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COMPOSITE OPERATORS FOR BCS SUPERCONDUCTOR

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The new fo.m of the composite operator generalizing the Cooper pairs for a BCS superconductor is introduced. The approach is similar to the derivation of the composite operator of the odd - frequency superconductors. The examples of the $d_{x^2-y^2}$, d_{xy} and p-wave composite operators for a $2D$ $t-J$ model are given.

Recently the notion of a composite operator as a generalization of the Cooper pair in the theory of superconductivity has been proposed [1, 2, 3, 4]. The examples of the composite operators are

$$\Delta = \begin{cases} \langle \mathrm{S(i)} c_\alpha(\mathrm{j}) c_\beta(\mathrm{k})\rangle (i\sigma^y \vec{\sigma})_{\alpha\beta}; & S=0 \\ \langle \mathrm{S(i)} c_\alpha(\mathrm{j}) c_\beta(\mathrm{k})\rangle (\sigma^x \vec{\sigma})_{\alpha\beta}; & S=1, \ S_z=0 \end{cases}, \tag{1}$$

which describe the bound state of the spin excitation at site i and a triplet or a singlet Cooper pair at j, k. Clearly these operators carry charge $2e$ and thus describe the superconducting ordering. This type of the condensate is inherent for the odd-frequency superconductors [1, 2, 3, 4], which might occur in strongly correlated systems such as $t-J$ and Hubbard models. Indeed, it has been argued that frustration of the magnetic degrees of freedom by carriers may produce enhanced composite pairing correlations, for the operators similar to Eq. (1) [2].

The composite operator in the theory of superconductivity represents a new level in the hierarchy of the possible superconducting condensate. Any number of particle-hole (i.e. neutral) operators composed together with the Cooper pair operator possess charge $2e$ and thus can, in principle, describe some superconducting state [5]. Because of this general argument we point out that the composite operators are possible for BCS superconductors as well as for odd frequency ones.

The purpose of this note is to show that composite operators similar to Eq. (1) can be constructed also in the case of BCS pairing. As an example we will consider $2D$ $t-J$ model on the square lattice. Previously, the composite operator for particular case of $d_{x^2-y^2}$ symmetry in a $2D$ $t-J$ model was considered by Poilblanc [8]. The most relevant for possible BCS (even-gap) superconductivity in this model are singlets: extended s-wave (identity representations of D_4 point group symmetry), $d_{x^2-y^2}$-wave (B_2) and d_{xy}-wave (B_1). We find that, apart from the standard choice of the pairing state in these channels as: $\Delta_{x^2-y^2} = \langle c_{\uparrow \mathrm{k}} c_{\downarrow -\mathrm{k}}\rangle \propto \cos k_x - \cos k_y$; $\Delta_{xy} \propto \sin k_x \sin k_y$, there is a set of composite operators which satisfy all the requirements of the symmetry and spin eigenvalues

$$\Delta^{d_{x^2-y^2}}_{R=\sqrt{2}} = \sum_{\mathrm{i}} \ \{ \ (\mathrm{S}_{\mathrm{i}+\hat{\mathrm{x}}} - \mathrm{S}_{\mathrm{i}+\hat{\mathrm{y}}})\mathrm{T}_{\mathrm{i,i}+\hat{\mathrm{x}}+\hat{\mathrm{y}}} + (\mathrm{S}_{\mathrm{i}-\hat{\mathrm{x}}} - \mathrm{S}_{\mathrm{i}-\hat{\mathrm{y}}})\mathrm{T}_{\mathrm{i,i}-\hat{\mathrm{x}}-\hat{\mathrm{y}}} + \tag{2}$$

$$+\quad (S_{i+\hat{x}} - S_{i-\hat{y}})T_{i,i+\hat{x}-\hat{y}} + (S_{i-\hat{x}} - S_{i+\hat{y}})T_{i,i-\hat{x}+\hat{y}}\},$$

$$\Delta^{d_{xy}}_{R=1} = \sum_i \quad \{ \quad (S_{i+\hat{x}+\hat{y}} - S_{i+\hat{x}-\hat{y}})T_{i,i+\hat{x}} - (S_{i-\hat{x}+\hat{y}} - S_{i-\hat{x}-\hat{y}})T_{i,i-\hat{x}} +$$

$$+\quad (S_{i+\hat{x}+\hat{y}} - S_{i-\hat{x}+\hat{y}})T_{i,i+\hat{y}} - (S_{i+\hat{x}-\hat{y}} - S_{i-\hat{x}-\hat{y}})T_{i,i-\hat{y}}\},$$

$$\Delta^{p_x}_{R=1} = \sum_i \quad \{ \quad (S_{i+\hat{x}} - S_{i-\hat{x}})(P_{i,i+\hat{y}} + P_{i,i-\hat{y}})\},$$

