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ON THE DISTRIBUTION OF NEUTRAL AND CHARGED PIONS THROUGH THE PRODUCTION OF A CLASSICAL PION FIELD

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High energy reactions may produce a state around the collision point that is best described by a classical pion field. Such a field might be an isospin rotated vacuum of the chiral σ-model or, as discussed in this work, a solution of the equations of motion resulting from the coupling of fields of this model to quarks produced in the collision. In such configurations all directions in isospin space are allowed leading to a sizable probability of events with, essentially, only charged particles (Centauros) or all neutral particles (anti-Centauros). (In more common statistical models of multiparticle production, the probability of such events is suppressed exponentially by the total multiplicity.) We find that the isospin violation due to the mass difference of the up and down quarks has a significant effect on these distributions and enhances the production of events consisting predominantly of neutral particles.

1. In recent years several authors [1]–[9] have suggested that the celebrated Centauro events [10], in which no π^0's have been observed versus a large number of charged hadrons, might be explained by the production at these high energies of a classical pion field; an interesting example is the "disoriented chiral condensate" [6, 7]. The idea is that such a process, considered event-by-event, would correspond to the field being along a given Cartesian isospin direction. In events where the isospin is oriented (almost) parallel to the 3-rd axis one would expect mainly neutral pions while in events where the isospin lies in the perpendicular plane predominantly charged pions would be produced. Let (π_1, π_2, π_3) be the three Cartesian isotopic amplitudes of the classical pion field. As all the orientations are equivalent, the distribution in the amplitude π_3 is

$$dw \sim d\pi_3; \qquad \pi^2 = \pi_1^2 + \pi_2^2 + \pi_3^2 = \text{const}\,. \tag{1}$$

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The number of neutral pions, n_0, is proportional to π_3^2 while the total number of produced pions, $n = n_o + n_+ + n_- \sim \pi^2$. With $f = n_0/n$, the fraction of neutral pions, one has from (1),

$$dw = \frac{df}{2\sqrt{f}}\,; \tag{2}$$

this distribution is normalized to unity.

Obviously (2) predicts many more events with a small number of neutrals than do usual statistical mechanisms for pion production. In the latter case one expects dw/df to peak at $f = 1/3$ ($n_+ = n_- = n_0 = 1/3$ as $n \to \infty$)) and to decrease exponentially with n as f deviates from this value. The distribution (2) corresponds to the limit $n \to \infty$ and gives for the relative number of events with the fraction of neutrals less than f

$$P(f) = \int_0^f \frac{dw}{df'}\ df' = \sqrt{f}\,. \tag{3}$$

For a typical Centauro event $f \sim 1/100$ and $P \sim 10\%$. This seems to be a reasonable number as the five "classic" Centauros represent about 1% of events with appropriate energies [10].

At the other end of the spectrum, near $f = 1$, the probability of an event having an anomalously large fraction of π^0's is

$$1 - P(f) = 1 - \sqrt{f} \sim \frac{1}{2}(1 - f)\,. \tag{4}$$

We do not have the square root enhancement exhibited in (3) and instead we find a linear dependence at the end of the spectrum; however, there still is a finite probability of finding events with a large number of π^0's. It is possible that such "anti-Centauro" events have been observed [11] and we shall present a mechanism for enhancing their probability over that of (4).

The distribution (2) results from exact isospin symmetry. At the quark level this symmetry is rather strongly violated due to the up-down quark mass difference, $m_u \neq m_d$. In this Letter we shall demonstrate that this mass inequality can enhance the probability of anti-Centauros.

2. A class of solutions for the pion field whose dynamics are governed by a non-linear chiral Lagrangian was presented in Ref. [3]. The results of that work may be understood in the following simple way. The Lagrangian is

$$\mathcal{L} = \frac{f_\pi^2}{2}\mathrm{Tr}\ (\partial_\mu U \partial^\mu U^\dagger)\,, \tag{5}$$

where $f_\pi = 93$ MeV and the unitary matrix U is connected to the pion fields by

$$U = \exp\left(\frac{i\tau \cdot \pi}{f_\pi}\right)\,. \tag{6}$$

For the particular form

$$U = \exp[i\tau_3\,\theta(\mathbf{r}, t)] \tag{7}$$

the Lagrangian (5) leads to the free equation of motion

$$\partial^2\theta = 0\,. \tag{8}$$

For constant unitary matrices V_L and V_R a generalization of (7) is

$$U = V_L^\dagger \exp(i\tau_3\,\theta)V_R\,; \tag{9}$$

this is a general class of solutions which has been studied in Ref. [5]. All other known solutions [5, 12] are particular cases of (9).

