Pis'ma v ZhETF, vol. 79, iss. 6, pp. 344 – 350 © 2004 March 25

Universal behavior of heavy-fermion metals near a quantum critical point

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Submitted 24 November 2003
Resubmitted 9 February 2004

The behavior of the electronic system of heavy fermion metals is considered. We show that there exist at least two main types of the behavior when the system is nearby a quantum critical point which can be identified as the fermion condensation quantum phase transition (FCQPT). We show that the first type is represented by the behavior of a highly correlated Fermi-liquid, while the second type is depicted by the behavior of a strongly correlated Fermi-liquid. If the system approaches FCQPT from the disordered phase, it can be viewed as a highly correlated Fermi-liquid which at low temperatures exhibits the behavior of Landau Fermi liquid (LFL). At higher temperatures T, it demonstrates the non-Fermi liquid (NFL) behavior which can be converted into the LFL behavior by the application of magnetic fields B. If the system has undergone FCQPT, it can be considered as a strongly correlated Fermi-liquid which demonstrates the NFL behavior even at low temperatures. It can be turned into LFL by applying magnetic fields B. We show that the effective mass M^* diverges at the very point that the Neél temperature goes to zero. The $B-T$ phase diagrams of both liquids are studied. We demonstrate that these $B-T$ phase diagrams have a strong impact on the main properties of heavy-fermion metals such as the magnetoresistance, resistivity, specific heat, magnetization, volume thermal expansion, etc.

PACS: 71.10.Hf, 71.27.+a, 75.30.Cr

In heavy-fermion (HF) metals with strong electron correlations, quantum phase transitions at zero temperature may strongly influence the measurable quantities up to relatively high temperatures. These quantum phase transitions have recently attracted much attention because the behavior of HF metals is expected to follow universal patterns defined by the quantum mechanical nature of the fluctuations taking place at quantum critical points (see e.g. [1, 2]). It is widely believed that the proximity of the electronic system of HF metal to quantum critical points may lead to non-Fermi liquid (NFL) behavior. The system can be driven to quantum critical points (QCPs) by tuning control parameters other then temperature, for example, by pressure, by magnetic field, or by doping. When a system is close to QCP we are dealing with the strong coupling limit where no absolutely reliable answer can be given on pure theoretical first principle grounds. Therefore, the only way to verify what type of quantum phase transition occurs is to consider experimental facts which describe the behavior of the system. Only recently, there appeared experimental facts which deliver experimental grounds to understand the nature of quantum phase transition producing the universal behavior of HF metals.

It is the very nature of HF metals that suggests that their unusual properties are defined by a quantum phase transition related to the unlimited growth of the effective mass at its QCP. Moreover, a divergence to infinity of the effective electron mass was observed at a magnetic field-induced QCP [3, 4, 5]. We assume that such a quantum phase transition to be the fermion condensation quantum phase transition (FCQPT), an essential feature of which is the divergence of the effective mass M^* at its QCP [6, 7]. FCQPT takes place when the density x of system tends to the critical density x_{FC} so that $M^* \propto 1/r$ where r is a distance from the QCP, $r = |x - x_{FC}|$. Such a behavior does not qualitatively depend on the system's dimensions and valid in both cases of two-dimensional (2D) and three-dimensional (3D) Fermi systems [8, 9]. Beyond FCQPT the system possesses fermion condensation (FC) and represents a new state of electron liquid with FC [7, 10]. As soon as FCQPT occurs, the system becomes divided into two quasiparticle subsystems: the first is characterized by quasiparticles with the effective mass M^*_{FC}, while the second one is occupied by quasiparticles with mass M^*_L. The quasiparticle dispersion law in systems with FC can

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be represented by two straight lines, characterized by the effective masses M^*_{FC} and M^*_L, and intersecting near the binding energy E_0. Properties of these new quasiparticles with M^*_{FC} are closely related to the state of the system which is characterized by the temperature T, pressure P, or by the presence of the superconductivity. We may say that the quasiparticle system in the range occupied by FC becomes very "soft" and is to be considered as a strongly correlated liquid. Nonetheless, the basis of the Landau Fermi liquid theory [11] survives FCQPT: the low energy excitations of the strongly correlated liquid with FC are quasiparticles, while this state can be considered as a quantum protectorate [6]. The only difference between the Landau Fermi-liquid and Fermi-liquid after FCQPT is that we have to expand the number of relevant low energy degrees of freedom by introducing a new type of quasiparticles with the effective mass M^*_{FC} and the energy scale E_0 [6]. It is possible to provide a consistent picture of high-T_c metals as the strongly correlated Fermi-liquid [12].

