

# THE S-CHANNEL APPROACH TO LIPATOV'S POMERON AND HADRONIC CROSS SECTIONS

*N.N.Nikolaev, B.G.Zakharov, V.R.Zoller*

*IKP(Theorie), KFA Jülich,  
5170 Jülich, Germany*

*+ L. D. Landau Institute for Theoretical Physics, GSP-1, 117940,  
117334, Moscow, Russia*

*\* Institute for Theoretical and Experimental Physics,  
Bolshaya Chermushkinskaya 25, 117259 Moscow, Russia.*

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We derive a generalized Balitskii-Fadin-Kuraev-Lipatov equation, which applies directly to the perturbative QCD component of total cross section. With the gluon correlation radius  $R_c \sim 0.4\text{fm}$  we reproduce the empirical rate of growth of the hadron-nucleon total cross sections. The simultaneous estimate of the triple-pomeron coupling also agrees with the experiment.

Lipatov and his collaborators have shown [1-3] that for short-distance interactions the QCD pomeron has the intercept

$$\alpha_{\mathbb{P}} = 1 + \Delta_{\mathbb{P}} = 1 + \frac{12 \log 2}{\pi} \alpha_S, \quad (1)$$

which gives a very large  $\Delta_{\mathbb{P}} \sim 1$  even with the reasonably small strong coupling  $\alpha_S = g_S^2/4\pi \sim 0.4$  appropriate for the already short distances ( $r \sim 0.15\text{fm}$ ) (here  $g_S$  is the color charge). On the other hand, the  $\sigma_{tot} \propto s^{\Delta_{\mathbb{P}}(hN)}$  fit of the hadronic total cross sections yields  $\Delta_{\mathbb{P}}(hN) \sim 0.1$  [4] (here  $s$  is the square of the c.m.s. energy).

In this paper we derive a particularly simple generalization of the Balitskii - Fadin - Kuraev - Lipatov (BFKL) equation directly for the total cross sections. We use the  $s$ -channel approach to the pomeron, which is based on the technique of light-cone multiparton wave functions introduced by the two of the present authors [5,6]. It allows us to easily introduce in the gauge-invariant manner the effect of a finite radius for gluon correlations  $R_c$  and to evaluate the effective intercept of the pomeron for the hadronic scattering regime. We relate the growth of the total cross section to the rise of the number of perturbative gluons in the lightcone hadrons. The lightcone wave functions of the multiparton states and their interaction cross sections were derived in our previous paper [6] and applied to an analysis of the diffractive deep inelastic scattering in the Double-Leading-Logarithm Approximation (DLLA) (see also [7]). In this paper we extend the considerations of [6] to the BFKL regime, discuss the connection between the BFKL and DLLA regimes, and estimate  $\Delta_{\mathbb{P}}$  for hadronic scattering processes.

Our starting point is the lowest order perturbative QCD cross section for the scattering of the two color dipoles  $r$  and  $R$  (here  $r, R$  are 2-dimensional vectors in the impact parameter plane)

$$\sigma_0(r, R) = \frac{32}{9} \int \frac{d^2k}{(k^2 + \mu_G^2)^2} \alpha_S^2 [1 - \exp(-ikr)][1 - \exp(ikR)]. \quad (2)$$

Here the effective mass of the gluon  $\mu_G$  serves as a reminder that the colour forces can not propagate beyond the gluon correlation radius  $R_c = 1/\mu_G$  and  $\alpha_S^2$  must be understood as  $\alpha_S(\max\{k^2, \frac{1}{r^2}\})\alpha_S(\max\{k^2, \frac{1}{R^2}\})$  [5]. At  $r \ll R \lesssim R_c$ , Eq. (2) gives the driving term of the DLLA cross section (for a detailed discussion of the DLLA regime see [6])

$$\sigma_0(r, R) \approx Cr^2 \alpha_S(r) L(R, r), \quad (3)$$

which is independent of  $R_c$ . Here  $L(R, r) \approx \log[\alpha_S(R)/\alpha_S(r)]$ . In terms of the dipole-dipole cross section (2) the perturbative part of the total cross section for the interaction of mesons  $A$  and  $B$  equals

$$\sigma^{(pt)}(AB) = \langle \langle \sigma(r_A, r_B) \rangle \rangle_{AB} = \int dz_A d^2 r_A dz_B d^2 r_B |\Psi(z_A, r_A)|^2 |\Psi(z_B, r_B)|^2 \sigma_0(r_A, r_B). \quad (4)$$

The advantage of the representation (4) is that it makes full use of the exact diagonalization of the scattering matrix in the dipole-size representation. Hereafter we discuss  $\sigma(r, R)$  averaged over the relative orientation of dipoles.

