Письма в ЖЭТФ, том 59, вып.8, стр.539 - 545 ©1994 г. 25 апреля

NUCLEAR SPIN RELAXATION VIA DIPOLAR INTERACTION IN TWO-DIMENSIONAL ELECTRON GAS

Yu.N.Ovchinnikov [†1)], *I.D.Vagner*[+], *A.M.Dyugaev*[+1)]

[†] *Centre des Recherches sur les Trés Basses Températures, CNRS, BP 166X, 38042 Grenoble, Cedex 9, FRANCE*

[+] *High Magnetic Field Laboratory, Max-Planck-Institut für Festkörperforschung and Centre National de la Recherche Scientifique, BP 166, F-38042 Grenoble, Cedex 9, FRANCE.*

Submitted 23 March, 1994

The nuclear spin-lattice relaxation in the two-dimensional eletron gas under quantizing magnetic fields is considered. We show that contrary to the case of three-dimensional conductors and two-dimensional systems with sufficient amount of disorder, where the hyperfine contact interaction is operative in the relaxation of the nuclear spins, in sufficiently pure two-dimensional systems the dipole-dipole interaction between the nuclear and the conduction electron spins can be the leading mechanism for the nuclear spin relaxation.

Nuclear spin-lattice relaxation in three-dimensional conductors is governed, usually, by the contact (Fermi) hyperfine interaction between the nuclear spins and the spins of conduction electrons [1]. This interaction conserves the total spin of interacting particles. Recently there is a growing experimental [2,3] and theoretical [4-6] activity in studies of the nuclear spin-relaxation in heterojunctions in strong magnetic fields, i.e. under the quantum Hall effect conditions. It was found, that the existence of the energy gaps in the spectrum of the two-dimensional electron gas (2DEG) under strong magnetic fields strongly reduces the effectiveness of the hyperfine contact interaction. This can be understood as follows. In the flip-flop process (simultaneous reversal of a nuclear spin and of an electron spin) the energy needed for crossing the electron Zeeman gap is larger, by a factor $\frac{M_i}{m_0}$ (where M_i is the nuclear mass and m_0 is the free electron mass), than the energy, which can be provided by a nuclear spin. As a result, the alternative relaxation channels may start to be operative.

Here we study the magnetic electron-nuclei interaction (dipolar), which do not conserve the total spin, and is not sensitive, therefore, to the existence of the Zeeman gap in the electron spectrum. In this process the spin momentum of nuclei is converted, as a result of the interaction, to the orbital momentum of electron gas. As we will see below a good agreement with the experiment [2] can be achieved without any adjusting parameter.

Let us consider a model two dimensional electron gas in quantizing magnetic field, interacting with the nuclear spins and phonons. The Hamiltonian $\hat{H}$ of such a system can be written in the form:

$$\hat{H} = \hat{H}_e^0 + \hat{H}_n^0 + \hat{H}_{int} \qquad (1)$$

where $H_{e,n}^0$ - are Hamiltonians of electron and nuclei system in magnetic field, and $\hat{H}_{int}$ is the dipolar interaction between the nuclear and conduction electron

[1)] Permanent address: L.D. Landau Institute for Theoretical Physics, Moscow, RUSSIA.

spins [1]

$$\hat{H}_{int} = \mu_n \mu_e \int d^3 r_e \int d^3 r_n \frac{\Psi_e^+ \Psi_n^+ \hat{S}_n \hat{S}_e \Psi_n \Psi_e}{|\mathbf{r}_e - \mathbf{r}_n|^3} -$$

$$- \frac{3\Psi_e^+ \Psi_n^+ (\hat{S}_n, r_n - r_e)(\hat{S}_e, r_n - r_e)\Psi_n \Psi_e}{|r_e - r_n|^5} - \frac{8\pi}{3}\delta(r_e - r_n)\Psi_e^+ \Psi_n \hat{S}_n \hat{S}_e \Psi_n \Psi_e. \quad (2)$$

Here $\psi_e, \hat{S}_e, \mathbf{r}$ and $\psi_n, \mathbf{S_n}, \mathbf{r_n}$ are the wave functions, spin operators and the position vectors for the interacting electrons and nuclei, respectively.