When a Fermi system approaches FCQPT from the disordered phase, its low energy excitations are Landau quasiparticles which can be characterized by the effective mass M^*. This mass strongly depends on the distance r, temperature and magnetic fields B [8]. At low temperatures, it becomes a Landau Fermi liquid with the effective mass M^* provided that $r > 0$. This state of the system, with M^* strongly depending on T, r and B, resembles the strongly correlated liquid. In contrast to the strongly correlated liquid, there is no energy scale E_0 and the system under consideration is the Landau Fermi liquid at $T \to 0$. Therefore, this liquid can be called a highly correlated liquid. Such a highly correlated Fermi-liquid was observed in non-superconducting $La_{1.7}Sr_{0.3}CuO_4$ [8, 13].

In this Letter, we continue to show that within the framework of FCQPT it is possible to understand the NFL behavior observed in different strongly and highly correlated Fermi liquids such as high-T_c superconductors [12] and heavy-fermion metals. We apply the theory of fermion condensation to describe the behavior of the electronic system of HF metals and to show that there exist at least two main types of the behavior. If the system approaches FCQPT from the disordered phase it can be viewed as the highly correlated electron liquid, and the effective mass M^* depends on temperature, $M^* \propto T^{-1/2}$. Such a dependence of M^* leads to the NFL behavior of the electronic system. The application of a magnetic field $(B - B_c) \geq B^*(T) \propto T^{3/4}$ restores Landau Fermi liquid (LFL) behavior. Here B_c is a critical magnetic field. At $(B - B_c) \geq B^*(T)$, the effective mass depends on the magnetic field, $M^*(B) \propto (B - B_c)^{-2/3}$, being approximately independent of the temperature at $T \leq T^*(B) \propto (B - B_c)^{4/3}$. At $T \geq T^*(B)$, the $T^{-1/2}$ dependence of the effective mass and the NFL behavior are re-established. At $T \to 0$, the system becomes LFL with the effective mass $M^* \propto 1/r$. When the system has undergone FCQPT, it becomes a strongly correlated electron liquid, and the effective mass behaves as $M^* \propto 1/T$ leading to the NFL behavior even at low temperatures. The application of a magnetic field $(B - B_c) \geq B^*(T) \propto T^2$ restores the LFL behavior. At $(B - B_c) \geq B^*(T)$, the effective mass $M^*(B) \propto (B - B_c)^{-1/2}$, being approximately independent of the temperature at $T \leq T^*(B) \propto \sqrt{B - B_c}$. At $T \geq T^*(B)$, both the $1/T$ dependence and the NFL behavior are re-established. We show that the effective mass M^* diverges at the very point that the Neél temperature goes to zero. It is demonstrated that obtained $B - T$ phase diagrams have a strong impact on the main properties of HF metals such as the magnetoresistance, resistivity, specific heat, magnetization, volume thermal expansion, etc.

We start with the case of a highly correlated electron liquid when the system approaches FCQPT from the disordered phase. FCQPT manifests itself in the divergence of the quasiparticle effective mass M^* as the density x tends to the critical density x_{FC}, or the distance $r \to 0$ [8, 9]

$$M^* \propto \frac{1}{|x - x_{FC}|} \propto \frac{1}{r}. \tag{1}$$

Since the effective mass M^* is finite, the system exhibits the LFL behavior at low temperatures $T \sim T^*(x) \propto |x - x_{FC}|^2$ [8]. The quasiparticle distribution function $n(\mathbf{p}, T)$ is given by the equation

$$\frac{\delta\Omega}{\delta n(\mathbf{p}, T)} = \varepsilon(\mathbf{p}, T) - \mu(T) - T \ln \frac{1 - n(\mathbf{p}, T)}{n(\mathbf{p}, T)} = 0. \tag{2}$$