The perturbative  $q\bar{q}g$  Fock state generated radiatively from the parent colour-singlet  $q\bar{q}$  state has the interaction cross section

$$\sigma_3(r, \rho_1, \rho_2) = \frac{9}{8} [\sigma_0(\rho_1) + \sigma_0(\rho_2)] - \frac{1}{8} \sigma_0(r),$$

where  $\vec{\rho}_{1,2}$  are separations of the gluon from the quark and antiquark respectively,  $\vec{\rho}_2 = \vec{\rho}_1 + r$  [6]. The increase of the cross section for the presence of gluons equals (we suppress the target variable  $R$ )

$$\Delta\sigma_g(r, \rho_1, \rho_2) = \sigma_3(r, \rho_1, \rho_2) - \sigma_0(r) = \frac{9}{8} [\sigma_0(\rho_1) + \sigma_0(\rho_2) - \sigma_0(r)], \quad (5)$$

The lightcone density of soft,  $z_g \ll 1$ , gluons in the  $q\bar{q}g$  state derived in [6] equals

$$|\Phi_1(r, \vec{\rho}_1, \vec{\rho}_2, z_g)|^2 = \frac{1}{z_g} \frac{1}{3\pi^3} \mu_G^2 \left| g_S(r_1^{(min)}) K_1(\mu_G \rho_1) \frac{\vec{\rho}_1}{\rho_1} - g_S(r_2^{(min)}) K_1(\mu_G \rho_2) \frac{\vec{\rho}_2}{\rho_2} \right|^2. \quad (6)$$

Here  $g_S(r)$  is the running colour charge,  $r_{1,2}^{(min)} = \min\{r, \rho_{1,2}\}$ ,  $K_1(x)$  is the modified Bessel function,  $z_g$  is a fraction of the (lightcone) momentum of  $q\bar{q}$  pair carried by the gluon, and  $\int dz_g/z_g = \log(s/s_0) = \xi$ . With allowance for the  $q\bar{q}g$  Fock state the dipole cross-section takes the form  $\sigma_{tot}(\xi, r) = \sigma_0(r) + \sigma_1(r)\xi$ , where [6]

$$\sigma_1(r) = \int d^2 \vec{\rho}_1 z_g |\Phi_1(r, \vec{\rho}_1, \vec{\rho}_2, z_g)|^2 \Delta\sigma_g(r, \rho_1, \rho_2) = \mathcal{K} \otimes \sigma_0(r). \quad (7)$$

To higher orders in  $\xi$ ,

$$\sigma(\xi, r) = \sum_{n=0} \frac{1}{n!} \sigma_n(r) \xi^n,$$

where  $\sigma_{n+1} = \mathcal{K} \otimes \sigma_n$ , so that

$$\frac{\partial \sigma(\xi, r)}{\partial \xi} = \mathcal{K} \otimes \sigma(\xi, r) \quad (8)$$

is our generalized BFKL equation for the dipole cross section. We emphasize that the introduction of the gluon correlation length  $R_c$  in the kernel  $\mathcal{K}$  does not conflict the gauge-invariance constraints  $\sigma(r) \rightarrow 0$  and  $|\Phi(\mathbf{r}_1, \vec{\rho}_1, \vec{\rho}_2, z_g)|^2 \rightarrow 0$  at  $r \rightarrow 0$ , and  $\Delta\sigma_g(r, \rho_1, \rho_2) \rightarrow 0$  at  $\rho_{1,2} \rightarrow 0$ . The essential ingredient of this derivation is the subtraction of  $\sigma_0(r)$  in Eq. (5), which in a simple and intuitively appealing form takes care of the virtual radiative corrections. Once the dipole-dipole scattering problem is solved, Eq. (4) gives the hadron-hadron cross section.

