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On the sigma-model structure of type IIA supergravity action in doubled field approach

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In this letter we describe how to string together the doubled field approach by Cremmer, Julia, Lü and Pope with Pasti–Sorokin–Tonin technique to construct the sigma–model–like action for type IIA supergravity. The relation of the results with that of obtained in the context of searching for Superstring/M-theory hidden symmetry group is discussed.

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The essential step towards making phenomenological sense of a higher-dimensional theory is to find the relevant scheme of reducing the extra dimensions and of a confining the Standard Model fields on a four-dimensional space-time submanifold. The idea of viewing this submanifold as a three-brane embedded into a five-dimensional space [1] has been proved to be useful in searching for the solution to some longstanding problems of phenomenology like the hierarchy problem and the value of cosmological constant. To shed a light on stringy origin of this Brane-World picture [2] one needs to deal with a domain wall coupling to ten-dimensional bulk fields that enter the low-energy effective action of a superstring theory. Constructing such a coupling one should have in mind that as a superbrane the domain wall is the source of antisymmetric tensor field of supergravity multiplet and as the higher brane with the dimension of worldvolume greater that five it couples to the conventional supergravity tensor fields as well as to their duals. Moreover, requirement of quantum consistency of a theory forces to take into account other higher branes as a latent source of new local anomalies. Hence coupling the higher branes to higher-dimensional maximal supergravity backgrounds and studying the issue of anomaly cancellations deserve of having a formulation for supergravities whose dynamics is managed by the standard and the dual fields entering the theory in a duality-symmetric way.

Another problem that is important from phenomenological point of view is to identify the underlying symmetry group of Superstring/M-theory in which the Standard Model group have to be embedded. Recently it was emphasized that duality-symmetric structure of maximal supergravities becomes important in searching for such a hidden symmetry group [3 – 10].

There are different routes to recover duality-symmetric structure. For instance one can purely interested in dynamics of different sub-sectors of duality-symmetric theory doubling the fields on-shell, i.e. at the level of fields' equations of motion [11]. After the doubling, applying a method akin to non-linear realization technique one can find elegant representation of the scalars' and antisymmetric tensors' duality relations and equations of motion in the form of a twisted self-duality condition and zero-curvature condition. Or one can construct a supersymmetric pseudo-action [2] supported by on-shell duality relations between the doubled fields and these relations have to be imposed by hands. There also is universal formulation which is off-shell and produces the duality relations as equations of motion [12 – 15]. This formulation is based on the ground of PST technique [16]. But either pseudo-action approach or PST formalism which are just the extensions of standard supergravity actions say nothing about the hidden symmetry structure of maximal supergravities. On the contrary, the doubled field approach of [11] is a higher-dimensional remnant of G/H coset structure of scalar group manifolds that appear after toroidal reduction of D=11 supergravity with subsequent dualising the higher rank tensor fields [17]. In general the group of invariance of equations of motion is more restrictive than the group of invariance of the action. Therefore the step toward recovering the true hidden symmetry group of Superstring/M-theory is the construction of relevant low-energy effective actions. Since the dynamics of scalars mentioned so far

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is described by G/H sigma–model action one can also expect the sigma–model–like structure of the doubled field action.

As a by-product of studying performed in [15] the doubled field formalism of [11] dealing with non-gravitational subsector of $D=11$ supergravity has been extended to the off-shell supersymmetric formulation of duality-symmetric $D=11$ supergravity [12]. There has also been claimed that the sigma–model–like structure of the doubled gauge field action in $D=11$ has a generic form that remains the same also for the doubled field formulations of type IIA and type IIB supergravities as well as for lower dimensional maximal supergravities considered in [11]. This point roughly seems to be clear at least for the sequence of maximal supergravities coming from $D=11$ supergravity since all of them are related to each other via dimensional reduction. However, there is no rigorously defined procedure of dimensional reduction for the sigma–model–like action. And having the duality-symmetric gauge field structure in $D=11$ is not enough to recover complete duality-symmetric structure of $D=10$ type IIA supergravity since one shall double the fields coming from the reduction of gravity sector. Hence the explicit supersymmetric sigma-model structure of low-dimensional maximal supergravities has to be shown in case by case manner. The aim of the present paper is to fill the gap for type IIA supergravity and to demonstrate how the generic form of the $D=11$ doubled field sigma-model action [15] is accommodated to describe its ten-dimensional counterpart.