Interaction with phonons is included in Hamiltonians $H^0_{e,n}$. Magnetic field is assumed along the z-axis. In the Landau gauge, $\mathbf{A} = H_0(0, x, 0)$, the single particle wave function is:

$$\Psi_e(\mathbf{r}) = \frac{1}{\sqrt{L_y}} \exp^{iky} \cdot \psi^n(x) \cdot \chi(z) \quad (3)$$

where L_y is size of the sample in y-direction and:

$$\psi^n(x) = \left(\frac{|eH|}{\pi}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n \Gamma_{(n+1)}}} H_n\left(\frac{x - x_0}{a_H}\right) \exp\left\{\frac{-(x - x_0)^2}{2a_H^2}\right\} \quad (4)$$

here: $x_0 = -a_H^2 k_y$ is the center of the electronic orbit and $a_H = \sqrt{\frac{\hbar c}{eH}}$ is the magnetic length. H_n are the Hermite polinoms, and $\chi(z)$ is Airy function:

$$\chi_{(z)} = C_1 \, A_i((\frac{2\alpha m_e}{\hbar^2})^{\frac{1}{3}}(z - \frac{\tilde{E}_{n'}}{\alpha})). \quad (5)$$

Here the coefficient C_1 is the normalization factor, the coefficient α is defined by the electric field in the heterojunction, confining the electron gas to move in two dimensions:

$$V(z > 0) = \alpha z \quad (6)$$

and the energy levels, $\tilde{E}_{n'}$, in this field, could be found from the following boundary condition:

$$\chi(z = 0) = 0. \quad (7)$$

Since in most practical cases the distance between neighbouring energy levels, $\tilde{E}_{n'}$, is much larger than the temperature and the distance between the Landau levels, we assume, in what follows, that only the ground, size quantized level: $n^1 = 1$ is occupied. In sufficiently clean systems at low temperatures: $\Gamma, k_B T < \hbar\omega_z < \hbar\omega_c; \Gamma$, where $\hbar\omega_z$ and $\hbar\omega_c$ are the electron Zeeman splitting and the distance between Landau levels respectively and Γ is the broadening of a Landau level, the nuclear spin relaxation is connected with the dipolar interaction between the nuclear and the electron spin, as is shown in the inset to Figure.

In the first order of perturbation theory the transition probability, the expression for W (the inverse relaxation time), is :

$$W = 2\frac{(\mu_n \mu_e)^2}{L^2} \mathrm{Im} \sum_{k_i, k_f} \frac{n^i_{e\uparrow}\left(1 - n^f_{e\uparrow}\right)}{2\mu_n H + \epsilon_{if} - i\left(\tilde{\gamma}_i + \tilde{\gamma}_f\right)} \cdot$$

$$\cdot \left| \int d^3 r_e |\chi(z)|^2 \exp^{i(k_i - k_f)y} \psi_{k_i} \psi_{k_f} \frac{(r_e - r_n)_z \left[(r_e - r_n)_x + i(r_e - r_n)_y\right]}{|r_e - r_n|^5} \right|^2 . \quad (8)$$

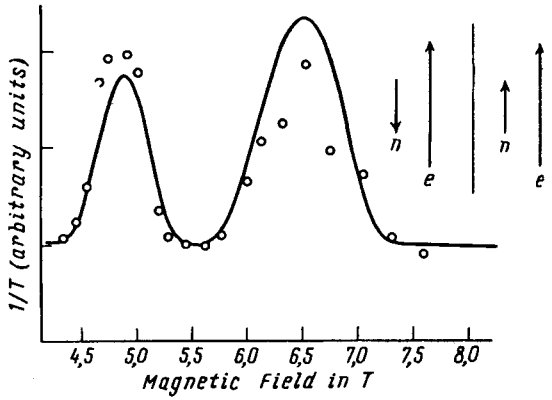

Fig. The magnetic filed dependence of the nuclear spin relaxation rate, T_1^{-1}. The curve presents our calculation, based on the Eq. (25) with the sample parameters corresponding to the experimental data from Ref.[2]. The dots are the experimental data from Ref.[2].
Inset
Relaxation of the nuclear spin via dipole interaction with the electron spin

Here $\gamma_{i,f}$ is the halfwidth of levels in initial (i), correspondently final (f) state. It is equal to the one half of sum of transition probabilities in all other states. In Eq. (8), ϵ_{if} is the difference of energies of an electron in initial and final states. At low temperatures only the transitions inside one, spin-splitted, Landau level are operative in relaxing the nuclear spins. In this case ϵ_{if} is connected with inhomogenous distribution of impurities. It will be estimated below.

