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SQUEEZED QUANTUM STATE OF DISORIENTED CHIRAL CONDENSATE

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We consider the quantum state describing the Disoriented Chiral Condensate (DCC) which may be produced in high energy collisions. Using the approach suggested by Rajagopal and Wilczek to describe the amplification of the long wavelength classical pion modes, we consider the quantum-mechanical evolution of the initial vacuum state into the final squeezed state describing the DCC. The obtained wave function has some interesting properties which are discussed.

The possibility of the Disoriented Chiral Condensate (DCC) production in high energy hadronic or heavy ion collisions attracted a lot of attention in the last years [1]-[14].

It is well known that QCD Lagrangian is invariant (approximately iif nonzero masses for the light N_f quarks are taken into account) under global chiral $SU(N_f)_L \times SU(N_f)_R$, where N_f is the number of the light flavours. This symmetry is spontaneously broken down to vector $SU(N_f)_V$ which leads to $N_f^2 - 1$ (quasi)goldstone bosons - pions (if $N_f = 2$) or pions, kaons and η meson (if $N_f = 3$). The order parameter for this breaking is the vacuum expectation value of quark condensate $< \bar{\psi}\psi >$. However one can imagine that under some special conditions, for example after high-energy collision, there is a "cool" region surrounded by a "hot" relatively thin expanding shell, which separates the internal region from the outer space. This picture was suggested by Bjorken, Kowalski and Taylor [6] and is called now the "Baked Alaska" scenario. In result the quark condensate may be

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disoriented in isotopical space - one gets the DCC. After hadronization the interior disoriented vacuum will collapse decaying into pions. The interesting signature of this process will be the coherent production of either charged or neutral pions. There are some arguments that the DCC was observed in the Centauro cosmic ray events [15].

It is convenient to consider the toy model describing the chiral dynamics - the linear sigma-model with four component field $\phi^a = (\sigma, \vec{\pi})$, where σ and $\vec{\pi}$ are an isoscalar and an isovector fields (here we use the same notation as in [5])

$$S = \int d^4x [\frac{1}{2}\partial_\mu \phi^a \partial_\mu \phi^a - \frac{\lambda}{4}(\phi^a \phi^a - v^2)^2 + H\sigma], \tag{1}$$

where $H \sim m_q$ describes the small explicit chiral symmetry breaking. The pion mass is $m_\pi^2 = H/f_\pi = \lambda(f_\pi^2 - v^2)$, where $f_\pi =<\sigma>$. The sigma meson mass is $m_\sigma^2 = 3\lambda f_\pi^2 - \lambda v^2 \approx 2\lambda f_\pi^2$. With $m_\pi = 135$ MeV, $m_\sigma = 600 MeV$ and $f_\pi = 92.5$ MeV one has [5] $\lambda = 20$ and $v = 87.4$ MeV. In the usual vacuum one has $<\sigma>= f_\pi$, $<\pi>=0$ and since σ is an isoscalar there is an isoscalar condensate $<\bar{\psi}\psi>$ only. However one can consider another configuration - $<\sigma>= f_\pi \cos\theta$ and $<\pi>= f_\pi \mathrm{n} \sin\theta$, here n is some unit vector in the isospace, which describes DCC, i.e. some classical pion field configuration, which is metastable and decays after some time into pions - the signature for this event will be the large number of either neutral (π^0) or charged ($\pi^\pm$) pions. Using the classical picture of DCC which predicts equal probability for all isotopical orientation of condensate one can get [1] - [4] the probability $1/\sqrt{f}$ for the fraction of neutral pion $f = N_{\pi^0}/(N_{pi^+} + N_{\pi^-} + N_{\pi^0})$.

The question we would like to discuss here is connected with the quantum description of the DCC. The simplest possibility is to describe it by the usual coherent state. However the usual coherent state wave function was critisised in [4] because such a description leads to creation of pion system with arbitrary large charge fluctuations. Instead it was suggested in [4] that the quantum state must be isosinglet. For the state with $2n$ neutral pions and total number of pions $2N$ it was suggested:

$$|\Psi> \sim (2a_+^\dagger a_-^\dagger - (a_0^\dagger)^2)^N |0> . \tag{2}$$

Then the probability to have 2n neutral pions is [16], [4]

$$P(n,N) = \frac{(N!)^2 2^{2N}}{(2N+1)!} \frac{(2n)!}{(n!2^n)^2} \sim \sqrt{N/n}, \quad n, N >> 1 \tag{3}$$

and corresponds to $1/\sqrt{f}$ distribution in classical picture.