Let us get started with recalling in brief the doubled field formulation of $D=11$ supergravity [11] and its extension at the level of proper action [15]. From the Lagrangian of $D=11$ supergravity [18] (see [15] for comprehensive list of our conventions)

$$\mathcal{L} = -\sqrt{g}\, R + \frac{i}{3!}\bar{\Psi}\wedge D[\frac{1}{2}(\omega+\tilde{\omega})]\Psi\Gamma^{abc}\wedge\Sigma_{abc} - \\ - \frac{1}{48}\sqrt{g}\, F_{mnpq}F^{mnpq} - \frac{1}{6}A^{(3)}\wedge F^{(4)}\wedge F^{(4)} - \\ - \frac{1}{2}(C^{(7)} + *C^{(4)})\wedge(F^{(4)} + (F^{(4)} - C^{(4)})), \quad (1)$$

with $F^{(4)} = dA^{(3)}$, $C^{(4)} = -1/4\, \bar{\Psi}\wedge\Gamma^{(2)}\wedge\Psi$, $C^{(7)} = i/4\,\bar{\Psi}\wedge\Gamma^{(5)}\wedge\Psi$, $\Gamma^{(n)} = 1/n!\, dx^{m_n}\wedge\cdots\wedge dx^{m_1}\Gamma^{(n)}_{m_1\ldots m_n}$, we get the second order equation of motion for the $A^{(3)}$ gauge field

$$d(*(F^{(4)} - C^{(4)}) - \frac{1}{2}A^{(3)}\wedge F^{(4)} - C^{(7)}) = 0 \quad (2)$$

that can be presented as the Bianchi identity for the dual field $A^{(6)}$

$$dF^{(7)} = \frac{1}{2}F^{(4)}\wedge F^{(4)}, \quad F^{(7)} = dA^{(6)} + \frac{1}{2}A^{(3)}\wedge F^{(4)}. \quad (3)$$

Now forget for a while the dynamical origin of $F^{(7)} = dA^{(6)} + \frac{1}{2}A^{(3)}\wedge F^{(4)}$ and introduce it as an independent partner of $F^{(4)}$. These field strengths are invariant under the local gauge transformations

$$\delta A^{(3)} = \Lambda^{(3)}, \qquad \delta A^{(6)} = \Lambda^{(6)} - \frac{1}{2}\Lambda^{(3)}\wedge A^{(3)} \quad (4)$$

with closed forms $\Lambda^{(3)}$, $\Lambda^{(6)}$ associated with the so-called large gauge transformations [19]. Because of the presence of "bare" $A^{(3)}$ in $F^{(7)}$ and therefore in $\delta A^{(6)}$ that is traced back to the presence of the Chern-Simons term in the Lagrangian (3) the large gauge transformations are non-abelian

$$[\delta_{\Lambda^{(3)}_1}, \delta_{\Lambda^{(3)}_2}] = \delta_{\Lambda^{(6)}}, \qquad [\delta_{\Lambda^{(3)}}, \delta_{\Lambda^{(6)}}] = [\delta_{\Lambda^{(6)}_1}, \delta_{\Lambda^{(6)}_2}] = 0. \quad (5)$$

These relations can be associated with a superalgebra generated by a "Grassmann-odd" generator t_3 and a commuting generator t_6

$$\{t_3, t_3\} = -t_6, \qquad [t_3, t_6] = [t_6, t_6] = 0 \quad (6)$$

after that one can realize an element of the supergroup

$$\mathcal{A} = e^{t_3 A^{(3)}} e^{t_6 A^{(6)}} \quad (7)$$

and introduce the Cartan form

$$\mathcal{G} = d\mathcal{A}\mathcal{A}^{-1} = F^{(4)}t_3 + F^{(7)}t_6, \quad (8)$$

which by definition satisfies the Maurer–Cartan equation called sometimes the zero-curvature condition

$$d\mathcal{G} + \mathcal{G}\wedge\mathcal{G} = 0. \quad (9)$$

To impose the duality relation between a priori independent field strengths and arriving therefore at the standard number of degrees of freedom one introduces the pseudo-involution operator $\mathcal{S}$ which interchanges the generators t_3 and t_6

$$\mathcal{S}t_3 = t_6, \qquad \mathcal{S}t_6 = t_3, \qquad \mathcal{S}^2 = 1. \quad (10)$$