The transition probability W, Eq. (8), can be presented in the form:

$$W_N = A_N(\tilde{z}_n^2)\frac{(\mu_n\mu_e)^2}{\hbar}a_H^{-6}n(1-n)^2\frac{(\tilde{\gamma}_i+\tilde{\gamma}_f)}{(2H\mu_n+\epsilon_{if})^2+4(1-n)^2(\tilde{\gamma}_i+\tilde{\gamma}_f)^2}\cdot$$

$$\cdot\left(\int_0^\infty dz|\chi(z)|^2\mathrm{sign}\,(z-z_n)\right)^2. \qquad (9)$$

Here $\tilde{z}_n = \frac{z_n}{a_H}$ is the nondimensional z-coordinate of the interacting nuclei, and the function A_N depends on the number N of filled Landau levels. In Eq. (9) we separate factor (1-n) from levels width and denote:

$$\gamma_{i,f} = (1-n)\tilde{\gamma}_{i,f}. \qquad (10)$$

The expression for functions $A_{1,2}$, (the first two Landau levels) are:

$$A_1(\tilde{z}_n) = \begin{cases} \frac{4}{9}, & \text{if} \quad \tilde{z}_n \ll 1 \\ \frac{1}{12\tilde{z}_n^2}, & \text{if} \quad \tilde{z}_n \gg 1 \end{cases},$$

$$A_2(\tilde{z}_n) = \begin{cases} \frac{4}{3}, & \text{if} \quad \tilde{z}_n \ll 1 \\ \frac{1}{12\tilde{z}_n^2}, & \text{if} \quad \tilde{z}_n \gg 1 \end{cases}. \qquad (11)$$

In order to estimate the value of $\tilde{\gamma}_{if}$, caused by interaction of electron with phonons and impurities, let us calculate the minimal effective Landau level width, needed to fulfill the energy conservation in the dipole-dipole interaction between the nuclear and the electron spins. We assume a model with a gaussian distribution of the electron energy levels within a single Landau level, caused by the elastic scattering with a random impurity potential. The excess electron energy (of the order of the nuclear Zeeman enrgy) is remouved by the long-wave phonons.

Assuming the following inequality:

$$\mathcal{N}_0(da_H^2) \gg 1, \qquad (12)$$

where $\mathcal{N}_0$ is the impurity density and $d = \frac{1}{\sqrt{\alpha m_e}}$ is the localization depth in z-direction, we arrive at the dispersion ϵ for the gaussian distribution of the electron energy levels in the Zeeman branch of a given Landau level:

$$\epsilon = V_a \frac{a^3 \mathcal{N}_0}{a_H \sqrt{d}}. \tag{13}$$

Here a is the interatomic distance in the sample and V_a is the potential of the electron-impurity interaction and Real transitions can take place only between $\{i, f\}$ states, that are spacially separated only on the distances of the order of the magnetic lenght:

$$R \propto a_H$$

Under these conditions, the energy differences ϵ_{if} are much smaller than ϵ. In what follows we assume that the two conditions:

$$\epsilon_{if} \leq \mu_n H \quad \text{and} \quad T > \epsilon \tag{14}$$

are fulfilled.

As was mentioned above, the value of $\gamma_{i,f}$ is connected with different processes. One of them is the transitions into localized impurity states. Such process is accompanied by emission or absorption of a phonon with the energy of order of T. The electron transitions inside the Zeeman branch of a given Landau level are accompanied by emission of real phonon with energy $2\mu_n H$. Since the nuclear Zeeman energy is very small, the wave vector of the corresponding phonons are large, and the deformation potential is not effective.