However let us note that the same distribution will be obtained and in the case of the arbitrary relative factor between charged and neutral creation operators $[2a_+^\dagger a_-^\dagger - \exp(i\theta)(a_0^\dagger)^2]^N$ and one can see that from this point of view the zero isospin condition is not so important - what was important indeed is that the wave function was constructed by the operators quadratic in $a^\dagger$ and by the construction there are equal numbers of π^+ and π^-.

It is interesting to understand what are the most natural class of these functions and what are the dynamical mechanisms leading to the generation of these functions. As we shall demonstrate here the quantum state of the DCC is the squeezed state. These quantum states known for a long time in quantum optics and measurement theory (for a review of squeezed states see, for example [17] -[20]). The simplest one-mode squeezed state is parametrized by the two parameters r and ϕ and can be obtained by acting on the vacuum by the squeezed operators $S(r,\phi)$

$$S(r,\phi)|0>= \exp[\frac{r}{2}(e^{-2i\phi}a^2 - e^{2i\phi}a^{\dagger 2})]|0> . \quad (4)$$

These states are minimum uncertainty states with $\Delta X_1 \Delta X_2 = 1/4$, where $a(a^\dagger) = X_1 \pm X_2$, as well as coherent states $\exp(\alpha a^\dagger)|0>$. However coherent states have some minimal quantum noise and $\Delta X_1 = \Delta X_2 = 1/2$. For the squeezed states one can reduce the quantum noise for one variable (increasing it for the conjugate one) and $\Delta Y_1 = e^{-r}/2$, $\Delta Y_2 = e^r/2$ where $Y_1 + iY_2 = (X_1 + iX_2)e^{-i\phi}$. The mean particle number $< N >= \sinh^2 r$.

To get the squeezed quantum state for the DCC let us consider the mechanism of the amplification of the long wavelength pion modes, suggested by Rajagopal and Wilczek in paper [5], where the the dynamics of the $O(4)$ linear sigma model after quenching was considered. The amplification of the long wavelength pion modes was found in the period immediately after quenching. This amplification leads to the coherent pion oscillations, i.e. to the creation of the DCC. Such a behaviour can be understood if one considers the equation of motion for the pion field [5]

$$\frac{\partial^2}{\partial t^2}\vec{\pi}(\mathbf{k},t) + [\mathbf{k}^2 + \lambda(<\phi^2>(t) - v^2)]\vec{\pi}(\mathbf{k},t) = 0, \quad (5)$$

where we substituted (as in [5]) the $\phi^a\phi^a$ in the nonlinear term in (5) by its spatial average $<\phi^2>(t)$ - this is nothing but the Hartree-Fock or mean field approximation. In the initial conditions one has $<\phi^2> < v^2$ and the long wavelength modes of the pion field with $\mathbf{k}^2 < \lambda(v^2 - <\phi^2>)$ start growing exponentially. The $<\phi^2>(t)$ starts to oscillate near the vacuum expectation value $<\sigma>$ and after some time the oscillations will be damped enough so that all the modes will be stable. Thus we see that at classical level each long wavelength mode is described by the equation for a parametrically excited oscillator and one get the DCC in result of amplification of the zero-point quantum fluctuations of the pion field.

This picture is similar to one which was discussed in [21], where the relic gravitons production from zero-point quantum fluctuations during the cosmological expansion was considered. For graviton mode with momentum n the equation $y'' + [n^2 - (R''/R)]y = 0$ was obtained, where $R(\eta)$ is the scale factor of the metric $ds^2 = R^2(\eta)(d\eta^2 - d\mathbf{x}^2)$ and a prime represents $d/d\eta$. One can see that this equation is equivalent to pion equation (5) if the scale factor R is connected with $<\phi^2>(t)$ as $\lambda(v^2 - <\phi^2>(t)) = R''/R$.