Using $\mathcal{S}$ and the Hodge operator one can immediately check that the following condition

$$*(\mathcal{G} + \mathcal{C}) = \mathcal{S}(\mathcal{G} + \mathcal{C}), \quad (11)$$

where we have introduced the superalgebra valued element $\mathcal{C} = -C^{(4)}t_3 + C^{(7)}t_6$, reproduces correctly the duality relations between the field strengths and therefore reduces tensors' degrees of freedom to the correct number. Moreover, when this condition holds the Maurer–Cartan equation amounts to second order equations of motion for $F^{(4)}$ and $F^{(7)}$.

Applying the PST technique the twisted self-duality condition is reproduced from the following action [15]

$$S = S_{EH} + S_\Psi - Tr \int_{\mathcal{M}^{11}} [\frac{1}{4}*\mathcal{G}\wedge\mathcal{G} + \frac{1}{2}(\mathcal{G}+\frac{1}{2}\mathcal{C})\wedge(\mathcal{S}-*)\mathcal{C} - \frac{1}{12}\mathcal{G}\wedge\mathcal{S}\mathcal{G} - \frac{1}{4}*i_v(\mathcal{S}-*)\mathcal{G}\wedge i_v(\mathcal{S}-*)\mathcal{G}], \quad (12)$$

where S_{EH} and S_Ψ stand for the Einstein-Hilbert and the Rarita-Schwinger actions,

$$v = \frac{da(x)}{\sqrt{-(\partial a)^2}} \quad (13)$$

is the one-form constructed out the PST scalar auxiliary field (cf. [16]) which ensures the covariance of the action and

$$\mathrm{Tr}(t_3 t_3) = -\mathrm{Tr}(t_6 t_6) = -1, \qquad \mathrm{Tr}(t_3 t_6) = 0. \quad (14)$$

As usual we have denoted by i_v the inner product of the vector field v_m with a form.

It is an instructive exercise to rewrite the sigma-model action (12) to the standard for $D = 11$ supergravity form. After some manipulations with taking into account the definitions of $\mathcal{G}$ and $\mathcal{S}$ one can arrive at

$$S = S_{CJS} + \int_{\mathcal{M}^{11}} \frac{1}{2} i_v\mathcal{F}^{(4)} \wedge *i_v\mathcal{F}^{(4)} \quad (15)$$

with S_{CJS} being the standard action by Cremmer, Julia and Scherk [18] and $\mathcal{F}^{(4)} = (F^{(4)} - C^{(4)}) - *(F^{(7)} + C^{(7)})$. This is the action for the duality-symmetric $D = 11$ supergravity [12] from which one can dynamically derive the duality condition $\mathcal{F}^{(4)} = 0$. Apparently, on the shell of the duality condition $\mathcal{F}^{(4)} = 0$ the action (15) coincides with the Cremmer–Julia–Scherk action.

Let us now turn to the type IIA supergravity which can be obtained from $D = 11$ supergravity by dimensional reduction. The corresponding Lagrangian has the following form [20–22]

$$\begin{aligned}\mathcal{L} = &-\sqrt{-g}R - \frac{i}{3!}\bar{\psi}\wedge D[\frac{1}{2}(\omega+\tilde{\omega})]\psi\wedge\Gamma^{abc}\Sigma_{abc} - \\ &- \frac{i}{2}\bar{\lambda}\Gamma^a D[\frac{1}{2}(\omega+\tilde{\omega})]\lambda\wedge\Sigma_a + \\ &+ (-)^{n+1}\sum_{n=1}^{4}[\frac{1}{2}\, e^{(-)^{n+1}\cdot\theta(n-1)\cdot\frac{(5-n)}{2}\phi}F^{(n)}\wedge *F^{(n)} + \\ &+ (-)^n(C^{(10-n)} - e^{(-)^{n+1}\cdot\theta(n-1)\cdot\frac{(5-n)}{2}\phi} * C^{(n)}) \times \\ &\times \wedge(F^{(n)} - \frac{1}{2}C^{(n)})] + \\ &+ \frac{1}{2}B^{(2)}\wedge dA^{(3)}\wedge dA^{(3)} + \mathcal{O}(f^4). \quad (16)\end{aligned}$$