At large distances from the collision point we require the normal structure of the vacuum, i. e. $U = 1$. Likewise we will pick solutions in which $\theta(\mathbf{r}, t) \to 0$ as $r \to \infty$. This forces $V_L = V_R$ and the solutions (9) reduce to isotopic rotations of (7). In other words, (9) takes the form

$$U = \exp[i\boldsymbol{\tau}\cdot\boldsymbol{n}\,\theta(x)] \tag{10}$$

for some direction $\boldsymbol{n}$ in isotopic spin space. A possible scenario for the production of a classical pion field discussed in [6, 7] is that inside a certain volume around the collision point a state corresponding to a constant (in the volume) θ is produced. This state is degenerate with the normal vacuum (in the limit $m_\pi = 0$) but is rotated with respect to it in isotopic spin space. In [6, 7] this situation is referred to as "disoriented chiral condensate". It follows from (10) that any solution of (8) describes chiral dynamics.

We now introduce interactions of pions with quarks keeping in mind that the pion field is the chiral phase of the quark field [13, 14]. In the presence of pion fields the quark fields should be modified

$$\begin{aligned} q_L(x) &\to \exp[\tfrac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{n}\,\theta(x)]q_L\,, \\ q_R(x) &\to \exp[\tfrac{-i}{2}\boldsymbol{\tau}\cdot\boldsymbol{n}\,\theta(x)]q_R\,. \end{aligned} \tag{11}$$

The quark mass terms give rise to the quark-pion interaction Hamiltonian

$$\mathcal{H} = m_u\bar{u}u + m_d\bar{d}d \to \bar{q}\exp(\tfrac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{n}\,\theta)\,(m_+ + m_-\tau_3)\exp(\tfrac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{n}\,\theta)q + \text{h. c.}\,, \tag{12}$$

where $m_\pm = \frac{1}{2}(m_u \pm m_d)$. For the solution (10)

$$\mathcal{H} = \bar{q}(m_+ + m_-\tau_3)q - (1-\cos\theta)\bar{q}\,(m_+ + m_-n_3\boldsymbol{\tau}\cdot\boldsymbol{n})\,q + \sin\theta\bar{q}i\gamma_5\,(m_+\boldsymbol{\tau}\cdot\boldsymbol{n} + m_-n_3)\,q\,. \tag{13}$$

Due to the quark masses the energy depends on θ and the probability of creation of different θ's will be different. This however will not lead, by itself, to the deviation from the inverse square root law as the three dimensional isotopic space remains isotropic. Quark mass differences do lead to an anisotropic distribution; we would like to know, qualitatively, the form of this deviation. In the normal vacuum (13) accounts for the pion mass term through the existence of the chiral condensate $\langle\bar{q}q\rangle \neq 0$ [14]. From (13) one sees that $m_\pi^2 = -m_+\langle\bar{q}q\rangle/f_\pi^2$, $\pi = f_\pi\theta$. In the presence of hadronic matter and at finite temperatures the changes in the condensate change the effective pion mass. We should also note that use of current quark masses in the above interactions does not imply that there are light quark excitations in the hadronic matter. The strength of their coupling to pions is, nevertheless, given by the current quark masses. The constituent quark mass, again, depends on the details of the chiral symmetry breaking [14]. Finite values of the other bilinear quark averages, besides providing us with an effective pion mass that depend on the surroundings, give the possibility that θ is non-zeo at the minimum of the potential.

The distributions, in the parameters θ and $\boldsymbol{n}$, of a classical pion field produced in a high energy collision are expected to depend on a production temperature T and have the form

$$dw \sim \int \prod_x d\theta(x) \exp\left(-\frac{1}{T}\int d^3x \mathcal{H}\right) d\boldsymbol{n}\delta(\boldsymbol{n}^2 - 1)\,. \tag{14}$$