In the BFKL scaling limit of  $r, \rho_1, \rho_2 \ll R_c$  and fixed  $\alpha_S$

$$\mu_G^2 \left| K_1(\mu_G \rho_1) \frac{\vec{\rho}_1}{\rho_1} - K_1(\mu_G \rho_2) \frac{\vec{\rho}_2}{\rho_2} \right|^2 = \frac{r^2}{\rho_1^2 \rho_2^2}, \quad (9)$$

the kernel  $\mathcal{K}$  becomes independent of  $R_c$  and with the fixed  $\alpha_S$  it takes on the scale-invariant form. The BFKL eigenfunctions of Eq. (8) are  $E(\omega, \xi, r) = (r^2)^{\frac{1}{2}+\omega} \exp[\xi \Delta(\omega)]$  with the eigenvalue (intercept) (here  $\mathbf{r} = r\mathbf{n}$ ,  $\vec{\rho}_1 = r\mathbf{x}$  and  $\vec{\rho}_2 = r(\mathbf{x} + \mathbf{n})$ )

$$\begin{aligned} \Delta(\omega) &= \frac{3\alpha_S}{2\pi^2} \int d^2x \frac{2(x^2)^{\frac{1}{2}+\omega} - 1}{x^2(x+\mathbf{n})^2} = \frac{3\alpha_S}{\pi} \int_0^1 dz \frac{z^{\frac{1}{2}-\omega} + z^{\frac{1}{2}+\omega} - 2z}{z(1-z)}, \\ &= \frac{3\alpha_S}{\pi} [2\Psi(1) - \Psi(\frac{1}{2} - \omega) - \Psi(\frac{1}{2} + \omega)], \end{aligned} \quad (10)$$

where  $\Psi(x)$  is the digamma-function. The final result for  $\Delta(\omega)$  coincides with eigenvalues of the BFKL equation. Indeed, in the scaling limit of  $\mu_G \rightarrow 0$  our Eq. (8) can be transformed to the same form as the original BFKL equation for the differential distribution of gluons [1,2].

When  $\omega$  is real and varies from  $-\frac{1}{2}$  to 0 and to  $\frac{1}{2}$ , also the intercept  $\Delta(\omega)$  is real and varies from  $+\infty$  down to  $\Delta(0) = \Delta_{\mathbb{P}}$  and back to  $+\infty$ , along the cut from  $j = 1 + \Delta_{\mathbb{P}}$  to  $+\infty$  in the complex angular momentum  $j$  plane. If  $\omega = i\nu$  and  $\nu$  varies from  $-\infty$  to 0 and to  $+\infty$ , then the intercept  $\Delta(i\nu)$  is again real and varies from  $-\infty$  up to  $\Delta(0) = \Delta_{\mathbb{P}}$  and back to  $-\infty$ , along the cut from  $j = -\infty$  to  $j = 1 + \Delta_{\mathbb{P}}$  in the complex  $j$ -plane. The choice of the latter cut is appropriate for the Regge asymptotics at  $\xi \gg 1$  and the counterpart of the conventional Mellin representation is

$$\sigma(\xi, r) = \int_{-\infty}^{+\infty} d\nu f(\nu) E(i\nu, r, \xi) = r \int_{-\infty}^{+\infty} d\nu f(\nu) \exp[2i\nu \log(r)] \exp(\Delta(i\nu)\xi), \quad (11)$$

where the spectral amplitude  $f(\nu)$  is determined by the boundary condition  $\sigma(\xi = 0, r)$ :

$$f(\nu) = \frac{1}{\pi} \int d\tau \frac{\sigma(0, r)}{r^2} \exp[-2i\nu \log(r)]. \quad (12)$$

In the BFKL regime, the rightmost  $j$ -plane singularity corresponds to the asymptotic cross section

$$\sigma_{\mathbb{P}}(\xi, r) \propto r \exp(\xi \Delta_{\mathbb{P}}). \quad (13)$$