Here we have denoted $F^{(1)} = d\phi$; $F^{(3)} = dB^{(2)}$ is the field strength of the NS 2-form field $B^{(2)}$ and the RR field strengths are defined as

$$\mathbf{F} = d\mathbf{A} - F^{(3)}\wedge\mathbf{A} \quad (17)$$

considering the formal sum $\mathbf{A} = \sum_{n=0}^{3} A^{(2n+1)}$ and extracting the relevant combinations. θ stands for usual step function taking the one in the case of a positive argument. The last term of (16) denotes the quartic fermion terms which do not involve into the gravitino, dilaton, dilatino and antisymmetric tensor gauge fields supercovariantization. The explicit form of the fermion bilinears $C^{(n)}$ can be read off [15].

Extracting the dual field strengths by use of equations of motion one arrives at

$$\begin{aligned}F^{(7)} &= dB^{(6)} - A^{(1)}F^{(6)} + \frac{1}{2}A^{(3)}dA^{(3)}, \\ F^{(9)} &= dA^{(8)} - \frac{3}{4}F^{(8)}A^{(1)} + \frac{1}{2}B^{(2)}dB^{(6)} - \frac{1}{4}F^{(6)}A^{(3)},\end{aligned} \quad (18)$$

while the RR field strengths $F^{(6)}$ and $F^{(8)}$ are defined as in (17).

Instead of repeating the analysis of large gauge transformations, constructing the associated superalgebra and introducing the Cartan form via non-linear realization of the supergroup element as it has been done before we will proceed further in a slightly different way.

Following [11] close inspection of the Bianchi identities for the dual field strengths shows that they can be obtained from the zero curvature condition for the Cartan form

$$\mathcal{G} = \frac{1}{2}d\phi\cdot t_0 + \sum_{n=2}^{4} e^{(-)^{n+1}\frac{(5-n)}{4}\phi}F^{(n)}\cdot t_{n-1} -$$

$$-\sum_{n=6}^{8} e^{(-)^{n+1}\frac{(5-n)}{4}\phi} F^{(n)} \cdot t_{n-1} - \frac{1}{2} F^{(9)} \cdot t_8,$$

with taking into account the following superalgebra of generators

$$[t_0,t_1] = \frac{3}{2}t_1, \qquad [t_0,t_2] = -t_2, \qquad [t_0,t_3] = \frac{1}{2}t_3,$$
$$[t_0,t_5] = -\frac{1}{2}t_5, \qquad [t_0,t_6] = t_6, \qquad [t_0,t_7] = -\frac{3}{2}t_7;$$
$$[t_1,t_2] = -t_3, \qquad \{t_1,t_5\} = t_6, \qquad [t_2,t_3] = t_5,$$
$$[t_2,t_5] = -t_7, \qquad \{t_3,t_3\} = t_6, \qquad \{t_1,t_7\} = \frac{3}{8}t_8,$$
$$[t_2,t_6] = \frac{1}{4}t_8, \qquad \{t_3,t_5\} = \frac{1}{8}t_8. \qquad (19)$$

Loosely speaking, the most part of the superalgebra can be recovered by extraction of the relevant Bianchi identities for dual field strengths from the zero-curvature condition (11). The rest of the algebra is restored from the graded Jacobi identities. Since we have required the zero-curvature condition, we can always present the Cartan form as $\mathcal{G} = d\mathcal{A}\mathcal{A}^{-1}$ for the supergroup element $\mathcal{A}$, the explicit expression for which can be read off [11].

To present the type IIA supergravity action in a sigma–model–like form one needs to write the former in the generating for such a representation following expression