Our problem now is to present the quantum mechanical formulation in terms of pion creation and annihilation operators and to get the wave function of the DCC. In the mean field approximation the wave function $|\Psi>= \prod_{i,\mathbf{k}}|\psi>_{i,\mathbf{k}}$ is the product of the wave functions for each mode with momentum $\mathbf{k}$ and isotopical index i. Later we shall omit i. The equation of motion (5) for each mode $\pi(\mathbf{k},t)$ can be derived from the lagrangian

$$L_k = \frac{1}{2}\dot{\pi}^2(\mathbf{k},t) - \frac{1}{2}\Omega^2(\mathbf{k},t)\pi^2(\mathbf{k},t),$$
$$\Omega^2(\mathbf{k},t) = \mathbf{k}^2 + \lambda(<\phi^2>(t) - v^2). \tag{6}$$

The wave function $|\psi>_{\mathbf{k}}$ obeys Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi>_{\mathbf{k}} = H_k(t)|\psi>_{\mathbf{k}} = [\frac{1}{2}\mathcal{P}_\pi^2 + \frac{1}{2}\Omega^2(\mathbf{k},t)\pi^2(\mathbf{k})]|\psi>_{\mathbf{k}}, \tag{7}$$

where $\pi(\mathbf{k})$ and $\mathcal{P}_\pi = -id/d\pi(\mathbf{k})$ are the quantum-mechanical coordinate and momentum for the mode with the spatial momentum $\mathbf{k}$. One can rewrite the Hamiltonian in (7) in terms of creation and annihilation operators which make it diagonal at any given moment. It is evident that we are interested to get the wave function in terms of creation and annihilation operators of ordinary pions, so we must diagonalise the Hamiltonian at $t \to \infty$, when the oscillation of the $<\phi^2>(t)$ will be damped. Thus we define

$$a(\mathbf{k}) = \frac{\mathcal{P}_\pi + i\omega(\mathbf{k})\pi(\mathbf{k})}{\sqrt{2\omega(\mathbf{k})}}, \qquad a^\dagger(\mathbf{k}) = \frac{\mathcal{P}_\pi - i\omega(\mathbf{k})\pi(\mathbf{k})}{\sqrt{2\omega(\mathbf{k})}}, \tag{8}$$

where $\omega(\mathbf{k}) = \Omega(\mathbf{k},\infty) = \sqrt{\mathbf{k}^2 + m_\pi^2}$. It is easy to see that the Hamiltonian is

$$H_k = \frac{1}{2}\omega(\mathbf{k})[1 + \frac{\Omega^2(\mathbf{k},t)}{\omega^2(\mathbf{k})}]a^\dagger(\mathbf{k})a(\mathbf{k}) + \frac{\omega^2(\mathbf{k}) - \Omega^2(\mathbf{k},t)}{4\omega(\mathbf{k})}[a^2(\mathbf{k}) + a^{\dagger 2}(\mathbf{k})]. \tag{9}$$

The $a^2(\mathbf{k})$ and $a^{\dagger 2}(\mathbf{k})$ terms in Hamiltonian is the reason that H transforms the initial vacuum state $|0>$ into a squeezed state $S(r,\phi)|0>$ (4).

To calculate the squeezing and phase parameters r and ϕ let us consider the solution of the Schrödinger equation (7) in the coordinate (here it is π) representation (we shall omit the label $\mathbf{k}$ for moment)

$$<\pi|\psi>(t) = C(t)\exp(-B(t)\pi^2) \tag{10}$$

and for $B = \omega/2$ this wave function describes the vacuum state. For all other values this wave function describes the squeezed state (4) where the parameters r and ϕ are connected with B by the relation [17] (see also [19] and [21])

$$B = \frac{\omega}{2}\frac{\cosh r + \exp(2i\phi)\sinh r}{\cosh r - \exp(2i\phi)\sinh r},$$
$$\cosh 2r = \frac{\omega^2 + 4|B|^2}{4\omega \mathrm{Re}B}; \qquad \sin 2\phi = \frac{1}{\sinh 2r}\frac{\mathrm{Im}B}{\mathrm{Re}B}. \tag{11}$$

Substituting (10) into (7) one gets equations for $B(t)$ and $C(t)$

$$i\dot{B}(t) = 2B^2(t) - \frac{1}{2}\Omega^2(t), \qquad i\frac{\dot{C}(t)}{C(t)} = B(t) \tag{12}$$

which means that $B(t)$ is related to the solution of the classical equation (5)

$$B(t) = -\frac{i}{2}\frac{\dot{\psi}(t)}{\psi(t)}, \qquad \ddot{\psi}(t) + \Omega^2(t)\psi(t) = 0 \tag{13}$$

and $C(t) = C(0)\exp(i\int_0^t B(\tau)d\tau)$ is an evident phase factor. The last equation can be viewed as a Schrödinger equation describing the wave function $\psi(t)$ of a "particle" with mass $m = 1/2$ on a line t having energy $\omega^2(k) = k^2 + m_\pi^2$ and moving through the potential barrier $V(t) = -\lambda(<\phi^2>(t) - f_\pi^2)$

$$-\frac{d^2\psi(t)}{dt^2} + \lambda(f_\pi^2 - <\phi^2>(t))\psi(t) = \omega^2(k)\psi(t). \tag{14}$$