Consider, now, two different two-dimensional electron systems: heterojunctions and the silicon MOSFET's. In GaAs, at low temperatures, the main contribution to the electron-phonon interaction is the piezoelectric one [7]. In silicon MOSFET, where the quantum Hall effect was initially observed, the two-dimensional electron gas is subjected to external electrical field of the gate voltage, which may strongly modify the electron-phonon interaction. In both cases the electron-phonon interaction is wave vector independent, as can be seen from the following Hamiltonian:

$$\hat{H}_{int} = \frac{g_1}{\sqrt{V \rho \omega_k}} \sum_k \sum_\alpha \int d^3 r \psi^\dagger \psi f^\alpha(\theta_k) \left(b_{k,\alpha} e^{ikr} + b^\dagger_{k,\alpha} e^{-ikr} \right), \tag{15}$$

where θ_k is the angle betwen k and the normal n to the surface, function depends on type of phonon and is equal to $(\alpha = 1, 2, 3)$,

$$f(\theta) = \begin{cases} \cos\theta_k, & \text{for} \quad \text{LP} \\ \sin\theta_k, & \text{for} \quad \text{TP with polarization in (k, n) plane.} \\ 0 & \text{for} \quad \text{polarization in [k, n] direction} \end{cases} \tag{16}$$

The coupling constant g_1 is equal to

$$g_1 = \frac{eE}{\sqrt{2}} + \text{piezoelectric term}, \tag{17}$$

where eE can be found from Eq. (6) to be $eE = \alpha$. The piezoelectric contribution to g_1 can be found in literature, see [7].

Consider, first, the width γ^{ph}_{imp}, connected with transitions to the impurity states. At low temperatures:

$$T \ll 2\mu_e H, \hbar\omega_c \tag{18}$$

the filling of impurity states is:

$$n_{imp} = \frac{n}{n + (1-n)\exp\left(\frac{\epsilon}{T}\right)}, \tag{19}$$

where energy ϵ is counted from the position of Landau level in a filled zone, n - is the electron filling factor in this zone. We introduce, now, the density $\tilde{\mathcal{N}}$ of the impurity states in the unit volume and energy by means of relation:

$$\tilde{\mathcal{N}}(\epsilon) = \frac{1}{V}\sum_{imp} |\int \psi_{e,imp} d^3r|^2 \theta(\epsilon_{eimp} < \epsilon),$$

$$\mathcal{N}(\epsilon) = \frac{\partial \tilde{N}(\epsilon)}{\partial \epsilon} \tag{20}$$

The sum in Eq. (20) is taken over impurity states in volume V with the energies smaller then E. In the second order of perturbation theory we obtain:

$$2\gamma_{imp}^{ph} = 2\pi g_1^2 \frac{\mathcal{N}}{\rho}(1-n)\int_{-\infty}^{\infty} d\epsilon \int \frac{d^3k}{(2\pi)^3}\sum_{\alpha}\frac{1}{\omega_k^\alpha} f_\alpha^2(\theta)\cdot$$

$$\cdot\left[\delta(\epsilon-\omega_k^\alpha)\frac{1}{e^{\frac{\omega_k^\alpha}{T}}-1} + \delta(\epsilon+\omega_k^\alpha)\frac{1}{1-e^{-\frac{\omega_k^\alpha}{T}}}\right]\frac{\exp(\frac{\epsilon}{T})}{n+(1-n)\exp(\frac{\epsilon}{T})}. \tag{21}$$

Finally after simple calculation we obtain:

$$2\gamma_{imp}^{ph} = \frac{g_1^2\mathcal{N}(1-n)T^2}{3\pi\rho}\left(\frac{2}{s_\perp^3}+\frac{1}{s_\parallel^3}\right)\int_0^{\frac{\mu_e H}{T}} \frac{x}{e^x-1}\left[\frac{1}{(1-n)+ne^{-x}}+\frac{1}{n+(1-n)e^{-x}}\right]dx, \tag{22}$$

where $s_{\perp,\parallel}$ are the velocities of the transverse and the longitudinal sound respectivily. We note, that the contribution of this process to γ is proportional to T^2 and is independent of the value of splitting energy, $2\mu_n$H, in the nuclear spin system.