$$S = S_{EH} + S_\psi + S_\lambda +$$
$$+(-)^{n+1}\int_{\mathcal{M}^{10}} \sum_{n=1}^{4}[\frac{1}{4}\, e^{(-)^{n+1}\cdot\theta(n-1)\cdot\frac{(5-n)}{2}\cdot\phi} F^{(n)}\wedge * F^{(n)} +$$
$$+\frac{(-)^n}{2}(C^{(10-n)} - e^{(-)^{n+1}\cdot\theta(n-1)\cdot\frac{(5-n)}{2}\cdot\phi} * C^{(n)}) \times$$
$$\times \wedge (F^{(n)} - \frac{1}{2}C^{(n)})] +$$
$$+\int_{\mathcal{M}^{10}} \sum_{n=6}^{9}[\frac{1}{4}\, e^{(-)^{n+1}\cdot\theta(9-n)\cdot\frac{(5-n)}{2}\cdot\phi} F^{(n)}\wedge * F^{(n)} +$$
$$+\frac{(-)^{n+1}}{2}(C^{(10-n)} - e^{(-)^{n+1}\cdot\theta(9-n)\cdot\frac{(5-n)}{2}\cdot\phi} * C^{(n)}) \times$$
$$\times \wedge (F^{(n)} + \frac{1}{2}C^{(n)})] +$$
$$+(-)^{n+1}\int_{\mathcal{M}^{10}} \frac{1}{4}\sum_{n=1}^{4} e^{(-)^{n+1}\cdot\theta(n-1)\cdot\frac{(5-n)}{2}\cdot\phi} i_v\mathcal{F}^{(n)}\wedge * i_v\mathcal{F}^{(n)}$$
$$-\int_{\mathcal{M}^{10}} \frac{1}{4}\sum_{n=6}^{9} e^{(-)^{n+1}\cdot\theta(9-n)\cdot\frac{(5-n)}{2}\cdot\phi} i_v\mathcal{F}^{(n)}\wedge * i_v\mathcal{F}^{(n)} +$$
$$+\frac{1}{2}\int_{\mathcal{M}^{10}} \sum_{n=1}^{4} \frac{1}{3^{[\frac{n+1}{4}]}}\, F^{(10-n)}\wedge F^{(n)} + \mathcal{O}(f^4), \qquad (20)$$

where S_{EH}, S_ψ, S_λ stand for the kinetic terms of graviton, gravitino and dilatino, and $[(n+1)/4]$ denotes the integer part of the number $(n+1)/4$. Here we have denoted by $\mathcal{F}^{(n)}$ the duality relations between the fields and their dual partners

$$\mathcal{F}^{(n)} = \hat{F}^{(n)} + e^{(-)^n\cdot\theta(n-1)\cdot\theta(9-n)\cdot\frac{5-n}{2}\cdot\phi} * \hat{F}^{10-n},$$
$$n = 1,\ldots,4,6,\ldots,9, \qquad (21)$$

with $\hat{F}^{(n)} = F^{(n)} - C^{(n)}$ for $n < 5$ and $\hat{F}^n = F^{(n)} + C^{(n)}$ for $n > 5$.

Defining the traces between the same generators as follows

$$\mathrm{Tr}(t_0t_0) = -\mathrm{Tr}(t_8t_8) = -4, \quad \mathrm{Tr}(t_1t_1) = \mathrm{Tr}(t_7t_7) = -1,$$
$$\mathrm{Tr}(t_2t_2) = -\mathrm{Tr}(t_6t_6) = -1, \;\mathrm{Tr}(t_3t_3) = \mathrm{Tr}(t_5t_5) = -1, \qquad (22)$$

and setting other traces to zero it is matter to check that eq. (20) is presented as

$$S = S_{EH} + S_\psi + S_\lambda -$$
$$-\mathrm{Tr}\int_{\mathcal{M}^{10}} [(-)^{\theta(5-n)}\{\frac{1}{4}\, *\mathcal{G}\wedge\mathcal{G} - \frac{1}{2}(\mathcal{S}-*)\mathcal{C}\wedge(\mathcal{G}+\frac{1}{2}\mathcal{C})\}$$
$$-\frac{1}{4} * i_v(\mathcal{S}-*)\mathcal{G}\wedge i_v(\mathcal{S}-*)\mathcal{G}] -$$
$$-\frac{1}{4}\mathrm{Tr}\int_{\mathcal{M}^{10}} \frac{1}{3^{[\frac{\min\,(n,k)+1}{4}]}}\,(-)^n\,\mathcal{G}\wedge\mathcal{S}\mathcal{G} + \mathcal{O}(f^4). \qquad (23)$$

