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Screening and inplane magnetoresistance of anisotropic two-dimensional gas

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In order to split the influence of the orbital and spin effects on the inplane magnetoresistance of a quasi two-dimensional (2D) gas we derive its linear response function and dielectric function for the case of anisotropic effective mass. This result is used for the calculation of elastic transport relaxation time of a quasi two dimensional system in a parallel magnetic field. The relaxation time is proved to be isotropic in the low density limit for the case of charged impurity scattering, allowing to separate the two contributions.

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Among a variety of experimentally used two dimensional semiconductor structures some possess the anisotropy of Fermi surface originating from that of a bulk material, including n-type Si-MOSFET's on other than (100) surfaces [1], AlAs heterostructures [2], p-type Si and GaAs structures [3]. The transport properties of such anisotropic semiconductors has been addressed largely [4–6] and it is well known, that the Fermi surface anisotropy gives rise in general to the anisotropy of relaxation time, even if the scattering potential is isotropic [5].

Another possibility is the externally introduced anisotropy through the application of parallel to a quasi 2D layer magnetic field, known to deform a Fermi surface [7] due to the so called orbital effect [8]. In this case, however, there is additionally a coupling of parallel field to the particles' spins leading to the partial spin polarization of the system [9, 10] and subsequent change of screening [11] (so called spin effect). Although substantial interest [2, 9, –15] has been recently paid to the longitudinal magnetoresistance (MR) studies of quasi two dimensional systems, it remained unclear so far how the two effects play together [12].

In this letter we address the screening properties of the anisotropic 2D gas deriving the linear response function for the case of elliptic Fermi surface. This result is then used to evaluate the transport relaxation time τ for elastic charged impurities scattering, which, surprisingly, turns out to be isotropic in the low density limit. Furthermore, for partially spin polarized anisotropic 2D system the relaxation time for each spin subband is also shown to be isotropic, allowing finally to reach the main result of the paper – to separate the influence of the orbital and spin effects on the longitudinal magnetoresistance of a diluted quasi 2D gas.

In the following we utilize the simplest possible representation of anisotropy – the elliptic Fermi surface in the effective mass approximation. The interactions are treated in the random phase approximation (RPA), so that the screened linear response function equals to linear response of a free particles gas [16]:

$$\chi^{scr.}(\mathbf{q},0) = g_v g_s \sum_{\mathbf{k}} \left|(\rho_q)_{n0}\right|^2 \frac{f_0(E_{\mathbf{k}}) - f_0(E_{\mathbf{k}+\mathbf{q}})}{E_{\mathbf{k}} - E_{\mathbf{k}+\mathbf{q}} - i0}, \tag{1}$$

where $E_{\mathbf{k}}$ is kinetic energy of a quasiparticle with momentum $\mathbf{k}$, $f_0(E_{\mathbf{k}})$ – the zero temperature Fermi-Dirac distribution function, g_v, g_s – the valley and spin degeneracies. The excited state $|n\rangle$ contains a single pair of a quasiparticle with momentum $\mathbf{k}+\mathbf{q}$ and a quasihole with momentum $\mathbf{k}$, and the matrix element of a density fluctuation operator $(\rho_q)_{n0}$ is equal to unity. The last property originates entirely from the Bloch type of Hamiltonian eigenfunctions in the effective mass approximation, similar to the isotropic case [16].

We perform the following change of coordinates to rewrite the integral (1) in a spherically symmetric form:

$$\begin{aligned} k_x &\to k_x^F \tilde{k}_x, \qquad q_x \to k_x^F \tilde{q}_x, \\ k_y &\to k_y^F \tilde{k}_y, \qquad q_y \to k_y^F \tilde{q}_y. \end{aligned} \tag{2}$$

The kinetic energy depends solely upon the length of the distorted wave vector $\tilde{\mathbf{k}}$:

$$E_{\mathbf{k}} = E_F((k_x/k_x^F)^2 + (k_y/k_y^F)^2) = E_F|\tilde{\mathbf{k}}|^2$$

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and for integral (1) we have:

$$\chi^{scr\cdot}(\mathbf{q},0) = g_v g_s \frac{k_x^F k_y^F}{4\pi^2 E_F} \int \frac{f_0(E_{\tilde{\mathbf{k}}}) - f_0(E_{\tilde{\mathbf{k}}+\tilde{\mathbf{q}}})}{-2\tilde{\mathbf{k}}\cdot\tilde{\mathbf{q}} - \tilde{\mathbf{q}}^2 - i0} d^2\tilde{\mathbf{k}}. \tag{3}$$