Let us consider, now, the contribution to γ due to electron transitions inside a given spin-splitted Landau level. In the first order of perturbation theory, using the Hamiltonian, Eq. (15), we obtain :

$$2\gamma_i^{ph} = \frac{2\pi g_1^2}{\rho}\frac{1}{L}\int\frac{d^3k}{(2\pi)^3}\int\frac{dk_f}{2\pi}|\int dxdy e^{i\vec{k}\vec{r}}\psi_{k_i}\psi_{k_f}\exp\{i(k_i-k_f)y\}|^2\cdot$$

$$\cdot\sum_\alpha \frac{1}{\omega_k^\alpha} f_\alpha^2(\theta_k)\delta(2\mu_n H-\omega_k)\frac{1-n}{1-e^{-\frac{\omega_k^\alpha}{T}}}. \tag{23}$$

Since $T \gg 2\mu_n H$, and correspondingly $k \ll \sqrt{eH}$, the Eq. (23) reduces to a simple expression:

$$2\gamma_i^{ph} = \frac{g_1^2 kT(1-n)}{3\pi\hbar\rho}\left(\frac{2}{s_\perp^3}+\frac{1}{s_\parallel^3}\right). \tag{24}$$

Note that the width γ_i^{ph} is proportional to the first power of temperature and the coupling constant g_1 in Eqs. (17) depends on the applied electrical field E.

Since both γ's are small and differ only by temperature dependence, consider only one of them and suppose that second mechanism is the essential one. We

arrive, then, to a quite simple temperature and magnetic field dependence of relaxation rate:

$$W = \beta^N \frac{Tn(1-n)^2 H}{1 + \frac{\alpha T^2 (1-n)^2}{H^2}} \tag{25}$$

where a coefficient α is independent of N, the Landau level number, while β is N-dependend. The relation between them is given, for the first two Landau levels by Eq. (11). In Fig. we compare the experimental data from [2], after substraction of the background values, with the theoretical one for the special values of α and β.

To complete our consideration we present here also the magnetic field dependence of the filling factor n. Assuming that $\tilde{\mathcal{N}}_0$, the number of impurity states for unit area and unit energy, is constant in considered energy region and small compared to $\frac{e}{\hbar c \mu_e}$, we can write the equation for chemical potential μ in the form [8]:

$$n_0 = \frac{H}{\Phi_0} \sum_{n=0}^{\infty} \left[\frac{1}{1 + \exp\left(\frac{\epsilon_n^+ - \mu}{T}\right)} + \frac{1}{1 + \exp\left(\frac{\epsilon_n^- - \mu}{T}\right)} \right] + \\ + \tilde{\mathcal{N}}_0 \int_0^{\infty} \frac{d\epsilon}{1 + \exp\left(\frac{\epsilon - \mu}{T}\right)}, \tag{26}$$

where $\Phi_0 = \frac{2\pi\hbar c}{e}$ is the elementary magnetic flux and

$$\epsilon_n^{\pm} = \pm \mu_e H + \tilde{E}_1 + \hbar\omega_c \left(n + \frac{1}{2}\right). \tag{27}$$

In the temperature region, Eq. (18), the Eq. (26) simplifies:

$$n_0 = \tilde{\mathcal{N}}_0 \mu + \frac{H}{\Phi_0}(N + n) \tag{28}$$

where N is the number of completelly filled Landau levels (including the spin splitting). The chemical potential coincides with the energy of the highest filled energy level (if n is different from to zero). It's easy to see that there exists finite regions in magnetic field, where n=0. Suppose that N_{2k+1} is odd and H is the boundary point. Then the equation (28) reads:

$$n_0 = \frac{H_{2k+1}}{\Phi_0}(2k+1) + \tilde{\mathcal{N}}_0 \epsilon_k^-. \tag{29}$$