The function $n(\mathbf{p}, T)$ depends on the momentum $\mathbf{p}$ and the temperature T. Here $\Omega = E - TS - \mu N$ is the thermodynamic potential, and μ is the chemical potential, while $\varepsilon(\mathbf{p}, T)$,

$$\varepsilon(\mathbf{p}, T) = \frac{\delta E[n(p)]}{\delta n(\mathbf{p}, T)}, \tag{3}$$

is the quasiparticle energy. This energy is a functional of $n(\mathbf{p}, T)$ just like the total energy $E[n(p)]$, entropy $S[n(p)]$ and the other thermodynamic functions. The entropy $S[n(p)]$ is given by the familiar expression

$$S[n(p)] = -2 \int \big[n(\mathbf{p}, T) \ln n(\mathbf{p}, T) + (1 - n(\mathbf{p}, T)) \ln(1 - n(\mathbf{p}, T))\big] \frac{d\mathbf{p}}{(2\pi)^3}, \tag{4}$$

which results from purely combinatorial considerations. Eq. (2) is usually presented as the Fermi-Dirac distribution

$$n(\mathbf{p},T)=\left\{1+\exp\left[\frac{(\varepsilon(\mathbf{p},T)-\mu)}{T}\right]\right\}^{-1}. \quad (5)$$

At $T\to 0$, one gets from Eqs. (2), (5) the standard solution $n_F(\mathbf{p},T\to 0)\to\theta(p_F-p)$, with $\varepsilon(p\simeq p_F)-\mu=$ $=p_F(p-p_F)/M^*$, where p_F is the Fermi momentum, $\theta(p_F-p)$ is the step function, and M^* is the Landau effective mass [11]

$$\frac{1}{M^*}=\frac{1}{p}\frac{d\varepsilon(p,T\to 0)}{dp}|_{p=p_F}. \quad (6)$$

It is implied that in the case of LFL M^* is positive and finite at the Fermi momentum p_F. As a result, the T-dependent corrections to M^*, to the quasiparticle energy $\varepsilon(p)$, and other quantities, start with T^2-terms being approximately temperature independent. The Landau equation relating the mass M of an electron to the effective mass of the quasiparticles is of the form [11]

$$\frac{\mathbf{p}}{M^*}=\frac{\mathbf{p}}{M}+\int F_L(\mathbf{p},\mathbf{p}_1,x)\nabla_{\mathbf{p}_1}n(\mathbf{p}_1)\frac{d\mathbf{p}_1}{(2\pi)^3}. \quad (7)$$

Applying Eq. (7) at $T<T^*(x)$, we obtain the common result

$$M^*=\frac{M}{1-N_0F_L^1(x)/3}. \quad (8)$$

Here N_0 is the density of states of the free Fermi gas and $F_L^1(x)$ is the p-wave component of the Landau interaction. At $x\to x_{FC}$, the denominator in Eq. (8) tends to zero and one obtains Eq. (1). The temperature smoothing out the step function $\theta(p_F-p)$ at $p_F=(x/3\pi^2)^{1/3}$ induces the variation of the Fermi momentum $\Delta p_F\sim TM^*/p_F$. We assume that the amplitude F_L has a short range $q_0\ll p_F$ of interaction in the momentum space. It is a common condition leading to the existence of FC and nearly-localized Fermi liquids [7, 14]. If the radius is such that $q_0\sim\Delta p_F\sim T_0M^*/p_F$, corrections to the effective mass are proportional to T at $T\sim T_0$. Here $T_0\propto|x-x_{FC}|$ is a characteristic temperature at which the system's behavior is of the NFL type. On the other hand, at $T^*(x)\ll T_0$, we have $q_0\gg T^*(x)M^*/p_F$, and the system behaves as LFL at $T\sim T^*(x)$, so that the corrections to the effective mass start with T^2 terms. We can also conclude that the transition region is rather large compared with $T^*(x)$ being proportional to T_0.

In the case of $T\sim T_0$, we again can use Eq. (8) with $F_L^1(p_F+\Delta p_F)\sim F_L^1(p_F)+A\Delta p_F$, where $A\propto$ $\propto dF_L^1(x)/dx$. Substituting this expansion of $F_L^1(p_F+$ $+\Delta p_F)$ into Eq. (8) we find that

$$M^*\sim\frac{M}{A\Delta p_F}\propto\frac{M}{M^*T}. \quad (9)$$

In deriving Eq. (9) we assumed that the system is close to FCQPT so that $(1-N_0F_L^1(p_F))\ll N_0A\Delta p_F$. We can say that at $T\sim T_0$, Δp_F induced by T becomes larger then the distance r from FCQPT, $\Delta p_F>|p_F^{FC}-p_F|$, where p_F^{FC} corresponds to x_{FC}. Solving Eq. (9) with respect to M^*, we obtain [8]

$$M^*(T)\propto\frac{1}{\sqrt{T}}. \quad (10)$$

The behavior of the effective mass given by Eq. (10) can be verified by measuring the thermal expansion coefficient which is given by [15]

$$\alpha(T)=\frac{1}{3}\left(\frac{\partial(\log V)}{\partial T}\right)_P=-\frac{x}{3K}\left(\frac{\partial(S/x)}{\partial x}\right)_T. \quad (11)$$

