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GLUON DISTRIBUTION AS FUNCTION OF F_2 AND $dF_2/d\ln Q^2$ AT SMALL x

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The paper presents a formula to extract the gluon distribution from deep inelastic structure function F_2 and its derivative $dF_2/d\ln Q^2$ at small x in the leading order of perturbation theory. The detailed analysis is given for new data of H1 group from HERA. The values of gluon distribution are found at $10^{-3} \leq x \leq 2\cdot 10^{-2}$ and $Q^2 = 20$ GeV^2.

Recently the small-x behaviour of the structure functions (SF) of deep inelastic scattering (DIS) was considered in connection with a possibility to provide experimental studies on new powerful colliders HERA [1] and LEP·LHC [2]. The analysis of SF gives the main information about the behaviour of parton (quarks and gluon) distributions (PD) of nucleon. The knowledge of PD is a basis to study other processes.

Let us introduce the standard parametrizations of singlet quark $s(x,Q^2)$ and gluon $g(x,Q^2)$ PD [2)] (see, for example, [3])

$$\begin{aligned} s(x,Q^2) &= A_s x^{-\delta}(1-x)^{\nu_s}(1+\epsilon_s\sqrt{x}+\gamma_s x) \equiv x^{-\delta}\tilde{s}(x,Q^2), \\ g(x,Q^2) &= A_g x^{-\delta}(1-x)^{\nu_g}(1+\gamma_g x) \equiv x^{-\delta}\tilde{g}(x,Q^2), \end{aligned} \tag{1}$$

with Q^2 dependent parameters in the r.h.s.. We use the similar small-x behaviour for gluon and sea quarks PD that follows from the form of the kernel of Gribov-Lipatov-Altarelli-Parisi (GLAP) equation (see also recent fits of experimental data in [4]).

The "conventional" choice is $\delta = 0$. It leads to nonsingular behaviour of PD (see D'_0 fit in [3]) when $x \to 0$. Another value $\delta \sim \frac{1}{2}$ has been obtained in papers

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[2)] We use PD multiplied by x and neglect the nonsinglet quark distribution at small x.

[5] as the sum of leading powers of $\ln(1/x)$ in all the orders of perturbation theory (PT) (see also D'_- fit in ref.[3]). Recent NMC data [6] agree with small values of δ . This choice corresponds to the present experimental data for pp and $\overline{\rm p}$p total cross-sections (see [7]) and the model of Landshoff and Nachtmann pomeron [8] with exchange of the pair of a nonperturbative gluons, yielding $\delta = 0.086$. However, the new H1 data [9] from HERA, prefers $\delta \sim 0.5$. With help GLAP equation some attempts (see [10]) have been undertaken to obtain an agreement between the results of NMC at small Q^2 and H1 group at large Q^2.

In the present letter we are studying the behaviour of gluon PD at small x using the new H1 data and the method (see [11]) of replacement of Mellin convolution by ordinary products.

1. Assuming the Regge-like behaviour for gluon and singlet quark PD (see eq.(1)), we get the following equation for Q^2 derivative of the SF F_2 [3]:

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = -\frac{\alpha(Q^2)}{2}\delta_s x^{-\delta} \sum_{p=s,g} \left(\tilde{\gamma}^{1+\delta}_{sp}(\alpha)\ \tilde{p}(0,Q^2) + \tilde{\gamma}^{\delta}_{sp}(\alpha)\ x\tilde{p}'(0,Q^2)\right) + O(x^{2-\delta}), \quad (2)$$

where $\tilde{\gamma}^{\eta}_{sp}(\alpha)$ are some combination of the Wilson coefficients and anomalous dimensions of the η "moment" of Wilson operators (i.e., the corresponding variables expanded from integer values of argument to noninteger ones) and

$$\tilde{p}'(0,Q^2) \equiv \frac{d}{dx}\tilde{p}(x,Q^2) \ \text{ at } \ x=0.$$

Here δ_s is the coefficient depending on the process and number of quarks f: $\delta_s = 5/18$ for ep collision and $f=4$.

Further we restrict our consideration to the leading order (LO) of perturbation theory (where $F_2(x,Q^2) \equiv \delta_s s(x,Q^2)$ and the $\tilde{\gamma}^{\eta}_{sp}(\alpha)$ are equal to the LO anomalous dimension γ^{η}_{sp}) and case $\delta = 0.5$ corresponding to Lipatov pomeron that is supported by H1 data. Both: including the case $\delta = 0$ corresponding to standard pomeron into our consideration and the expansion of this analysis to the next-to-leading (NLO) order of perturbation theory, will be done in future.