When the magnetic field decreses: $H = H_{2k+1} - \delta H$ the chemical potential will increese and the filling factor n on the 2k+2 -th level equals zero till the point:

$$\delta H \leq \delta H_{2k+1}^{cr}, \tag{30}$$

where

$$\delta H_{2k+1}^{cr} = \frac{2\mu_e H_{2k+1}}{\mu_e + \frac{e\hbar}{mc}\left(k + \frac{1}{2}\right) + \frac{(2k+1)}{\Phi_0 \tilde{\mathcal{N}}_0}}. \tag{31}$$

After that the filling factor starts growing untill it reaches unity at the magnetic field value H_{2k+2}, which can be found from the equation:

$$n_0 = \frac{H_{2k+1}}{\Phi_0}(2k+1) + \tilde{\mathcal{N}}_0 \epsilon_k^+. \tag{32}$$

In the region

$$H_{2k+2} - \delta H^{cr}_{2k+2} < H < H_{2k+2}; \tag{33}$$

$$\delta H^{cr}_{2k+2} = \frac{\left(\frac{e}{mc} - 2\mu_e\right) H_{2k+2}}{-\mu_e + \frac{e\hbar}{mc}\left(k + \frac{3}{2}\right) + \frac{(2k+2)}{\Phi_0 \mathcal{N}_0}} \tag{34}$$

the filling factor of the 2k+2 th Landau level is equal to unity and of the next Landau level to zero.

The factor $n(1-n)$, appearing in Eq. (25), results, therefore, in the windows in magnetic field where the dipole-dipole interaction between the nuclear and the electron spins in a two dimensional electron gas under strong magnetic fields is suppresed, see Fig. These are the regions, where the Landau levels are either completely occupied or empty, i.e. the chemical potential lies in the localised states between the Landau levels. Finite temperature will lead to smearing of this picture. The width of this windows depend on the parity of N and roughly decreases with k as k^{-2}. Note that Eqs. (9) and (25) are in a good agreement with experimental data of [2] even without adjusting parameters.

To conclude we have suggested a new mechanism for the nuclear spin relaxation in the quantum Hall effect conditions, based on the dipole-dipole interaction between the nuclear and the electronic spins. This to be contrasted with the previous theoretical work on two-dimensional electron systems with large amount of disorder, [4–5], where the hyperfine contact interaction was considered. We outline that the principle physical difference between these two mechanisms is in the fact that in the contact interaction the total spin should be conserved and the an electron has to flip his spin in order to relax the nuclear spin. Since the nuclear Zeeman splitting is orders of magnitude smaller than the electron Zeeman energy, sufficient amount of disorder is needed in order to generate finite nuclear relaxation times. This mechanism is operative in the integer quantum Hall effect conditions.

By the other hand the conserved quantity in the dipole-dipole interaction is the total (spin plus orbital) momentum, therefore an electron can relax a nuclear spin without changing its own spin state, just by shifting its center of orbit. This mechanism, therefore, should be prevailing in clean samples, i.e. under the fractional quantum Hall effect conditions.

We acknowledge very fruitfull discussions with Yu. A. Bychkov, G. Kventsel, Y. Levinson and T. Maniv. I.D.V. acknowledges the support of the GIF, German Israely Foundation for Scientific Research and developement, No. I 0222-136.07/91.

---

1. J.Winter, Magnetic resonance in metals, Oxford, 1971.
2. A.Berg, M.Dobers, R.R.Gerhardts and K. von Klitzing, Phys. Rev. Lett. **64**, 2563 (1990).
3. B.E.Kane, L.N.Pfeiffer and K.W. West, Phys. Rev. **B46**, 7264 (1992).
4. I.D.Vagner and T.Maniv, Phys. Rev. Lett. **61**, 1400 (1988).
5. D.Antoniou and A.MacDonald, Phys. Rev. **B43**, 11686 (1991).
6. S.V.Iordanskii, S.V.Meshkov and I.D.Vagner, Phys. Rev. **B44**, 6554 (1991).
7. V.F.Gantmacher and I.B.Levinson, Carrier scattering in metals and semiconductors, North-Holand, Amsterdam, 1987.
8. I.D.Vagner and T.Maniv, Phys. Rev. **B32**, 8398 (1985).