Here, P is the pressure and V is the volume. Substituting Eq. (4) into Eq. (11), one obtains that in the LFL theory coefficient is of the order $\alpha(T)\sim M_L^*T/p_F^2K$. By employing Eq. (10), one obtains that at $T\sim T_0$ [16]

$$\alpha(T)\propto a\sqrt{T}+bT, \quad (12)$$

with a and b being constants. This result is in good agreement with experimental facts obtained in measurements on $CeNi_2Ge_2$ [17].

The application of magnetic fields B leads to Zeeman splitting of the Fermi level. As a result, two quasiparticle distribution functions with Fermi momenta p_F^1 and p_F^2 appear, so that $p_F^1<p_F<p_F^2$ and $\Delta p_F=(p_F^2-p_F^1)\sim$ $\sim\mu_0BM^*/p_F$. Here μ_0 is the electron magnetic moment. In the same way Eq. (10) was derived, we can obtain the equation determining $M^*(B)$ [8]. The only difference is that there are no contributions coming from the terms proportional to Δp_F, and we have to take into account terms proportional to $(\Delta p_F)^2$. Assuming that the system is close to the critical point, we obtain

$$M^*(B)\sim M\left(\frac{\varepsilon_F}{B\mu}\right)^{2/3}. \quad (13)$$

At $T\sim T^*(x)$, Eq. (13) is valid as long as $M^*(B)\le$ $\le M^*(x)$, otherwise we have to use Eq. (1). It follows from Eq. (13) that the application of magnetic fields reduces the effective mass. If there exists a magnetic order in the system which is suppressed by magnetic field $B=B_{c0}$, then the quantity $(B-B_{c0})$ plays the role

of zero field, and Eq. (13) has to be replaced by the equation,

$$M^*(B) \propto \left(\frac{1}{B - B_{c0}}\right)^{2/3}. \qquad (14)$$

At high magnetic fields, we expect Eq. (14) to be invalid because Δp_F becomes too large so that $\Delta p_F > q_0$. In that case, the effective mass still depends on the magnetic field but the proportionality given by Eq. (14) is not preserved, and the dependence on the magnetic field becomes weaker.

At elevated temperatures $T \sim T_0$, the effective mass starts to depend on both the temperature and the magnetic field. A cross over from the B-dependent effective mass $M^*(B)$ to the T-dependent effective mass $M^*(T)$ takes place at a transition temperature $T^*(B)$ as soon as $M^*(B) \simeq M^*(T)$. This requirement and Eqs. (10) and (14) give that

$$T^*(B) \propto (B - B_{c0})^{4/3}. \qquad (15)$$

At $T > T^*(x)$, Eq. (15) determines the line on the $B - T$ phase diagram which separates the region of the LFL behavior taking place at $T < T^*(B)$ from the NFL behavior occurring at $T > T^*(B)$. At $T < T^*(B)$, the system behaves like LFL with the effective mass $M^*(B)$, and corrections to the effective mass start with T^2 terms. In accordance with the LFL theory, the specific heat $c \simeq \gamma T$, with

$$\gamma(B) \propto M^*(B) \propto (B - B_{c0})^{-2/3}. \qquad (16)$$

The resistivity ρ behaves as $\rho = \rho_0 + A(B)T^2$, where the coefficient

$$A(B) \propto (M^*(B))^2 \propto (B - B_{c0})^{-4/3}. \qquad (17)$$

It follows from Eq. (16) and (17) that the Kadowaki-Woods ratio $K = A/\gamma^2$ [18] is conserved. All these results obtained from Eqs. (14)–(17) are in good agreement with experimental facts observed in measurements on the HF metal YbAgGe single crystal [19]. The critical behavior of the coefficient $A(B) \propto (B - B_{c0})^{\beta}$ at $B \to B_{c0}$ described by Eq. (14) with $\beta = -4/3$ is in accordance with experimental data obtained in measurements on $CeCoIn_5$ which displayed the critical behavior with $\beta = -1.37 \pm 0.1$ [4].