The solution of Eq. (8) can be written as

$$\sigma(\xi, r) = r \int \frac{dr'}{(r')^2} K(\xi, r, r') \sigma(\xi = 0, r'), \quad (14)$$

where in the BFKL regime the evolution kernel equals

$$K(\xi, r, r') = \frac{1}{\pi} \int d\nu \exp \left[ 2i\nu \log \frac{r}{r'} \right] \exp[\xi \Delta(i\nu)] \\ \propto \frac{\exp(\Delta_{\mathbb{P}} \xi)}{\sqrt{\xi}} \exp \left( -2 \frac{(\log r - \log r')^2}{\xi \Delta''(0)} \right). \quad (15)$$

The 'diffusion' kernel (15) makes it obvious that starting with  $\sigma(\xi = 0, r)$  which was concentrated at the perturbative small  $r \lesssim R \ll R_c$  one ends up at large  $\xi$  with  $\sigma(\xi, r)$  which extends up to the nonperturbative  $r \sim R \exp(\sqrt{\xi \Delta''(0)}) > R_c$ . This 'diffusion' towards large  $r$  is further accelerated if the running coupling is introduced. Thus, the scattering of even very small dipoles of size  $r \sim R \ll R_c$  and/or deep inelastic scattering at  $Q^2 \gg R_c^{-2}$ , will eventually be dominated by interactions of the perturbative gluons sticking out of small dipoles at a distance  $\rho \sim R_c$ , and  $\Delta_{\mathbb{P}}$  will be determined by interactions at the scale  $R_c$  and by the frozen coupling  $\alpha_S^{(f r)}$ .

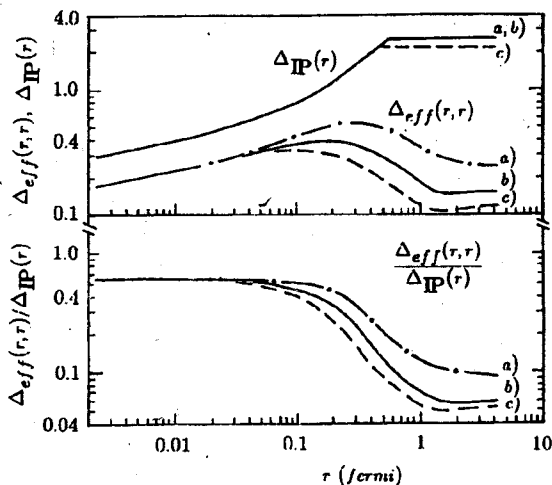


Fig.1.

Fig.1. The effective intercept  $\Delta_{eff}(r, r)$  for the scattering of two identical dipoles of size  $r$  in comparison with  $\Delta_{\mathbb{P}}(r)$  Eq. (16). The lower box shows the ratio  $\Delta_{eff}(r, r)/\Delta_{\mathbb{P}}(r)$ . The curves a, b and c are for  $\mu_G = 0.3, 0.5, 0.7$  GeV, respectively

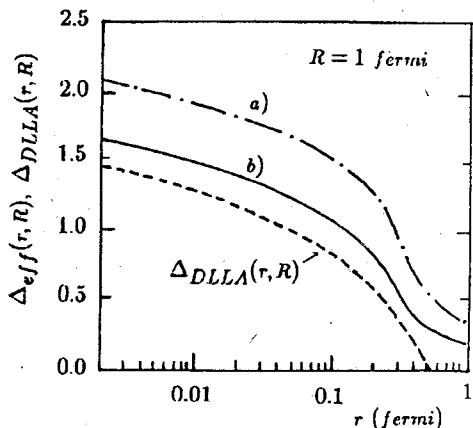


Fig.2.

Fig.2. The effective intercept  $\Delta_{eff}(r, R)$  for the scattering of unequal dipoles of size  $r$  and  $R$  in comparison with DLLA formula  $\Delta_{DLLA}(r, R)$ . The curves a, b are for  $\mu_G = 0.3, 0.5$  GeV, respectively