A question arises as to what temperature to use and even as to the extent we can consider thermal equilibrium [8]. We assume that in the region of a very high energy collision a disoriented chiral vacuum, in contact with thermalized quark matter [9], is produced. The temperature of the matter is larger than T_c, the temperature of chiral symmetry is restoration [6, 7]. The average of θ over the whole collision region will vanish; however smaller regions may have non-zero values for this average. As time progresses this region grows and the temperature decreases. In Refs. [6, 7, 9] the system is assumed to stay in thermal equilibrium as it cools down through T_c and to hadronize at some $T < T_c$. In Ref. [8] this transition is assumed to take the system out of equilibrium by quenching the high temperature configuration and then letting it evolve by equations of motion at fixed energy (micro-canonical ensemble). For the present discussion it is not important to know whether the quark matter does or does not stay in equilibrium, as long as it is correlated over large regions; this assumption will be needed further on. The probability of finding a distribution of $\theta(x)$ will be assumed to be thermal and determined by the Hamiltonian of (13) and some temperature $T < T_c$. In the rest of the paper we shall take $T = T_c$ and note that this assumption under-estimates the size of the effects we are interested in.

If the quark density in the collision is not too high $\langle\bar{q}q\rangle$ should be set equal to its usual vacuum value. Expanding around $\theta = 0$ (14) becomes

$$dw \sim \int \prod_x d\theta(x) \exp\left(-\frac{m_+|\langle\bar{q}q\rangle|}{2T}\int d^3x\theta^2\right) d\boldsymbol{n}\delta(\boldsymbol{n}^2 - 1)\,. \tag{15}$$

For $T = T_c \sim 140$ MeV [15] and a volume $V \sim 100$ fm^3 the above is $\exp[-4 < \theta^2 >]$; large values of θ will not be excited. However, after the functional θ integration the distribution in isospin directions remains uniform leading immediately to (2).

3. Our critical assumption is that in the high density medium created by such collisions the quark density and other bilinears in $q, \bar{q}$ acquire classical values that may be comparable to or larger than the vacuum chiral condensate $\langle\bar{q}q\rangle \simeq -(250\ \text{MeV})^3$. From the explicit dependence of (13) on n_3 we see that isospin rotation symmetry is broken. We consider two possibilities: either $I(x) = \langle\langle\bar{q}\tau_3 q\rangle\rangle \neq 0$ or $P(x) = \langle\langle\bar{q}i\gamma_5 q\rangle\rangle \neq 0$, in addition to $S(x) = \langle\langle\bar{q}q\rangle\rangle \neq 0$ and are sizable. $\langle\langle\cdots\rangle\rangle$ denotes the averaging over quantum fluctuations and we allow for a smooth (on the microscopic scale) position dependence. The value of $S(x)$ may differ significantly from the vacuum value of $\langle\bar{q}q\rangle$.

We first consider the first case, $I(x) \neq 0$; although it has less interesting consequences it is simpler to analyze. The functional integration over $\theta(x)$ in (14) (in the quadratic approximation) yields

$$dw \sim \frac{1}{\sqrt{|m_+S(x) + m_-I(x)n_3^2|}} dn_3 . \tag{16}$$

For the dependence of the above on n_3 to be significant it is necessary for the second term in the square root to be comparable in magnitude to the first one. This is, however, unlikely as their ratio is (even for $f = n_3^2 = 1$)

$$\frac{m_-I(x)}{m_+S(x)} = \frac{m_u - m_d}{m_u + m_d} \frac{\langle\langle u\bar{u} - d\bar{d}\rangle\rangle}{\langle\langle u\bar{u} + d\bar{d}\rangle\rangle} ; \tag{17}$$

with $m_u - m_d/m_u + m_d \sim -0.3$ and the second factor less than unity the n_3 dependence will be insignificant. We reach the same conclusion if we allow other components of $\bar{q}\tau q$ to acquire some classical value.

The situation is significantly different if we assume that $P(x)$ has a sizable value. Below, we shall return to see whether this is feasible, but first discuss the consequences of this assumption. We are now asked to evaluate

$$dw \sim \int \prod_x d\theta(x) \exp\left\{\frac{1}{T}\int d^3x \exp\left[m_+S(x)(1-\cos\theta) - m_-n_3P(x)\sin\theta\right]\right\} dn\delta(n^2-1) . \tag{18}$$

The exponent has a minimum for a non-zero θ obtained from $\tan\theta = m_-n_3P(x)/m_+S(x)$. The functional integral can be done (again in a quadratic approximation) and, aside from a prefactor, yields

$$dw \sim \exp\frac{1}{T}\int d^3x \left[+\sqrt{m_+^2S^2(x) + m_-^2P^2(x)n_3^2} + m_+S(x)\right] dn_3 . \tag{19}$$