In the LFL theory, the magnetic susceptibility $\chi \propto \propto M^*/(1 - F_0^a)$. Note, that there is no ferromagnetic instability in Fermi systems related to the growth of the effective mass, and the relevant Landau amplitude $F_0^a > -1$ [14]. Therefore, at $T < T^*(B)$, the magnetic susceptibility turns out to be proportional to the effective mass

$$\chi(B) \propto M^*(B) \propto (B - B_{c0})^{-2/3}, \qquad (18)$$

while the static magnetization $M_B(B)$ is given by

$$M_B(B) \propto B\chi \propto (B - B_{c0})^{1/3}. \qquad (19)$$

At $T > T^*(B)$, as it follows from Eq. (10), Eq. (18) has to be rewritten as

$$\chi(T) \propto M^*(T) \propto \frac{1}{\sqrt{T}}. \qquad (20)$$

The behavior of $\chi(B)$ and $M_B(B)$ as a function of magnetic field B given by Eqs. (18) and (19) and the behavior of $\chi(T)$, see Eq. (20), are in accordance with facts observed in measurements on $CeRu_2Si_2$ with the critical field $B_{c0} \to 0$ [20].

Consider the system when $r \to 0$. Then its properties are determined by the magnetic fields B and the temperature T because there are no other parameters to describe the state of the system. At the transition temperatures $T \simeq T^*(B)$, the effective mass depends on both T and B, while at $T \ll T^*(B)$, the system is LFL with the effective mass being given by Eq. (14), and at $T \gg T^*(B)$, the mass is defined by Eq. (10). Instead of solving Eq. (8), it is possible to construct a simple interpolation formula to describe the behavior of the effective mass over all the region,

$$M^*(B,T) \propto F(B,T) = \frac{1}{c_1(B - B_{c0})^{2/3} + c_2 f(y)\sqrt{T}}. \qquad (21)$$

Here, $f(y)$ is a universal monotonic function of $y = \sqrt{T}/(B - B_{c0})^{2/3}$ such that $f(y \sim 1) = 1$, and $f(y \ll \ll 1) = 0$. It is seen from Eq. (21) that the behavior of the effective mass can be represented by a universal function F_M of only one variable y if the temperature is measured in the units of the transition temperature $T^*(B)$, see Eq. (15), and the effective mass is measured in the units of $M^*(B)$ given by Eq. (14), $F_M(y) = aF(B,T)/M^*(B)$ with a being a constant. This representation describes the scaling behavior of the effective mass. As seen from Eqs. (19) and (21), the scaling behavior of the magnetization can be represented in the same way, provided the magnetization is normalized by the saturated value at each field given by Eq. (19)

$$\frac{M_B(B,T)}{M_B(B)} \propto \frac{1}{1 + c_3 f(y) y}, \qquad (22)$$

where c_3 is a constant. It is seen from Eq. (22), that magnetization is a monotonic function of y. Upon using the definition of susceptibility, $\chi = \partial M_B/\partial B$, and differentiating both sides of Eq. (22) with respect to B, we arrive at the conclusion that the susceptibility also exhibits the scaling behavior and can be presented as a universal function of only one variable y, provided it is normalized by the saturated value at each field given by Eq. (18)

$$\frac{\chi(B,T)}{\chi(B)} \propto \frac{1}{1+c_3 f(y)y} + 2c_3 y \frac{f(y)+ydf(y)/dy}{(1+c_3 f(y)y)^2}. \quad (23)$$

It is of importance to note that the susceptibility is not a monotonic function of y because the derivative is the sum of two contributions. The second contribution on the right hand side of Eq. (23) makes the susceptibility have a maximum. The above behaviors of the magnetization and susceptibility are in accordance with the facts observed in measurements on $CeRu_2Si_2$ [20]. Note, that the magnetic properties of $CeRu_2Si_2$ do not show any indications of the magnetic ordering at the smallest temperatures and in the smallest applied magnetic fields [20], that is $B_{c0} \to 0$ in that case. As a result, we can conclude that the QCP is driven by the divergence of the effective mass rather then by magnetic fluctuations, and FCQPT is the main cause of the NFL behavior. We can also conclude that the Neél temperature is zero in this case, because the magnetic susceptibility diverges at $T \to 0$, as it is seen from Eq. (20). A more detailed analysis of this issue will be published elsewhere.