The analytic solution of Eq. (8) at finite  $R_c$  and with the running coupling is not available, and we resort to the numerical analysis. We use the running QCD coupling  $\alpha_S(r) = 6\pi/[(33 - 2N_f) \log(1/\Lambda r)]$  with  $\Lambda = 0.2$  GeV, and at large  $r$  we impose the simplest freezing  $\alpha_S(r) = \alpha_S^{(f r)} = \min\{\alpha_S(R_c), 1\}$ . Firstly, we calculate  $\Delta_{eff}(r, R) = \sigma_1(r, R)/\sigma_0(r, R)$  for the boundary condition  $\sigma(\xi = 0, r)$  given by the dipole-dipole cross section (2). The first interesting case is the scattering of the equal-size dipoles (Fig. 1). Here the kernel  $\mathcal{K}$  suggests the quasiclassical estimate

for the effective intercept

$$\Delta_{\mathbb{P}}(r) = \frac{12 \log 2}{\pi} \alpha_S(r). \quad (16)$$

Indeed, we find that at small  $r$  the ratio  $\beta = \Delta_{eff}(r, r) / \Delta_{\mathbb{P}}(r)$  tends to a constant  $\beta \approx 0.57$  independent of the gluon correlation radius  $R_c$ . The effective intercept  $\Delta_{eff}(r, r)$  rises with  $r$  up to  $r \sim R_c$ , then decreases and flattens at  $r \gg R_c$ , where both  $\sigma_{0,1}(r) \propto R_c^2$ .

Another interesting regime is the DLLA of unequal dipoles  $r \ll R$ . In this limit Eq. (7) takes the form [6]

$$\sigma_{n+1}(r) = \mathcal{K} \otimes \sigma_n(r) = \frac{3r^2 \alpha_S(r)}{\pi^2} \int_{r^2}^{R_c^2} \frac{d^2 \vec{\rho}}{\rho^4} \sigma_n(\rho), \quad (17)$$

which is equivalent to the GLDAP evolution equation [8] and to the first order in  $\xi$  gives  $\Delta_{DLLA}(r, R) = \frac{2}{3} \log[\alpha_S(R) / \alpha_S(r)]$ . Like any logarithmic estimate, this formula works up to a constant term  $\sim 1$ . In Fig. 2 we show our results for  $\Delta_{eff}(r, R)$  for  $R = 1\text{fm}$ . The difference  $\delta = \Delta_{eff}(r, R) - \Delta_{DLLA}(r, R)$  flattens at  $r/R \lesssim 0.2$ , which is a signal of the onset of DLLA.

The position of the rightmost singularity in the  $j$ -plane can easily be estimated from the asymptotic behavior of the numerical solution of Eq. (7). At  $R_c = 0.4, 0.28, 0.22$  fm, i.e., at the frozen coupling  $\alpha_S^{(fr)} = 1.0, 82, 0.63$ , we find estimates  $\Delta_{\mathbb{P}} \approx 0.52, 0.41, 0.36$ , respectively. The detailed discussion of the convergence to, and properties of, the limiting cross section  $\sigma_{\mathbb{P}}(\xi, r)$  will be presented elsewhere. We only notice, that these estimates are significantly below the Collins-Kwiecinski lower bound  $\Delta_{\mathbb{P}} > 3.6 \alpha_S^{(fr)} / \pi$  [9]. (The derivation of this bound in [9] is flawed by the infrared cutoff which breaks the initial symmetry of the BFKL kernel in the momentum space.)

The two cases of certain theoretical, although of little practical, interest are worth of mention. The first is the case of finite  $R_c$  at fixed  $\alpha_S$ , the second case is of  $\mu_G \rightarrow 0$  in the wave function (6) while keeping finite  $R_c$  in the strong coupling. Both share the property of restoration of the scaling invariance of the kernel  $\mathcal{K}$  on the infinite semiaxis  $\log r < \log R_c$  or  $\log r > \log R_c$ , where  $\alpha_S(r)$  freezes, respectively. On the corresponding semiaxis, the eigenfunctions are essentially identical to the BFKL set, the spectrum of eigenvalues will evidently be continuous and the  $j$ -plane partial waves will have the cut in the  $j$ -plane. Here we differ from Lipatov [3], who concluded that the running coupling leads to the discrete spectrum of eigenvalues and to a sequence of poles in the  $j$ -plane. The numerical analysis shows that the tip of the cut in the  $j$ -plane is very close to  $\Delta_{\mathbb{P}}$  as given by Eq. (1) with  $\alpha_S = \alpha_S^{(fr)}$ ; the more detailed analysis is needed to check a possibility of a finite departure from Eq. (1) which depends on the value of  $\alpha_S^{(fr)}$ .