In view of spherical symmetry the integral value in (3) is invariant to rotation of vector $\tilde{\mathbf{q}}$, hence the linear response function depends upon its length $\tilde{q}$ only. The integral in (3) would be the same for isotropic Fermi surface as well, thus the only difference from Stern's linear response function [17] is due to the normalizing prefactor proportional to the density of states of anisotropic Fermi gas $D = g_v g_s \sqrt{m_x m_y}/2\pi\hbar^2$. Finally, we get for χ:

$$\chi^{scr\cdot}(\mathbf{q},0) = -D\cdot\phi(\tilde{q}),$$
$$\phi(\tilde{q}) = \begin{cases} 1, & \tilde{q} < 2 \\ 1 - (1 - 4/\tilde{q}^2)^{1/2}, & \tilde{q} \geq 2 \end{cases}. \tag{4}$$

The linear response function (4) of anisotropic system depends upon the direction of perturbation wave vector $\mathbf{q}$ through the variable $\tilde{q} = ((q_x/k_x^F)^2 + (q_y/k_y^F)^2)^{1/2}$, so that the screening does become anisotropic, in contrast to the case of isotropic Fermi surface [17]. Note, however, that this anisotropy is the same as that of kinetic energy as a function of momentum, since $\tilde{q}^2 = E_{\mathbf{q}}/E_F$.

At the same time we are able to find the RPA static dielectric function [16]:

$$\varepsilon(\mathbf{q},0) = 1 - V(q)\cdot\chi^{scr\cdot}(\mathbf{q},0) = 1 + \frac{q_{TF}}{q}\phi(\tilde{q}) \tag{5}$$

where $V(q) = 2\pi e^2/q$ is the 2D Fourier transform of the bare Coulomb interaction potential, $q_{TF} = 2\pi e^2 D$ – the Thomas-Fermi screening parameter [1].

We now turn to the application of our results to the transport properties of anisotropic 2D Fermi gases. Expression (5) for dielectric function enables one to find the elastic transport relaxation time. In general $\tau(\mathbf{k})$ is anisotropic and calculation of this could be a rather complicated procedure [6, 18]. As we show below, for the case of screened charged impurity scattering, τ is still isotropic in the low density regime and can be obtained analytically. One has for elastic scattering transport relaxation time:

$$\frac{1}{\tau(\mathbf{k})} \propto$$
$$\propto \int \left(\frac{V(q)}{\varepsilon(\mathbf{q},0)}\right)^2 \left(1 - \frac{\mathbf{v_k}\cdot\mathbf{v_{k'}}}{\mathrm{v}_{\mathbf{k}}^2}\right) \delta(E_{\mathbf{k}'} - E_F)\, d^2\mathbf{k}', \tag{6}$$

where $\mathbf{v_k} = \hbar^{-1} dE_{\mathbf{k}}/d\mathbf{k}$ is the particle's group velocity. Note that the relaxation time isotropy is already implicit in this expression and is verified in what follows. First term in the integrand is the square of a scattering matrix element in the Born approximation [5], where we have neglected all the form-factors of the real Coulomb interaction between a quasi 2D electron (hole) and a charged impurity [1]. Applying the change of coordinates (2) to the $\mathbf{k}'$-space we find from (5): $V(q)/\varepsilon(\mathbf{q},0) = 2\pi e^2 (q + q_{TF}\phi(\tilde{q}))^{-1}$ which in the low density limit ($k_x^F, k_y^F \ll q_{TF}$) reduces to[2]:

$$V(q)/\varepsilon(\mathbf{q},0) = 2\pi e^2 q_{TF}^{-1}\phi^{-1}(\tilde{q}) \tag{7}$$

The second term in (6) accounts for a loss of initial velocity in a scattering event and is similar to the factor $1 - \cos(\widehat{\mathbf{k},\mathbf{k}'})$ in the isotropic case [18]. Changing the coordinates to polar ones $\tilde{k}_x, \tilde{k}_y \to \tilde{k}, \theta$ we rewrite this in terms of a scattering angle θ: $1 - \mathbf{v_k}\cdot\mathbf{v_{k'}}/\mathrm{v}_{\mathbf{k}}^2 = 1 - \cos\theta + A(\mathbf{k})\sin\theta$, where number $A(\mathbf{k})$ depends upon the initial particle momentum $\mathbf{k}$. Hence in the low density limit we find:

$$\frac{1}{\tau(\mathbf{k})} \propto \frac{1}{g_v g_s D} \int_{-\pi}^{\pi} \phi^{-2}(\cos\theta)(1 - \cos\theta)\, d\theta, \tag{8}$$

since $\tilde{q} = (2 - 2\cos\theta)^{1/2}$, and the odd in θ part of the integrand gives no contribution to the integral. This final expression for the elastic scattering time in the low density limit is essentially the same as in the isotropic case, the only difference represented by the reciprocal density of states D^{-1} in the prefactor. This finding has two important consequences: in the low density limit, within the elliptic deformation of the Fermi surface, the transport relaxation time (i) remains isotropic and (ii) increases proportionally to the density of states D[3].

