Письма в ЖЭТФ, том 59, вып.9, стр.599 - 604 ©1994 г. 10 мая

MAGNETIC FIELD DEPENDENCE OF LOCALIZATION RADIUS ON THE DIELECTRIC SIDE OF METAL-INSULATOR TRANSITIONS

*B. Spivak, Hui Lin Zhao *1), Shechao Feng**

Department of Physics, University of Washington, Seattle, Washington 98195

**Department of Physics, University of California, Los Angeles, California 90024*

Submitted 10 March 1994

Magnetic field dependence of negative magnetoresistance (MR) in variable-range-hopping conduction is discussed. Above four dimensions, the negative MR saturates at large magnetic fields. In dimensions lower than four, the negative MR corresponds to a field-dependent correction to localization length. A qualitative coarse-graining picture is presented for such behaviors.

It is well known that the low field magnetoresistance (MR) on the dielectric side of a metal-insulator transition in the variable-range-hopping (VRH) regime can be negative (see Ref. [1] for a review).

In three dimensions, the weak localization theory, [2] which works well in metals, predicts that the magnetic field induces a correction to the conductivity $\langle\delta\sigma\rangle = \langle\sigma(H)\rangle - \langle\sigma(0)\rangle = \frac{e^2}{\hbar L_H}$. Here L_H is the magnetic length, and brackets $<>$ correspond to averaging over random realizations of scattering potential. The expression for this correction does not contain length scales other than L_H itself. It is connected with the fact that main contribution comes from self-crossing diffusion paths with a characteristic size of loops of order L_H.

In Ref. [3], it was assumed that this correction dominates the magnetoresistance in all metallic region including the vicinity of a metal-insulator transition. This leads to $\sigma(H) \sim \frac{e^2}{\hbar\xi(0)} - A\frac{e^2}{\hbar L_H}$, where A is a coefficient about which nothing is known. This equation is equivalent to a change of correlation radius $\delta\xi = \xi(H) - \xi(0) \sim -\frac{\xi_0^2}{L_H}$. This behavior was interpreted in Ref. [3] as a shift of mobility edge due to magnetic fields.

In the spirit of scaling theory of metal-insulator transition, the behavior of the localization radius below the transition and the correlation radius above the transition should be the same and it is natural to expect that on the dielectric side of the transition, the magnetic field correction to the correlation radius has the same form on both sides of the transition. On the other hand, on the dielectric side of the transition weak localization theory does not work because at $L_H \gg \xi$, the contribution to the conductivity from self-crossing paths of size L_H is exponentially small and can be neglected.

A qualitative picture and a simple model for MR in the VRH regime, based on the interference of directed tunneling paths, was proposed in Refs. [4] and [1]. The numerical simulations of MR for relatively small hopping lengths in the framework of this model yielded a MR of order of unity. [4,1] Further numerical works in Ref. [5] and Ref. [6] for samples much larger than those in Ref. [4,1] have led to the conclusion that in two dimensions the negative MR corresponds to corrections to localization radius.

1)Present address: Department of Physics, University of Florida, Gainesville, FL 32611.

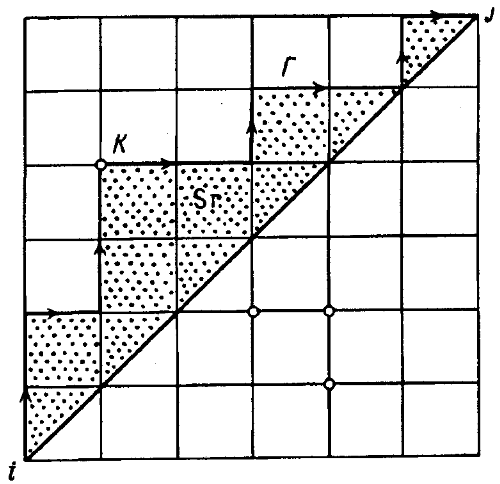

Рис.1

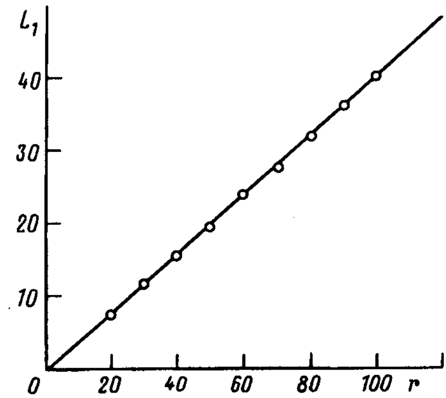

Рис.2

Fig.1. The region of coherent tunneling. Γ is a typical directed path, S_Γ is the area covered by the path