The presence of the coefficient with theta-function in front of the second integral over a ten-dimensional manifold which after evaluating the traces from (22) becomes proportional in particular to the sum of $F^{(n)}\wedge * F^{(n)}$ manages the sign flips in the dilaton and gauge fields' kinetic terms, and the coefficient in the last term of (23) gives the correct signs and coefficients to obtain the last term of (20). To recover the structure of the supercovariant terms for the $F^{(n)}$s we have introduced the superalgebra valued element

$$\mathcal{C} = -\frac{1}{2}C^{(1)}\cdot t_0 - \sum_{n=2}^{4} e^{(-)^{n+1}\frac{(5-n)}{4}\phi} C^{(n)}\cdot t_{n-1} -$$
$$-\sum_{n=6}^{8} e^{(-)^{n+1}\frac{(5-n)}{4}\phi} C^{(n)}\cdot t_{n-1} - \frac{1}{2}C^{(9)}\cdot t_8.$$

The twisted self-duality condition

$$*(\mathcal{G}+\mathcal{C}) = \mathcal{S}(\mathcal{G}+\mathcal{C}) \qquad (24)$$

with the pseudo-involution $\mathcal{S}$ exchanging the generators

$$\mathcal{S}t_n = t_{8-n}, \quad n = 0, \ldots, 3, 5, \ldots, 8; \qquad \mathcal{S}^2 = 1 \quad (25)$$

is reproduced from the action (23) as an equation of motion. As soon as eq. (24) holds the zero-curvature condition encodes the second order equations of motion for fields and their duals.

Therefore, we have found the explicit form of the sigma-model representation of the type IIA duality-symmetric supergravity action and have demonstrated that the form of this representation obtained previously for the $D = 11$ duality-symmetric supergravity [15] is generic and needs just slight modifications to accommodate the structure of type IIA supergravity. The main difference between eleven and ten-dimensional cases is the structure of quartic fermion terms. In the former case the terms of this type can be absorbed into the corresponding supercovariant quantities, while in the latter case there are additional quartic fermion terms which do not involve into the supercovariantization (cf. [20 – 22]).

In conclusion, let us discuss the relation of the results with that of obtained in the context of searching for Superstring/M-theory hidden symmetry group [4 – 10]. It was realized long ago [3] that the global symmetry groups of toroidally compactified up to four space-time dimensions $D = 11$ supergravity fall into the class of exceptional groups E_n with $n \leq 7$. Discovering the exceptional geometry of $D = 3$ maximal supergravity [8] gave one more evidence in favor of previously conjectured [10] (and Refs. therein), [5] E_{10} hidden symmetry group of "small tension limit" of M-theory [9] compactifying to one dimension. Recently it was demonstrated that the bosonic sector of $D = 11$ supergravity can be reformulated as a non-linear realization of E_{11}/F_{11} coset space with maximal non-compact group F_{11} containing $SO(1,10)$ as a subgroup [6]. To realize such a formulation and to require representatives of such a coset space belong to the Borel subgroup of E_{11} one is forced to involve dual partners for graviton and 3-index antisymmetric tensor gauge field from the beginning and the action obtained is expected that of a sigma-model. Additional strong arguments in favor of the E_{11} M-theory hidden symmetry group conjecture [6] follow from careful analysis of group structure relevant to realize non-linearly the bosonic sector of type IIA supergravity. The approach based on the non-linear realization is rigorous and consistent but possesses the real drawback of absence of fermions into the game. On the contrary, though the doubled gauge field approach looks artificial it allows one to take fermions into account. In common the abovementioned approaches seem to be tightly related to each other, and the puzzle for both approaches is to construct duality-symmetric action for graviton and its dual partner. The necessity of making this step can be viewed for instance under the derivation of duality-symmetric action for $D = 10$ type IIA supergravity [15]. As it has been mentioned above the Kaluza-Klein reduction of the duality-symmetric w.r.t. the 3- and 6-index photons $D = 11$ supergravity can not provide the duality-symmetric structure of type IIA supergravity in the subsector of fields coming from the reduction of metric tensor. This fact forces to make additional efforts to recover complete duality-symmetric formulation of $D = 10$ type IIA supergravity. On the other hand, having a formulation for $D = 11$ supergravity which is completely duality-symmetric w.r.t. the all fields including the graviton, the complete duality-symmetric structure of $D = 10$ type IIA theory could be recovered in a straight way. The searching for the extension of a duality-symmetric formulation with graviton field will shed a light on the relation between the doubled field and non-linear realization approaches and will be helpful in overcoming the drawbacks of the latter.

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