Although we could analyze this result it is simpler to consider the situation where $|m_-P/m_+S| < 1$. Keeping only the first term in the expansion of the square root we obtain (ignoring, in the case $S(x)$ is positive, terms not depending on n_3)

$$dw \sim \exp\left[\frac{1}{2T}\frac{m_-^2}{m_+}\int d^3x \frac{P^2(x)}{|S(x)|}n_3^2\right] dn_3 . \tag{20}$$

Remembering that $f = n_3^2$ we find

$$dw = N(A)\, e^{Af} \frac{df}{2\sqrt{f}} , \tag{21}$$

where

$$A = \frac{1}{2T}\frac{m_-^2}{m_+}\int d^3x \frac{P^2(x)}{|S(x)|} , \tag{22}$$

and the normalization factor

$$N^{-1}(A) = \int_0^1 dx e^{Ax^2} . \tag{23}$$

We used the probability of creation of given θ to be proportional to the Boltzmann factor, (14). Equilibrium or not quite equilibrium, we believe that, in average, this it is not far from the truth; we then immediately arrive at (21) as a parametrization. Evidently the change in the distribution is important only if A is large enough. However, as we shall see A is proportional to the collision volume and we will argue that it cannot be small. In this case the distribution (21) has a minimum at $f = 1/2A$ and, contrary to the situation described by (2), grows as f approaches 1. For $A >> 1$ an approximate evaluation of (23) yields

$$dw \simeq Ae^{-(1-f)A} \frac{df}{\sqrt{f}} . \tag{24}$$

This distribution has a peak at $f = 1$ and is enhanced near that value by a factor $2A$ over that of (2) making anti-Centauros more probable.

We shall now try to estimate possible values for A. Note that (22) depends on the absolute value of $S(x)$ and on the square of $P(x)$; thus, all spatial regions will add to the value of A. In a region of significant nuclear density $S(x)$ will differ from its vacuum value and will be related to the sum of the quark and anti-quark densities. We would like to compare it to $\rho(x) = \langle \bar{q}\gamma^0 q \rangle$, which is the difference of quark and anti-quark densities. Although the collision region of interest will contain an equal number of quarks and anti-quarks, $\rho(x)$ may be either positive or negative and large over sizable regions (see discussion following (14)). For $|\rho(x)| \geq (250 \text{ MeV})^3$, $S(x)$ will coincide with $\rho(x)$ rather than with its vacuum value. $P(x)$ can be represented as

$$P(x) = \xi R \sigma(x) \cdot \nabla \rho(x) . \tag{25}$$

Here $\sigma(x)$ is some spin density, R is a characteristic linear size of the effective volume (or characteristic time before hadronization) and ξ is a constant, probably smaller than one.

Integrating (22) we get

$$\int d^3x \frac{P^2(x)}{|S(x)|} = 4\pi R^2 r \frac{\xi^2 \rho^2 R^2}{r^2} \frac{1}{\rho} = \frac{4}{3}\pi R^3 \frac{3R}{r} \xi^2 \rho . \tag{26}$$

We use r as a characteristic length for the gradient; this variation in density is likely to be confined to the surface of the quark matter produced in the collision. We assume that the volume over which $P(x)$ does not vanish is $4\pi R^2 r$. The spin densities are averaged approximately to unity. Thus for the parameter A we have:

$$A = \frac{\xi^2}{2T} \; \frac{m_-^2}{m_+} \frac{3R}{r} N \simeq \frac{1}{70} \frac{R}{r} \xi^2 N . \tag{27}$$

Here $N = 4\pi R^3 \rho/3$ is the number of quarks produced. We believe one could expect $N \geq 200$ in a sphere of $R \simeq 3$ fm (note that for the vacuum $\rho = < \bar{q}q > = 2 \text{ fm}^{-3}$, so that $N \simeq 200$). For $R/r \simeq 5$ we find $A \sim 15\xi^2$ and for $\xi \geq 0.25$ A will be sizable enough to enhance the probability of anti-Centauros. Note that $\xi \leq 0.8$ is required for the approximation in going from (19) to (20). We are well aware of the crudeness of these estimates and the purpose of this exercise was only to show that values of $A \geq 1$ are not excluded. As A is proportional to the volume it is unlikely to be very small; we have presented almost a dimensional estimate.

The whole change in the distribution of neutrals is due to the violation of isotopic spin invariance; the parameter A in (21) is proportional to $(m_u - m_d)^2$. Can we claim that the anti-Centauro events are caused by the mass difference of light quarks?

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