Fig.2. In two dimensions, magnetoconductance L_1 defined by Eq.(4) versus mean hopping distance r

In this paper we show, that in the framework of the model based in the interference of directed paths the MR is universal and only depends on dimensionality. A coarse-grained picture of negative MR is introduced, in which the tunneling area is divided into blocks of the size of the magnetic length, and the global MR is attributed entirely to magnetic field reduction of the variance of tunneling transmission probability of each block. Each plaquette in the coarse-grained lattice represents a block of the size of the magnetic length in the original model. The solution of the coarse-graining model shows that for dimensions $d < 4$ the MR corresponds to a correction to localization length of the same form as H-dependent corrections to correlation radius in metallic regime. This means that the results of the model [4] are also universal in a sence that they do not contain other parameters, but L_H itself and depend only on dimensionality of space. In the cases $d \geq 4$, MR saturates at large H.

We start with a brief discussion of the model. In VRH conduction, electrons hopping between localized states are associated with absorption or emission of phonons. At low temperatures, the typical electron hopping distance r is much larger than the localization length and the average distance between impurity atoms. As a result, in the course of hopping between localized states, a hopping electron undergoes multiple elastic scatterings with impurities. The lattice model proposed in Ref. [4] (see Fig. 1) takes into account the interference of different tunneling paths between initial and final sites "i" and "j". Tunneling paths containing returns and loops are neglected because of their exponentially small contribution to the total probability of a hop. A typical path is shown in Fig. 1 for a two dimensional lattice. Generalization to higher dimensions is straightforward. Let Γ denote a direct path in the figure. The tunneling amplitude associated with Γ is given by

$$A_\Gamma = \exp(i2\pi\phi_\Gamma/\phi_0)\prod_{k\in\Gamma}\alpha_k, \tag{1}$$

where α_k's are independent random variables associated with each site, representing the random scattering amplitudes of impurities encountered by the electron in the course of tunneling. The index k in Eq. (1) runs over sites belonging to the path Γ, ϕ_Γ is the magnetic flux through the shaded area enclosed by Γ and the diagonal, and $\phi_0 = hc/e$ is the flux quantum. The hopping probability is

$$\sigma_{ij}(H) = |A|^2 = |\sum_\Gamma A_\Gamma|^2. \tag{2}$$

Eq. (1) and Eq. (2) can be derived directly from the Anderson model, using the locator expansions [1]. By percolation theory, the macroscopic conductance can be obtained by performing a logarithmic average of Eq. (2) over impurity ensembles [1,7]. We define the magnetoconductance (MC) as,

$$L = \left\langle \ln\frac{\sigma_{ij}(H)}{\sigma_{ij}(0)}\right\rangle. \tag{3}$$

The directed path model can also be used to describe the insulating regime near metal-insulator transition, where the localization length ξ is much larger than the length of individual impurity state wave functions. The idea is to divide the region of tunneling into "blobs" of size ξ and assume that it is possible for tunneling paths to take backward steps within a given blob but not between blobs [1]. In this analog, each site in Fig. 1 should then be understood as a blob. Results we shall present are valid when the magnetic length is larger than the localization length. In Ref. [8], this picture was also extended to superconductor-insulator transitions.

It was shown in Ref. [1] that if the random variable α_k in Eq. (1) has a large enough probability to take negative values, the total tunneling amplitude $A = \sum_\Gamma A_\Gamma$ will have a random sign at $H = 0$ for large enough tunneling distances. We can introduce a sign persistence length r_s beyond which the tunneling amplitude A becomes random in sign. Then one can coarse-grain the lattice so that each site of the new lattice represents a block of size r_s. This corresponds to a renormalized α_k with random signs (i.e., $\langle\alpha_k\rangle = 0$) in the new coarse-grained lattice. In the following, our discussion will be focused on such coarse-grained lattices.

The qualitative picture of negative MR proposed in Ref. [1] is the following. If α_k's are of random signs, then $\langle|A|^2\rangle = \langle|\sum_\Gamma A_\Gamma|^2\rangle$ does not depend on H while all higher moments decrease with H because the variance of a sum of real numbers with random sign is *larger* than the variance of a sum of the corresponding complex numbers which are generated from the random real numbers by multiplying random phases. As a result, the magnetoconductance defined in Eq. (3) is an increasing function of H. It is of order of unity when the magnetic phases in Eq. 1 for typical A_Γ's become of order π. It was proposed in Ref. [1] that this happens when $H \geq H_c = \phi_0/(r^3\xi)^{1/2}$, here $(r^3\xi)^{1/2}$ is the area covered by typical directed paths.