For the gluon part from r.h.s of eq.(2) with the accuracy of $O(x^2)$ we have the following form:

$$\gamma^{3/2}_{sg}\tilde{g}(x/\xi_{sg},Q^2) \ \text{ with } \ \xi_{sg} = \gamma^{3/2}_{sg}/\gamma^{1/2}_{sg}. \quad (3)$$

For the quark part the similar simple form is absent because the corresponding anomalous dimensions $\gamma^{3/2}_{ss}$ and $\gamma^{1/2}_{sg}$ have the opposite signs. However, with the accuracy of $O(x^2)$ it may be represented as a sum of two terms like eq.(3) with some coefficients and arguments shifts. Choosing the shifts as 1 and $\xi_{sg}{}^{-1}$ we have the following representation for the quark part:

$$c_1\ \tilde{s}(x,Q^2) + c_2\ \tilde{s}(x/\xi_{sg},Q^2),$$

where

$$c_1 = \frac{\gamma^{3/2}_{ss}\gamma^{1/2}_{sg} - \gamma^{1/2}_{ss}\gamma^{3/2}_{sg}}{\gamma^{1/2}_{sg} - \gamma^{3/2}_{sg}} \ \text{ and } \ c_2 = \gamma^{3/2}_{sg}\frac{\gamma^{1/2}_{ss} - \gamma^{3/2}_{ss}}{\gamma^{1/2}_{sg} - \gamma^{3/2}_{sg}}. \quad (4)$$

[3]Hereafter contrary to the standard case we use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

Thus, from eq.s (2)-(4) using the exact values of anomalous dimensions, we get

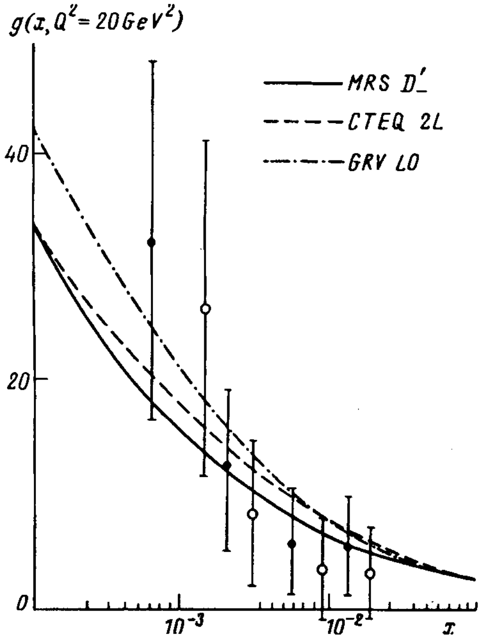
$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = 8\alpha(Q^2)\times$$

$$\times\left[\frac{\sqrt{253}}{30\sqrt{7}}\left(eg(\frac{77}{23}x,Q^2)+\frac{497}{81}F_2(\frac{77}{23}x,Q^2)\right)-\frac{4}{3}(\frac{413}{360}-\ln 2)F_2(x,Q^2)\right]+O(x^{2-\delta}), \quad (5)$$

where $e=\sum_i e_i^2$ is the sum of squares of quark charges. From eq.(4) with the accuracy of $O(x^{2-\delta})$, for gluon PD we obtain:

$$g(x,Q^2) = \frac{0.56}{\alpha(Q^2)}\frac{dF_2(0.3x,Q^2)}{d\ln Q^2} + 2.72F_2(0.3x,Q^2) - 5.52F_2(x,Q^2). \quad (6)$$

2. Let us study the predictions inspired by eq.(6). We use the values of SF F_2 and its Q^2 derivative found by H1 collaboration (see [9] and [12], respectively). The similar analysis has been given by H1 group themselves and presented in [12], where the results of paper [13] were used. Note, that our basic formula (2) coincides with the corresponding one from [13] when we use LO approximation, $\delta = 1$ and neglect the singlet quark contribution. However, since it has been studied in a recent preprint [14], the result from [13], exact for $\delta = 1$, is not quite a good approximation for δ from interval $0 \leq \delta \leq 0.5$, especially at $\delta \sim 0$. Moreover, the addition of the NLO corrections violates Prytz results very much (see [14]).



The gluon distribution $g(x,Q^2)$ at $Q^2 = 20\,\text{GeV}^2$. The white and black circles indicate the values extracted with the help our (see eq.(6)) and Prytz (see [13]) formulae, respectively. Only statistical errors are presented. The curves represent different parametrizations of $g(x,Q^2)$ [3, 15, 16]. The *CTEQ* and *GRV* curves are leading order parametrization, and the *MRS* parametrization is given in the *DIS* renormalization scheme.

We present the extracted gluon PD values into Fig.1 and compare them with [12]. As it was in [12], the hypothesis concerning the approximate linear lnQ^2 dependence of F_2 at small x and the value of QCD scale $\Lambda^{f=4}_{\overline{MS}} = 200\,\text{MeV}^2$, have been used. As one can see in Fig.1, we found the gluon PD values to be very similar to the results in [12]. Some difference in our results and in paper [12], happens due to the singlet quark contribution, which is important for $x \leq 10^{-2}$. Indeed, the singlet quark distribution reduces $g(x, Q^2)$ from several percents at $x \approx 10^{-3}$ to 20% at $x \approx 2 \cdot 10^{-2}$.

Resume. We have presented formula (2) to extract a gluon distribution at small x from SF F_2 and its Q^2 derivative. This formula generalizes the previous one found earlier by Prytz (see [13]) to the case of the arbitrary values of pomeron intercept and includes the singlet quark contribution. Moreover, the addition of NLO contribution into eq.(2) can be done in analogy with paper [11].

The application of eq.(6) to the analysis of H1 data from HERA has been performed. The values of gluon distribution for small x: $10^{-3} \leq x \leq 2 \cdot 10^{-2}$ have been found. The expansion of this analysis for the case $\delta \sim 0$ which is in agreement with NMC data and the evaluation of the NLO contributions will be done in the future.

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