \lambda(\frac{k+p}{2}) \lambda(\frac{k-p}{2}),$$
$$\eta = \int_\Lambda d^2 w V_5^{(-3)}(w, q), \quad (19)$$

with $\sigma = 1.534...$ There are other examples of vertex operators with ghost-matter mixing, and they also lead to stochastic terms in the beta-function of the dilaton. In particular, we have also considered the dilaton field in the background of closed string operators of higher ghost cohomologies:

$$W_5 \sim \int d^2 z e^{-4\phi-\bar{\phi}} \partial X_{(m_1} \ldots \partial X_{m_3)} \times \times \bar{\psi}_{m_6} e^{ik^{\perp} X} G_{m_1 \ldots m_5 m_6}, \quad (20)$$

where the G-tensor is symmetric and traceless in $m_1, ..., m_5$ (round brackets imply the symmetrization in space-time indices) and $k^{\perp}$ is transversely to $m_1, ..., m_6$ directions and

$$U_5 \sim \int d^2 z e^{-4\phi-\bar{\phi}} \partial X_{(m_1} \ldots \partial X_{m_4)} \times \times \psi_{m_5} \psi_{m_6} \bar{\psi}_{m_7} e^{ik^{\perp} X} G_{m_1 \ldots m_7}. \quad (21)$$

We have found that, even though the OPE details are quite different in each case, nevertheless in the end one always gets the beta-function equations in the form (19). The crucial point is that the σ factor, reflecting the stochasticity of the beta-function, appears to be universal and its value is independent on details of the ghost-matter mixing. Namely, we have found [8] $\sigma = 1.541...$ for the W_5 insertion and $\sigma = 1.538...$ for the of U_5 case. What is even more remarkable, can easily check that in fact

$$\sigma = \ln \delta, \quad (22)$$

where $\delta = 4.669...$ is the famous Feigenbaum universality constant describing the universal scaling of the iteration parameter in a huge variety of dynamical systems under bifurcations and transitions from order to chaos [7].

To understand the physical meaning behind the appearance of the Feigenbaum constant in (19), it is necessary to analyze the non-Markovian Fokker-Planck (FP) equation describing the stochastic process which can be straightforwardly derived from the Langevin equation (19). We shall present here FP equation for scaling functions $\lambda(q) = \lambda_0 / q^4$

$$\frac{\partial P_{FP}(\varphi, \tau)}{\partial \tau} = = -\int d^4 p \int d^4 q \frac{\delta}{\delta \varphi(p, \tau)} \left(\frac{\delta S_{\varphi}}{\delta \varphi(q, \tau)} P_{FP}(\varphi, \tau)\right) + + \sigma^2 \lambda_0^6 \int d^4 k_1 \int d^4 k_2 \int \frac{d^4 p}{p^4} \int \frac{d^4 q}{q^4} \int d\xi \times \times \alpha_{-3|1} C_{[-3|-3]}\left(\frac{k_1 + p}{2}\right) \alpha_{-3|1} C_{[-3|-3]}\left(\frac{k_2 + q}{2}\right) \times \times \frac{\delta}{\delta \varphi(p, \tau)} G_5(\xi, \tau) \frac{\delta}{\delta \varphi(q, \xi)} P_{FP}(\varphi, \tau), \quad (23)$$

where $\tau = \log \Lambda$ now plays the role of the stochastic time variable. The Green's function $G_5(\xi, \tau, p, q)$ is defined by the cutoff dependence of the two-point correlator of the V_5-vertices:

$$G_5(\xi, \tau) = \int_{\Lambda_1} d^2 z \int_{\Lambda_2} d^2 w |z - w|^{-4} \delta(p + q) = = \left(\frac{1 + e^{\xi - \tau}}{1 - e^{\xi - \tau}}\right)^2 \delta(p + q), \quad \xi = \log \Lambda_1, \quad \tau = \log \Lambda_2. \quad (24)$$

We shall look for the anzats solving this equation in the form (for more details see [3] and references therein):

$$P_{FP}(\varphi, \tau) = \exp[-H_{ADM}(\varphi, \tau)] = = \exp\left[-\int d^4 p \{g(\tau)(\partial_{\tau} \varphi)^2 + f(\tau) p^2 \varphi^2\}\right]. \quad (25)$$

Substituting it into (23) we find that (25) solves the FP equation provided that the functions $f(\tau)$ and $g(\tau)$ satisfy the following differential equations:

$$g'(\tau) + 4g(\tau) + \frac{\sigma^2 \lambda_0^6}{2} = 0,$$
$$\frac{1}{4} f'' + \left(1 + \frac{1}{4\tau}\right) f' + \left(1 + \frac{1}{4\tau} + \right. + \left. \frac{1}{4\sigma^2 \lambda_0^6}\left(1 - \frac{1}{\tau^2}\right)\right) f - \left(1 - \tfrac{1}{\tau^2}\right)\left(e^{-2\tau} + \frac{1}{4\sigma^2 \lambda_0^6}\right) = 1. \quad (26)$$