Comparing to previous studies we find that our result for the relaxation time isotropy recovers the one derived earlier for the short range scatterers [6], since for the case of zero spin polarization considered so far the relevant Fourier components of the screened impurity potential (7) do not depend on wave vector at all, according to (4). The predicted increase of the relaxation time with the Fermi surface deformation implies

[2] The low density limit determined by inequality $k_x^F, k_y^F \ll q_{TF}$ is essentially the limit of strong interactions $r_S \gg 1$, where r_S is a Wigner-Seitz radius [1]. Strictly speaking, the utilized random phase approximation is expected to fail in this limit and the local field corrections should be taken into account [16]. The simplified treatment we use is easily solved and seems to catch a major effect of Fermi surface anisotropy.

[3] One can easily see that these conclusions are also valid for the elastic scattering life time, which corresponds to omitting the second term in the integrand of (6).

of course the increase of conductivity, i.e. the negative magnetoresistance in the parallel to field direction caused by the orbital effect, as the effective mass in this direction remains unchanged [7]. This is in contrast to positive MR found for orbital effect in Ref. [8], where the change of screening has been neglected. Our prediction could be easily verified in the experiments on wide quantum wells, where the inplane magnetoresistance is mostly due to the orbital effect.

As was mentioned earlier, apart from the Fermi surface deformation the parallel magnetic field couples to particles' spins, resulting in a partial spin polarization of a system. Similar to the isotropic case [11] there is no more a single Fermi surface, but different ones for different spin projections onto magnetic field axis. We would like to treat only the spin conserving processes, which means that the linear response of such partially polarized system is simply a sum of the responses from different spin subbands [11]. Similar to the case of unpolarized system (expression (4)), the anisotropy of the linear response of partially polarized system is again the same as that of the kinetic energy as a function of wave vector $\mathbf{q}$, which leads to the isotropy of the relaxation time for each spin subband in the limit of low density, as we show below.

Calculating the transport relaxation times $\tau^{\uparrow,\downarrow}$ one should write the integrals of type (6) for major ($\uparrow$) and minor ($\downarrow$) spin subbands separately. In the low density limit, when the Fermi wave vectors of both subbands satisfy $k_x^{F\uparrow,\downarrow}, k_y^{F\uparrow,\downarrow} \ll q_{TF}$, the Fourier image of the screened impurity potential $V(\mathbf{q})/\varepsilon(\mathbf{q})$ has the anisotropy of the linear response function and kinetic energy, according to (7). This means that the change of coordinates (2) applied to the integrands leads to the same expressions for $\tau^{\uparrow,\downarrow}$ as one gets in the isotropic case [11]. Thus the relaxation times are isotropic, the only effect of anisotropy been again to normalize the absolute value of $\tau^{\uparrow,\downarrow}$ through the density of states dependent prefactor (8). We are now able to write down the conductivity tensor of the partially polarized anisotropic system:

$$\hat{\sigma} = \hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow} = n_S e^2 \tau_0 \hat{m}^{-1} \frac{D}{D_0} F_{DG}(\xi), \qquad (9)$$

where $\hat{\sigma}^{\uparrow}, \hat{\sigma}^{\downarrow}$ are the major and minor spin subbands conductivity tensors, e, n_S, τ_0, D_0 – respectively, electron charge, density of 2D particles and the zero field isotropic relaxation time and the density of states. D and $\hat{m}$ are respectively the density of states and the effective mass tensor in magnetic field. The last term in (9) stands for the Dolgopolov-Gold's calculated change of the conductivity of isotropic system as a function of its degree of spin polarization $\xi = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$ [11]. Note that for the case of a quasi 2D system with zero field anisotropic mass this result is valid only for a parallel field applied along the main axes of symmetry, otherwise the Fermi surface loses its ellipticity in magnetic field. Let us show finally how this simple expression allows to separate immediately the contributions from spin and orbital effects on the longitudinal magnetoresistance of a diluted quasi 2D system.

We focus on the recent magnetoresistance studies of the 2D electron gas of AlGaAs/GaAs heterostructure [12]. This system is isotropic in zero magnetic field, hence, according to (9) the anisotropy of experimental MR [12] is due to the effective mass change in a perpendicular to magnetic field direction [8]:

$$m_\perp/m_0 = \rho_\perp(B)/\rho_\parallel(B),$$

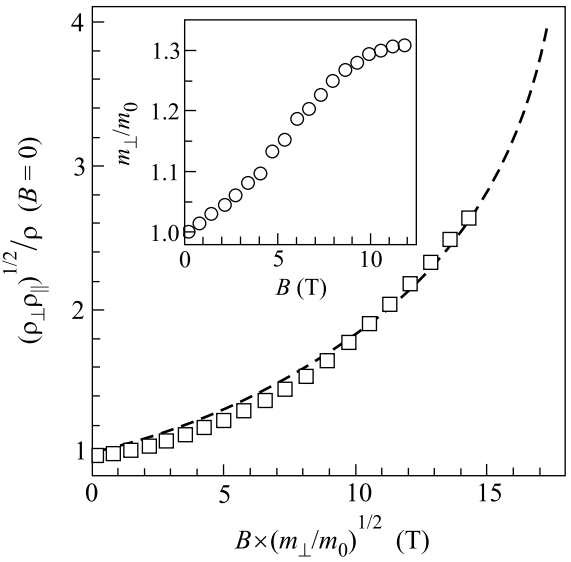
where $\perp$ and $\parallel$ mark the resistances measured in perpendicular and parallel to inplane magnetic field directions, respectively. The spin effect contribution in (9) is given thus by:

$$F_{DG}^{-1}(\xi) = \sqrt{\frac{\rho_\perp(B)\cdot\rho_\parallel(B)}{\rho_\perp(B=0)\cdot\rho_\parallel(B=0)}}. \qquad (10)$$

The degree of spin polarization depends upon both the Zeeman energy $g\mu_B B$ and the effective mass at a given field value: $\xi(B) = (E_{F\uparrow} - E_{F\downarrow})/(E_{F\uparrow} + E_{F\downarrow}) = g\mu_B B \cdot D/n_S$, where $E_{F\uparrow,\downarrow}$ are the kinetic parts of the Fermi energy for two spin subbands, g, μ_B – the Landé factor and Bohr magneton. Equivalently the last equation reads: $\xi(B) = B/B_P^0 \cdot \sqrt{m_\perp(B)/m_0}$, where $B_P^0 = 2E_F^0/g\mu_B$ is the full spin polarization field in the absence of orbital effect [11].

In the inset to Figure we show the effective mass growth extracted in the above manner from the $\rho_\perp, \rho_\parallel$ data of Ref. [12]. The effective mass in perpendicular to field direction grows by about 30% in moderate fields, as caused by the orbital effect [8]. For such a slight deformation the utilized approximation of elliptic Fermi surface should work reasonably, in contrast to general case [7]. The partial spin polarization has major effect on MR, as reflected by its small anisotropy [12], and leads to a roughly threefold resistance increase as is shown in the body of Figure. The full spin polarization has not been reached in [12] that's why the saturation [11] of the geometrical mean of the parallel and perpendicular to inplane field resistivities predicted by (10) is not seen in Figure. Fitting to calculation [11] we obtain the B_P^0 value 17.3 T which corresponds to a Landé factor $g^* \approx 2.1$ at $n_S \approx 3 \times 10^{11}\mathrm{cm}^{-2}$, in agreement with previous studies [10, 12].

We would like to add a note here, concerning the applicability of the MR data analysis presented above to



In the inset the effective mass growth caused by the orbital effect [8] is shown, extracted from the longitudinal MR data of Ref. [12] ($n_S \approx 3\cdot 10^{10}\,\mathrm{cm}^{-2}$) as described in the text. The spin effect contribution is shown in the figure body. The fit to theoretical dependence [11] (dashed line) gives full spin polarization field in the absence of orbital effect $B_P^0 \approx 17.3\,\mathrm{T}$ corresponding to a Landé factor $g^* \approx 2.1$

the real interacting quasi 2D systems. Apart from the so far considered single particle effect of Fermi surface deformation, the squeezing of the 2D layer by the parallel magnetic field additionally changes the form-factors of Coulomb interaction between particles [1] and increases the Wigner-Seitz ratio [15], which can in principle lead to the renormalization of the zero field effective mass and g-factor [19]. In presence of such many-body effects the spin effect contribution [11] cannot be extracted with formula (10). Experimentally this means that the geometrical mean of resistivities $\sqrt{\rho_\perp(B)\cdot\rho_\parallel(B)}$ does not saturate upon the reach of full spin polarization. The inplane magnetoresistance anisotropy, however, should still give the anisotropy of effective mass $\rho_\perp/\rho_\parallel = m_\perp/m_\parallel$, similar to the single-particle picture, although the independent measurement is required to find the full spin polarization field B_P^0 [14, 15].

In conclusion, we have derived the linear response function and dielectric function of 2D Fermi gas with anisotropic effective mass. In the low density limit the screened charged impurity potential is shown to possess the same symmetry as the kinetic energy as a function of wave vector. As a result the elastic transport relaxation time τ is isotropic in this limit, even if the 2D system is partially spin polarized. This finding allows us to separate for the first time the influence of the orbital [8] and spin [11] effects on the inplane magnetoresistance of a diluted quasi two dimensional system.

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