In the following, we shall show that at larger fields the MR at $d < 4$ is large and corresponds to a correction to localization radius.

Consider a two dimensional lattice, let us first divide the sample of size r into subsquares of size L_H. This guarantees that the phase of the tunneling amplitude A_Γ for a typical path Γ through each subsquare is of order π. It is also evident that the width of distribution function of the transmission probability through subsquares is reduced by a factor of order one compared to the corresponding zero field value, while the average of the transmission probability remains the same. For the reason we have discussed, this leads to an increase of the logarithmic average of the transmission probability for each subsquare. However, whether or not this leads to an increase of the everall $\langle \ln |A|^2 \rangle$ is not clear. We check this by a coarse-graining model described below.

According to the arguement above, after dividing a sample into blocks of size L_H, the only effect of magnetic fields is to reduce the variance of the transmission probability of each block. If we coarse-grain the lattice such that each block is reduced to an elementary plaquette as in Fig. 1, an interesting question arises: if we forget about the magnetic field and only consider a change of $Var\{|\alpha_k|^2\}$ while keeping $\langle \alpha_k \rangle = 0$ and $\langle |\alpha_k|^2 \rangle$ the same, how will the change of $\langle \ln |A|^2 \rangle$ depend on r? In other words, we can mimic the effect of applying a magnetic field by a change in the variance of $|\alpha_k|^2$. In the following, we will consider a quantity analogous to L in Eq. (3) for a bond model,

$$L_1 = \left\langle \ln \left| \frac{A\{\alpha_k^{(1)}\}}{A\{\alpha_k^{(2)}\}} \right|^2 \right\rangle , \tag{4}$$

where $\{\alpha_k^{(1)}\}$, $\{\alpha_k^{(2)}\}$ have different distribution functions satisfying the constraints stated above.

For $d = 1$, the answer is obvious: $L_1 = \beta r$, where β depends on the difference

$$\delta = Var\{|\alpha_k^{(2)}|^2\} - Var\{|\alpha_k^{(1)}|^2\} \ .$$

Above four dimensions, L_1 does not depend on r asymptotically. This can be proved exactly because the distribution function of A can be calculated from its moments. In calculating moments $\langle A^m \rangle = \langle \sum_{\Gamma,\Gamma',\Gamma''} A_\Gamma A_{\Gamma'} A_{\Gamma''} \ldots \rangle$, because of the sign randomness of α_k, only even moments survive the average and paths in the summation should be "paired", i.e., any site on the lattice should be visited by an even number of paths. The crucial point is that above four dimensions any two typical paired paths do not, statistically speaking, intersect. In another word, paired paths propagate independently. In the absence of magnetic fields, a simple combinatorial counting gives $\langle A^{2m} \rangle = (2m-1)!!\,(z\langle \alpha^2 \rangle)^{nm}$, where z is the number of neighbors in the forward direction, n is the number of steps on the lattice from site i to j, which is proportional to hopping length. This leads to a Gaussian distribution of A with a zero average and a variance equal to $(z\langle \alpha^2 \rangle)^n$. Under the conditions imposed on calculating L_1, it should vanish. Correspondingly, in the framework of the model of Ref. [4], there is no corrections to the localization length due to magnetic fields and MC saturates to a number of order unity above four dimensions.

This behavior is different from $d = 1$, and $d = 2, 3$ (discussed below) where MC increases linearly with the hopping length which corresponds to a correction to localization length. The relation between the saturation of MR and the assumption about the independence of tunneling paths has been discussed in Ref. [9] in two dimensions where the approximation is not valid. Only above four dimensions, where two diffusive lines do not intersect, the independent paths approximation is valid.

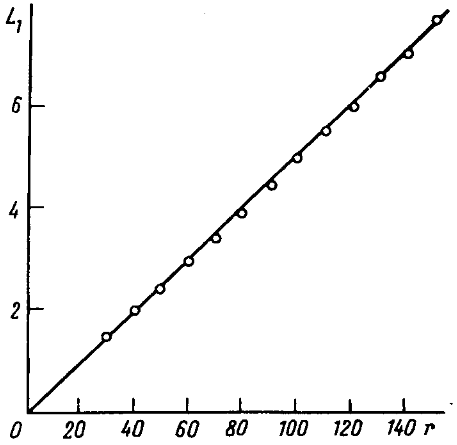

Fig.3. In three dimensions, magnetoconductance L_1 defined by Eq.(4) versus mean hopping distance r