Consider the case when the system has undergone FCQPT. Then, there exist special solutions of Eq. (2) associated with the so-called fermion condensation [7]. Being continuous and satisfying the inequality $0 < n_0(\mathbf{p}) < 1$ within some region in p, such solutions $n_0(\mathbf{p})$ admit a finite limit for the logarithm in Eq. (2) at $T \to 0$ yielding [7]

$$\varepsilon(\mathbf{p}) - \mu = 0, \quad \text{if} \quad 0 < n_0(\mathbf{p}) < 1;\ p_i \le p \le p_f, \quad (24)$$

where $\varepsilon(\mathbf{p})$ is given by Eq. (3). At $T = 0$, Eq. (24) defines a new state of electron liquid with FC [7, 10], which is characterized by a flat spectrum in the $(p_f - p_i)$ region, and which can strongly influence measurable quantities up to temperatures $T \ll T_f$. In this state, the order parameter of the superconducting state $\kappa(\mathbf{p}) = \sqrt{(1-n_0(\mathbf{p}))n_0(\mathbf{p})}$ has finite values in the $(p_f - p_i)$ region, whereas the superconducting gap $\Delta_1 \to 0$ in this region, provided that the pairing interaction tends to zero. Such a state can be considered as superconducting, with an infinitely small value of Δ_1, so that the entropy $S(T=0)$ of this state is equal to zero [6, 7].

When $p_f \to p_i \to p_F$ the flat part vanishes, and Eq. (24) determines QCP at which the effective mass M^* diverges and FCQPT takes place. When the density approaches QCP from the disordered phase, Eq. (24) possesses non-trivial solutions at $x = x_{FC}$ as soon as the effective inter-electron interaction as a function of the density, or the Landau amplitude, becomes sufficiently strong to determine the occupation numbers $n(\mathbf{p})$ which delivers the minimum value to the energy $E[n(p)]$, while the kinetic energy can be considered as frustrated [6]. As a result, the occupation numbers $n(\mathbf{p})$ become variational parameters and Eq. (24) has non-trivial solutions $n_0(\mathbf{p})$, because the energy $E[n(p)]$ can be lowered by alteration of the occupation numbers. Thus, within the region $p_i < p < p_f$, the solution $n_0(\mathbf{p})$ deviates from the Fermi step function $n_F(\mathbf{p})$ in such a way that the energy $\varepsilon(\mathbf{p})$ stays constant, while outside this region $n(\mathbf{p})$ coincides with $n_F(\mathbf{p})$ [7]. Note, that a formation of the flat part of the spectrum has been confirmed in Ref. [21–23].

At $r > 0$ when the system is on the disordered side, that is $\kappa(\mathbf{p}) \equiv 0$, and the density x moves away from QCP located at x_{FC}, the Landau amplitude $F_L(p = p_F, p_1 = p_F, x)$ as a function of x becomes smaller, the kinetic energy comes into a play and makes the flat part vanish. Obviously, Eq. (24) has only the trivial solution $\varepsilon(p = p_F) = \mu$, and the quasiparticle occupation numbers are given by the step function, $n_F(\mathbf{p}) = \theta(p_F - p)$.

At $\Delta_1 \to 0$, the critical temperature $T_c \to 0$. We see that the ordered phase can exist only at $T = 0$, and the state of electron liquid with FC disappears at $T > 0$ [6]. Therefore, FCQPT is not the endpoint of a line of finite-temperature phase transitions. This conclusion is in accordance with Eq. (2) which does not admit the existence of the flat part of spectrum at finite temperatures. As a result, the quantum to classical crossover upon approaching a finite-temperature phase transition does not exist. In the considered case, one can expect to observe such a cross-over at $T \sim T_f$. On the other hand, Δ_1 becomes finite if we assume that the pairing interaction is finite, and the corresponding $x - T$ phase diagram becomes richer. Moving along this line, we can consider the high-T_c superconductivity as well, see e.g. [6, 7].