Far from the barrier, i.e. at $t \to \pm\infty$ one has $V(\pm\infty) = 0$ and general solution of the Schrödinger equation at $t \to \pm\infty$ is the superposition of the left and right moving waves

$$\begin{aligned} \psi^+(t) &= S_R^+ e^{-i\omega(k)t} + S_L^+ e^{+i\omega(k)t}, && t \to +\infty \\ \psi(t) &= S_R^- e^{-i\omega(k)t} + S_L^- e^{+i\omega(k)t}, && t \to -\infty \end{aligned} \tag{15}$$

Due to the unitarity the total fluxes at $t \to \pm\infty$ must be equal $|S_L^-|^2 - |S_R^+|^2 = |S_L^-|^2 - |S_R^-|^2$ and one can find

$$\begin{aligned} S_R^+ &= \cosh r S_L^- - e^{2i\theta}\sinh r S_L^- \\ S_L^+ &= -e^{-2i\theta}\sinh r S_R^- + \cosh r S_L^- \end{aligned} \tag{16}$$

where θ is the scattering phase and factor r is defined by the probability of the transition through the barrier.

Let us remember that we are starting from the vacuum at $t \to -\infty$, i.e. from $B = \omega/2$, so one must have $S_R^- = 0$. This means that at the left (large negative t) we have only left moving outcoming wave $S_L^- e^{+i\omega(k)t}$. At the right (large positive t) one has both left and right moving waves, i.e. the incoming $S_L^+ e^{+i\omega(k)t}$ and reflected $S_R^+ e^{-i\omega(k)t}$ waves. The transition coefficient can be obtained from (16) by puting $S_R^- = 0$

$$\frac{|S_L^-|^2}{|S_L^+|^2} = \frac{1}{\cosh^2 r}. \tag{17}$$

Now let us calculate $B(t) = -(i/2)(\dot\psi/\psi)$ at large positive t. Using (15) one can find after simple calculations (restoring the the k dependence) :

$$B(k) = \frac{\omega(k)}{2}\frac{\cosh r(k) + \exp[2i(\theta - \omega(k)t)]\sinh r(k)}{\cosh r(k) - \exp[2i(\theta - \omega(k)t)]\sinh r(k)} \tag{18}$$

in complete agreement with (11), with the phase factor $\phi(t)$ depending on time as $\phi = \theta - \omega(k)t$.

The squeezing parameter $r(k)$ depends on the absolute value k of the mode spatial momentum **k** and is determined by the probability of tunneling through the potential barrier $V(t) = \lambda[f_\pi^2 - <\phi^2>(t)]$. Let us note that tunneling takes place precisely when $\omega^2(k) - V(t) < 0$, i.e. when the classical long wavelength modes are exponentially amplified and we see once more that squeezing is ultimately connected with the exponential growth of the classical long wavelength modes and the squuezing for each mode k is determined by the function $<\phi^2>(t)$ - this is the only input information we must know to calculate the DCC wave function.

In the quasiclassical approximation it is easy to calculate the squeezing parameter $r(k)$:

$$r(k) = 2\mathrm{Re}\int dt\sqrt{\lambda\left[v^2 - <\phi^2>(t)\right] - k^2}, \tag{19}$$

which is valid for small k when $r(k) >> 1$ The average number of particles in each mode is $< N_k >= \sinh^2 r(k)$ and we see that $< N_k >$ sharply decreases with the increase of k. To make some rough estimate let us consider the simple model for $< \phi^2 > (t)$ assuming that $< \phi^2 > (t) = 0$ in interval $t \in (0, \tau)$ and equal to its usual v.e.v. $< \phi^2 > (t) = f_\pi^2$ outside this interval. Such a behavior of $< \phi^2 > (t)$ is a very rough approximation to a more realistic behavior obtained in [5] in numerical experiment, nevertheless one can use it for some preliminary estimates. The transmission coefficient in this case is wellknown (see for example [22]) and in our notation takes the form