The first equation is elementary, its solution is given by

$$g(\tau) = \frac{\sigma^2 \lambda_0^6}{2}\left(e^{-4\tau} - 1\right), \quad \tau < 0. \quad (27)$$

The second equation on $f(\tau)$ can be reduced to the Bessel type equation by substituting

$$f(\tau) = \rho(\tau) e^{-2\tau} + 1/\sigma^2 \lambda_0^6.$$

The solution is given by

$$f(\tau) = 1 + \sigma^2 \lambda_0^6 e^{-2\tau}\left(1 + J_{1/\sigma\lambda_0^3}(\tau/\sigma\lambda_0^3)\right) \quad (28)$$

where $J_1/\sigma\lambda_0^3(\tau/\sigma\lambda_0^3)$ is the Bessel's function. In terms of the τ coordinate, the stochastic process, describing the RG flow in ghost-matter mixing backgrounds, evolves in the direction of $\tau = -\infty$. Next, let us study the behaviour of the Hamiltonian (25), (27), (28) in the conformal limit of $\tau \to -\infty$. In this limit the exponents become very large and moreover

$$J_1/\sigma\lambda_0^3\left(\frac{\tau}{\sigma\lambda_0^3}\right) \sim O\left(\frac{1}{\sqrt{\tau}}\right) << 1 \quad (29)$$

and after rescaling the Hamiltonian reduces to

$$H(\varphi, \tau) = R^2 \int d^4 p \{e^{-4\tau}(\partial_{\tau}\varphi)^2 + p^2 e^{-2\tau} \varphi^2\} \quad (30)$$

which is just the ADM Hamiltonian for the AdS_5 gravity in the temporal gauge [9] it is easy to see that the λ_0^6 parameter has the meaning of the square of the radius R^2 of the metric.

Let us now analyze in more details the solution (25), (27), (28) of the non-Markovian FP equation, leading to the new space geometry. Let us note first of all that the limit $\lambda_0 \to 0$ is not the same as $\lambda_0 = 0$ (ghost-matter mixing absent). The RG flow described by the effective metric (27), (28) must be single-valued; since Bessel's functions at zero argument are single-valued for the integer orders only, this leads to the quantization condition:

$$(\sigma\lambda_0^3)^{-1} = N. \qquad (31)$$

Moreover, since $J_\nu(\tau) \sim \tau^\nu$ as $\tau \to 0$, the absence of unphysical singularities at $\tau = 0$ requires N to be positive. The quantization condition (31) implies that

$$((\lambda_0)_N)^{-3} = N\sigma, \quad e^{(\lambda_0)_N^{-3}} = \delta^N \qquad (32)$$

implying the iteration law:

$$\frac{e^{(\lambda_0)_{N+1}^{-3}} - e^{(\lambda_0)_N^{-3}}}{e^{(\lambda_0)_N^{-3}} - e^{(\lambda_0)_{N-1}^{-3}}} = \delta \qquad (33)$$

with δ being the Feigenbaum number.

Therefore the Feigenbaum iteration rule (33) determines the scaling of characteristic curvatures of geometries emerging at the fixed points of the stochastic renormalization group. The role of iteration parameter characterizing the bifurcations is played by $\sim e^{-1/R^2}$, vanishing at $R = 0$ and being finite at large R, as it should be the case for the scaling parameter of the Feigenbaum iteration scheme.

From the quantization condition (31) it is clear that the stochastic renormalization group (19) has fixed points exists for $0 < \lambda_0 < 1$, i.e. correspond to *large* curvatures. Moreover the period doublings that lead to the transition to chaos corresponds to $N \to \infty$, i.e. $\lambda_0 \to 0$, which is a singularity. So we reached an amazing conclusion that precisely near singularity our RG flow becomes chaotic. It is tempting to assume that this may be the mechanism which can solve the problem of singularities in string theory.

In this Letter we discussed how matter-ghost mixing can radically modify the nature of the world sheet RG flows and lead to the emergence of chaos near curvature singularities. Here we analyzed only dilaton evolution, but the similar picture can be obtained for other massless fields, for example metric [8].

Amusingly, recently the chaotic behaviour of metric was discussed in [10] (for earlier papers see [11] and references therein) where the emergence of chaos in supergravity near cosmological singularity was demonstrated in the presence of higher rank antisymmetric tensor fields, i.e. $R{-}R$ fields. It will be extremely interesting to understand how chaos emerging during cosmological evolution in supergravity can be related to the chaotic nature of RG flows in underlying string theory in the presence of the sources of the background $R{-}R$ fields.

It is tempting to assume that the resolution of the singularities problem is transition to chaos and emergence of smooth distributions of fields, not restricted on-shell. One can imagine that curvature R is some new "Reynolds" number in string theory and for large R one have transition to chaotic behaviour in a similar fashion like in hydrodynamics there is a transition from a laminar to a turbulent flow. These ideas definitely need further investigation.

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