At finite temperatures $T \ll T_f$, the occupation numbers in the region $(p_f - p_i)$ are still determined by Eq. (24), and the system becomes divided into two quasiparticle subsystems: the first subsystem is occupied by normal quasiparticles with the finite effective mass M_L^* independent of T at momenta $p < p_i$, while the second subsystem in the $(p_f - p_i)$ range is characterized by the quasiparticles with the effective mass $M_{FC}^*(T)$ [6, 24]

$$M^*_{FC} \simeq p_F \frac{p_f - p_i}{4T}. \quad (25)$$

There is an energy scale E_0 separating the slow dispersing low energy part, related to the effective mass M^*_{FC}, from the faster dispersing relatively high energy part, defined by the effective mass M^*_L. It follows from Eq. (25) that E_0 is of the form [6]

$$E_0 \simeq 4T. \quad (26)$$

The described system can be viewed as a strongly correlated one, it has the second type of the behavior and demonstrates the NFL behavior even at low temperatures. By applying magnetic fields, the system can be driven to LFL with the effective mass [25]

$$M^*(B) \propto \frac{1}{\sqrt{B - B_{c0}}}. \quad (27)$$

In the same way as it was done above, we find from Eqs. (25) and (27) that a cross over from the B-dependent effective mass $M^*(B)$ to the T-dependent effective mass $M^*(T)$ takes place at a transition temperature $T^*(B)$

$$T^*(B) \propto \sqrt{(B - B_{c0})}. \quad (28)$$

Equation (28) determines the line in the $B - T$ phase diagram which separates the region of the LFL behavior at $T < T^*(B)$ from the NFL behavior occurring at $T > T^*(B)$. The existence of the $B - T$ phase diagram given by Eqs. (15) and (28) can be highlighted by calculating the resistivity and the magnetoresistance [8]. The resistivity, which at $T > T^*(B)$ demonstrates the NFL behavior, at $T < T^*(B)$, exhibits the LFL behavior, $\rho = \rho_0 + A(B)T^2$. The $B - T$ diagram of the dependence of the effective mass on the magnetic field can be highlighted by calculating the magnetoresistance. At $(B - B_{c0}) > B^*(T)$, the magnetoresistance is negative, and at $(B - B_{c0}) < B^*(T)$, it becomes positive. This behavior of both the magnetoresistance and the resistivity is in agreement with measurements on $YbRh_2Si_2$ [3], when the system exhibits the second type of the behavior, see Eq. (28), while $CeCoIn_5$ and YbAgGe demonstrate the first type of behavior consistent with that given by Eq. (15) [4, 19].

At $T < T^*(B)$, the coefficients $\gamma \propto M^*(B)$, $\chi(B) \propto$ $\propto M^*(B)$, and $A(B) \propto (M^*(B))^2$, and we find that the Kadowaki ratio K and the Sommerfeld-Wilson ratio $R \propto \chi(B)/\gamma(B)$ are preserved due to Eq. (27). The obtained $B - T$ phase diagram and the conservation of both the Kadowaki and the Sommerfeld-Wilson ratios are in full agreement with data obtained in measurements on $YbRh_2Si_2$ and $YbRh_2(Si_{0.95}Ge_{0.05})_2$ [3, 5, 17]. Taking into account Eqs. (11) and (25), we obtain that in the case of the two quasiparticle subsystems the thermal expansion coefficient $\alpha(T) \propto a + bT + c\sqrt{T}$, with a, b and c being constants. Here, the first term a is determined by the FC contribution, the second bT is given by normal quasiparticles with the effective mass M^*_L, and the third $c\sqrt{T}$ comes from a specific contribution related to the spectrum $\varepsilon_c(\mathbf{p})$ which insures the connection between the dispersionless region $(p_f - p_i)$ occupied by FC and normal quasiparticles [16, 24]. At finite temperatures, the contribution coming from the third term is expected to be relatively small because the spectrum $\varepsilon_c(\mathbf{p})$ occupies a relatively small area in the momentum space. Since at $T \to 0$, the main contribution to the specific heat $c(T)$ comes from the spectrum $\varepsilon_c(\mathbf{p})$, the specific heat behaves as $c(T) \propto a_1\sqrt{T} + b_1T$, with a_1 and b_1 being constants. The second term b_1T comes from the contribution given by FC and normal quasiparticles. Measurements for $YbRh_2(Si_{0.95}Ge_{0.05})_2$ show a power low divergence of $\gamma = c/T \propto T^{-\alpha}$ with $\alpha = 1/3$ [17]. This result is in a reasonable agreement with our calculations giving $\alpha = 0.5$. At lower temperatures, the relative contribution of the first term $a_1\sqrt{T}$ becomes bigger, and we expect that the agreement will also become better. Now we obtain that the Grüneisen ratio $\Gamma(T) = \alpha(T)/c(T)$ diverges as $\Gamma(T) \propto 1/\sqrt{T}$ [16]. This results is in good agreement with measurements on $YbRh_2(Si_{0.95}Ge_{0.05})_2$ [5, 17].