where $T_{i,j} = \frac{1}{i} c_{i,\sigma}(\sigma^y \vec{\sigma})_{\sigma\sigma'} c_{j,\sigma'}$ and $P_{i,j} = c_{i,\sigma}(\sigma^x \vec{\sigma})_{\sigma\sigma'} c_{j,\sigma'}$. We used real space representation with $\hat{x}$ and $\hat{y}$ being unit vectors in x and y direction respectively. In the last of three equations in Eq. (2) we present for completeness also p_x-wave triplet $S_z = 0$ composite operator. It is interesting to note, that Cooper pairs in Eqs.(2) lie on symmetry axes of given symmetries, i.e. $x^2 - y^2 = 0$, $xy = 0$, $x = 0$, respectively.

The symmetry of the above order parameters is exactly the same as of the standard operators and corresponds to one of the symmetry representations on the square lattice.[7] To describe the derivation of the composite operators Eq. (2) for BCS superconductor we will review the derivation of the composite operators for odd-gap superconductors [1, 2, 3, 4]. The general form of the two-particle gap function can be written as

$$\Delta_{ij}(t) = \langle T_t c_{i\alpha}(t) c_{j\beta}(0)\rangle \sigma^y_{\alpha\beta}. \tag{3}$$

Assuming analyticity of the gap function at small $t \to 0$, the latter can be expanded for both odd-frequency and BCS channels as [2]

$$\Delta^{odd}_{ij}(t) = \Delta^{(1)}_{ij} t + \mathcal{O}(t^3) \tag{4}$$

$$\Delta^{even}_{ij}(t) = \Delta^{(0)}_{ij} + \frac{1}{2}\Delta^{(2)}_{ij} t^2 + \mathcal{O}(t^4). \tag{5}$$

For odd-frequency pairing after taking time derivative

$$\frac{\partial \Delta^{odd}_{ij}(t)}{\partial t}|_{t=0} = \Delta^{(1)}_{ij} \propto \langle [H, c_{i\alpha}]\, c_{j\beta}\rangle \sigma^y_{\alpha\beta}$$

we arrive at the composite operator. Here we limit ourselves to the singlet case. The generic form of the composite operator is always as in Eq.(1), however the details will depend on the Hamiltonian H (for more details in the case of $t - J$ model see for example [2]).

To obtain the composite operators for even frequency or BCS pairing (2), we have to take the second order time derivative

$$\Delta^{(2)}_{ij} = \frac{\partial^2 \Delta^{even}_{ij}(t)}{\partial t^2}|_{t=0} \propto \langle [H, [H, c_{i\alpha}]]\, c_{j\beta}\rangle \sigma^y_{\alpha\beta}. \tag{6}$$

We arrive at the composite operator in the form of Eq. (2) by taking for the first commutator the hopping term $H_t = t\sum_{ij} c^\dagger_{i\sigma} c_{j\sigma}$ and for the second commutator the Heisenberg term $H_J = J\sum_{ij} S_i S_j$. The commutator $[H_t, c_{i\alpha}]$ moves particle to the neighboring site which may lie on the main symmetry axis.

The second commutator with $[H_J, [H_t, c_{i\alpha}]]$ produces extra spin operator. From $[\mathrm{S_i}, c_{i\alpha}] = (-\frac{1}{2})\vec{\sigma}_{\alpha\nu}c_{i\nu}$ we find $[H_J, c_{i\alpha}] = -J\sum_{<ik>} \mathrm{S_k}\vec{\sigma}_{\alpha\nu}c_{i\nu}$ from where directly follows the general structure of the operators in Eq.(2) as composite operators of a Cooper pair with an attached spin operator. Direct check also reveals that these composite operators obey the required symmetry conditions under D_4 point group transformations.

In conclusion, we present the list of composite operators for BCS (even frequency) pairing, using $2D$ $t - J$ model as an example. The structure of these composite operators is analogous to the composite operators, introduced for the odd-frequency pairing. Important difference between these operators for odd-frequency and BCS pairing is that the composite operator for BCS pairing comes from the dressing of the quasiparticle operator assuming that standard equal time BCS gap function is nonzero. Although this dressing might improve overlap with the ground state, it does not represent new physics. Situation changes drastically if the usual BCS gap function has very small or even zero expectation value. In this case the composite BCS operator corresponds to the real "pairing" processes. For the odd-frequency pairing the composite operator indeed represents the equal time "pairing" in odd-frequency superconductors. As it has been pointed out in [2], the closeness to the instability in the $t - J$ model helps the composite channel because of a soft spin fluctuations in the system. Presumably the same holds for the composite BCS channels in the frustrated correlated systems.

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