Figs. 2 and Fig. 3 shows the results of numerical simulations of L_1 for $d = 2, 3$. We chose that $\delta \approx 2$. Results in these figures clearly show that $L_1 \propto r$, which, according to our coarse-graining model, means that the MR scales with the number of blocks of size L_H over the length r,

$$L = \left\langle \ln \frac{\sigma_{ij}(H)}{\sigma_{ij}(0)} \right\rangle \sim \frac{r}{L_H}. \tag{5}$$

This corresponds to a field correction to the localization radius

$$\delta\xi = \xi(H) - \xi(0) \sim \xi^2/L_H. \tag{6}$$

Here we deliberately use the same symbol for localization length on dielectric side of the transition as we have used for correlation radius on the metallic side.

Thus we arrive at the following conclusion: within the framework of the model Eq. (3), there are magnetic field corrections to localization length in all dimensions less than four. This originates from the fact that in considering MR, one can divide a sample into blocks of size L_H such that a magnetic field will reduce the variance of transmission probability of each block by a factor of order one. The field effect on each block will be magnified into a correction to localization length (Eq. (5) and Eq. (6)) in dimensions lower than four. It is in agreement with the concept of a shift of mobility edge by magnetic fields.

Completely different situation arises in the presence of spin-orbit scatterings. In this case, the magnetic field correction to the localization radius originates from the model of Ref. [4] is positive and analytic in H when $L_H \gg \xi$. On the metallic side of the transition, however, the correction to the correlation radius has the same non-analytical form as before but with opposite sign, i.e., $\delta\xi \sim \xi_0/L_H$.

Many experimental groups have measured magnetoresistance in hopping conductivity for almost twenty years. Not all of them (especially in three dimensional samples) observed negative magnetoresistance. The reason of this is not clear at the moment. One of the explanation is that the existence of paramagnetic spins suppresses the orbital interference effect (see corresponding discussions in Ref. [1,6] Following to this point of view, experiments showing negative magnetoresistance may correspond to the absence of unpaired spin. In most experiments where negative magnetoresistance in VRH was observed it was less or of the order of unity. [10] These experiments correspond to relatively small hopping lengths, where calculations performed in Ref. [1] are relevant. However, there are more recent experiments showing that the measured magnetoresistance can be much larger than unity [11,12]. In these cases, the model we presented is more suitable to explain the experimental data.

We are grateful to B. Shklovskii for useful discussions. Work in UCLA is supported in part by the ONR under grant number N00014-90-J-1829, the DOE under grant number DE-FG03-88ER45378, and the Alfred P. Sloan Foundation.

---

1. B.I.Shklovskii and B.Z.Spivak, in Hopping Transport in Solids, edit. by M. Pollak and B.I.Shklovskii (Elsevier, 1991).
2. B.L.Altshuler, D.E.Khmel'nitskii, A.I.Larkin, and P.A.Lee, Phys. Rev. **B22**, 5142 (1980).
3. D.E.Khmel'nitskii and A.I.Larkin, Solid State Commun. **39**, 1069 (1981).
4. V.L.Nguyen, B.Z.Spivak, and B.I.Shklovskii, JETP Lett. **41**, 42 (1985); Sov. Phys. JETP **62**, 1021 (1985); JETP Lett. **43**, 44 (1986).
5. E.Medina, M.Kardar, Y.Shapir, and X.R.Wang, Phys. Rev. Lett. **64**, 1816 (1990).
6. Hui Lin Zhao, B.Z.Spivak, M.P.Gelfand, and S.Feng, Phys. Rev. B44, 10760 (1991).
7. B.I.Shklovskii and A.L.Efros, Electronic Properties of Doped Semiconductors (Springer, 1984).
8. S.Kivelson and B.Z.Spivak, Phys. Rev. **B45**, 10490 (1992).
9. U.Sivan, Entin-Wolman, and Y.Imry, Phys. Rev. Lett. **60**, 1566 (1988).
10. A.O.Orlov, A.K.Savchenko, JETP Lett. **44**, 41 (1986); Z.Ovadyahu, Phys. Rev. B **33**, 6552 (1986); O.Ye, Qiu-Yi, A.Zreniver, F.Koch, K.Ploog, Semicond. Sci. and Tech. **4**, 500 (1989); F.Tremblay, M.Pepper, R.Newbury et al. Phys. Rev. B **40**, 10052 (1989-I).
11. F.P.Milliken and Z. Ovadyahu, Phys. Rev. Lett. **65**, 911 (1990).
12. H.W.Jiang, C.E.Johnson and K.L.Wang, Phys. Rev. **B46**, 12830 (1992).