As it follows from Eq. (27), the static magnetization behaves as $M_B(B) \propto \sqrt{B - B_{c0}}$ in accordance with measurements on $YbRh_2(Si_{0.95}Ge_{0.05})_2$ [5]. We can also conclude that Eqs. (21), (22), and (23) determining the scaling behavior of the effective mass, static magnetization and the susceptibility are also valid in the case of strongly correlated liquid, but the variable y is now given by $y = T/\sqrt{B - B_{c0}}$, while the function $f(y)$ can be dependent on $(p_f - p_i)/p_F$. This dependence comes from Eq. (25). As a result, we can obtain that at $T < T^*(B)$, the factor $d\rho/dT \propto A(B)T$ behaves as $A(B)T \propto T/(B - B_{c0})$, and at $T > T^*(B)$, it behaves as $A(B)T \propto 1/T$. These observations are in good agreement with the data obtained in measurements on $YbRh_2(Si_{0.95}Ge_{0.05})_2$ [5].

It is worthy to note that in zero magnetic fields the Neél temperature is zero at FCQPT, because, as it follows from Eq. (10), the effective mass tends to infinity at FCQPT and makes the susceptibility be divergent. On the other hand, if there is the magnetic order and the Neél temperature is not equal to zero, the effective mass is finite and there is no FCQPT. As soon as the magnetic order is suppressed at $B \to B_{c0}$, that is the Neél temperature tends to zero, the effective mass $M^*(B) \to \infty$ as it follows from Eq. (14). If the system has undergone

FCQPT, again at $B \to B_{c0}$, the Neél temperature goes to zero and $M^*(B) \to \infty$, see Eq. (27). If $B_{c0} = 0$, the effective mass diverges at $T \to 0$, see Eq. (25), and the susceptibility χ tends to infinity being proportional to the effective mass. In this case, the Neél temperature is equal to zero as well. Therefore, one may say that the effective mass M^* diverges at the very point that the Neél temperature goes to zero.

A few remarks are in order at this point. To describe the behavior of heavy-fermion metals, we have introduced the system of quasiparticles, as it is done in the Landau theory of normal Fermi-liquids, where the existence of fermionic quasiparticles is a generic property of normal Fermi systems independent of microscopic details. As we have seen, at $T \ll T_f$, these quasiparticles have universal properties which determine the universal behavior of heavy-fermion metals. One can use another approach constructing the singular part of the free energy, introducing notions of the upper critical dimension, hyperscaling, etc., see e.g. [1, 2]. Moving along this way, one may expect difficulties. For example, having the only singular part, one has to describe at least the two types of the behavior. We reserve a consideration of these items for future publications.

In conclusion, we have shown that our simple model based on FCQPT explains the critical behavior observed in different HF metals. In the case of such HF metals as $CeNi_2Ge_2$, $CeCoIn_5$, YbAgGe, $CeRu_2Si_2$, etc., the behavior can be explained by the proximity to FCQPT, where their electronic systems behave like highly correlated liquids. In the case of such HF metals as $YbRh_2(Si_{0.95}Ge_{0.05})_2$ and $YbRh_2Si_2$ the critical behavior is different. This can be explained by the presence of FC in the electronic systems of these metals, i.e., by the fact that the electronic systems have undergone FCQPT and behave as strongly correlated liquids. We have shown that the basis of the Landau Fermi liquid theory survives in the both cases: the low energy excitations of both strongly correlated Fermi-liquid with FC and the highly correlated Fermi-liquid are quasiparticles. It is also shown that the effective mass M^* diverges at the very point that the Neél temperature goes to zero. The $B - T$ phase diagrams of both the highly correlated liquid and the strongly correlated one have been studied. We have shown that these $B - T$ phase diagrams influence strongly the effective mass and such important properties of HF metals as magnetoresistance, resistivity, specific heat, magnetization, susceptibility, volume thermal expansion, etc.

I thank P. Coleman for valuable comments. I am grateful to the CTSPS for hospitality during my stay in Atlanta. This work was supported in part by the Russian Foundation for Basic Research, # 04-02-16136.

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