$$\frac{1}{\cosh^2 r(k)} = \frac{4\omega^2(k)(m_\sigma^2/2 - \omega^2(k))}{4\omega^2(k)(m_\sigma^2/2 - \omega^2(k)) + (m_\sigma^2/2)^2 \sinh^2 \tau\sqrt{m_\sigma^2/2 - \omega^2(k)}}, \tag{20}$$

where we used relation $m_\sigma^2 = 2\lambda f_\pi^2$. Then the average number of particles with momentum k is

$$< N(k) >= \sinh^2 r(k) = \left(\frac{m_\sigma^2}{2}\right)^2 \frac{\sinh^2 \tau\sqrt{m_\sigma^2/2 - \omega^2(k)}}{4\omega^2(k)(m_\sigma^2/2 - \omega^2(k))}. \tag{21}$$

One can estimate the average number of particles $< N(k) >$ for different k. For τ we shall use an estimate $\tau = 3 - 6\ m_\sigma^{-1}$ which comes from the results [5] $\tau = 1 - 2\ (200\mathrm{MeV})^{-1}$. Then for $k = 0$ one gets $< N(0) >\approx 10^2 - 10^3$, at $k = m_\pi$ the $< N(m_\pi) >$ is approximately twice smaller but at $k = 3m_\pi \approx 400\,\mathrm{MeV}$ it is by two orders of magnitude smaller $< N(3m_\pi) >\approx 1 - 10$. Of course this is the very crude estimate, however it demonstrates the qualitative features of the phenomenon - the sharp exponential dependence on k and large amplification factor in τ which is of order of $m_\sigma\tau$, where τ is the characteristic time of damping of $< \phi^2 > (t)$ oscillations.

In modelling a realistic "Baked Alaska" scenario one should include the effect of the expansion. In [5] it was suggested to describe it as inclusion of the term $\dot{a}\pi/a$ in the equations of motion, where $a(t)$ is a scale factor for the expanding plasma. It means that we are considering the problem in the space-time with metric $ds^2 = dt^2 - a^2(t)dx^2$. It is easy to show that choosing the new time $\tilde{t}(t) = \int dt/a^3(t)$ one gets the same Schrödinger equation as in (7) but with new $\tilde{\Omega}^2(k,\tilde{t}) = a^6(\tilde{t})\Omega^2(k,\tilde{t})$. Thus taking into account the expansion one gets a squeezed state again but with parameters which depend on the details of an expansion - the scale factor $a(t)$.

Another interesting feature is the phenomenon of bunching and super-Poissonian statistics [18], [20]. One may consider the second-order correlation function

$$g^2(t) = \frac{< N(t)N(0) >}{< N >^2} \tag{22}$$

which gives us the relative probability to measure two particles in an interval t and $g^2(0)$ measures the probability of simultaneous detection. For coherent state one has $g^2(t) = 1$ which means that for coherent state the detection events are not statistically dependent. For squeezed state with large r one gets[2)] [18] $g^2(0) = 2+\coth^2 r > 3$ and $g^2(0)$ is increasing with increasing of k. The experimental measurement of $g^2(0)$ for pions with different k will be extremely interesting.

Let us note that the picture we have considered here may have applications not only to DCC but to the gluon condensate too. In a recent paper [23] it was shown that a high frequency standing wave in $SU(2)$ gauge theory is unstable against decay into long wavelength modes which gives the mechanism for energy transfer from initial high momentum modes to final states with low momentum excitations. The obtained picture is similar to the amplification of the pion soft modes and it is possible to obtain ther same squeezed description of the final state for the soft gluon modes. It is interesting that because the squeezed final state is quadratic in field one can get gluon condensate $< G^2_{\mu\nu} >$.

In conclusion we would like to stress that in the mean field (Hartree-Fock) approximation the wave function of DCC is the product of the the squeezed state wave functions of all three pions

$$|\Psi >_{DCC} = \prod_{i=1}^{3} [\prod_{\mathbf{k}} S(r(\mathbf{k}), \phi(\mathbf{k}))|0>_i], \tag{23}$$

where the universal functions $r(\mathbf{k})$ and $\phi(\mathbf{k})$ are completely determined (in the mean field approximation) by the only function $< \phi^2 > (t)$. This wave function leads to some interesting predictions about k^2 dependence of the observables which will be interesting to check in experiment. One of the most important problem is to find a way of analytical calculation of $< \phi^2 > (t)$ which now is known only from numerical experiments. It is also interesting to go beyond the Hartree-Fock approximation and get the effects of correlations between different pion modes.

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