Finally, let us consider the  $\pi N$  interaction as the typical hadronic scattering. The plausible assumption is that the growth of the hadronic cross sections is dominated by the perturbative gluons. In this case,  $\sigma_1^{(pt)}(\pi N) = \langle \langle \sigma_1(r_\pi, r_N) \rangle \rangle_N \pi$  and  $\Delta_{\mathbb{P}}(\pi N) \approx \sigma_1^{(pt)}(\pi N) / \sigma_{tot}(\pi N)$ . In Fig. 3 we present our prediction for the perturbative QCD contribution to the total cross section  $\sigma_0(\pi N)$  and the  $\Delta_{eff}(\pi N)$  vs. the gluon correlation radius  $R_c$ . We reproduce the empirical value  $\Delta_{\mathbb{P}}(hN) \sim 0.1$  at  $R_c \sim 0.4\text{fm}$ , when  $\sim 40\%$  of  $\sigma_{tot}(\pi N)$  is of the perturbative origin.

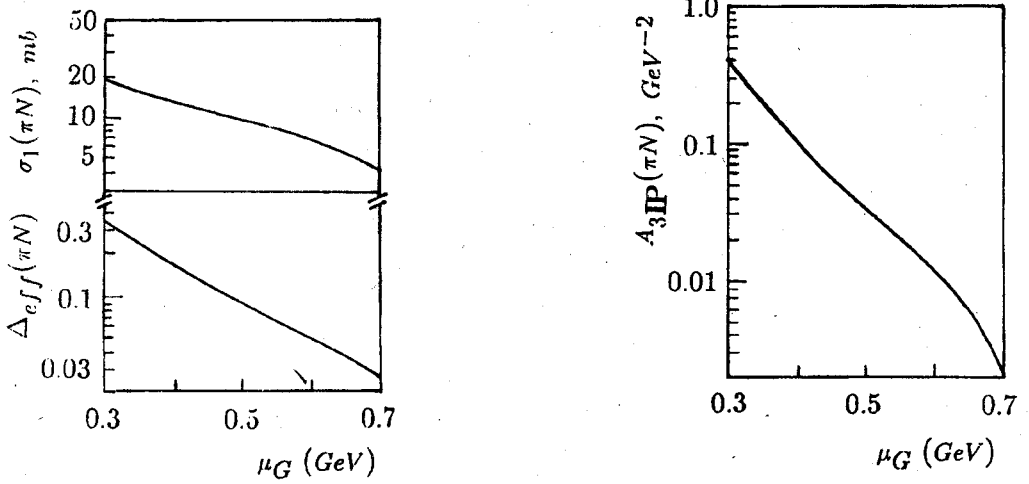


Fig.3. The effective intercept  $\Delta_{IP}(\pi N)$ , the perturbative QCD contribution to the total cross section  $\sigma_0(\pi N)$  and the effective triple-pomeron coupling for the pion-nucleon scattering vs.  $\mu_G$ .

Besides the rise of the total cross section, variations of the dipole cross section for the presence of gluons also contribute to the triple-pomeron coupling  $A_{3IP}$ . The method for calculating  $A_{3IP}$  was presented in [6,7]. With a correlation length  $R_c \sim 0.4$  fm, we find  $A_{3IP}(\pi N) \sim 0.04$   $GeV^{-2}$ , which is consistent with the experimental determinations [10].

In conclusion, we have derived generalized BFKL equation for the total cross sections with allowance for the finite gluon correlation radius  $R_c$ . We presented the first estimates of the perturbative QCD contribution to the rate of the growth of  $\sigma_{tot}(pN)$  and to the triple-pomeron coupling, which are consistent with experiment if  $R_c \sim 0.4$  fm. (Incidentally, the instanton model of the QCD vacuum and the lattice QCD calculations give very close value of  $R_c$  [11].) The irrefutable advantage of having the equation for the total cross section and of using the dipole-size representation which diagonalizes the scattering matrix, is that they allow an easy incorporation of the unitarity constraints. To this end, we recall that consideration of the unitarity corrections in the DLLA limit has already lead to an important conclusion [6] that the unitarity correction to structure functions in the diffractive deep inelastic scattering at small  $x$  satisfies the linear GLDAP evolution equations.

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