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ROTATION OF A RAY BY A MAGNETIC FIELD

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An experiment is suggested in which the application of a magnetic field H along the light propagation direction causes the rotation $\propto H$ of the output intensity pattern. Thus we approach to the Faraday's claim "to magnetize the ray", i. e. the transversely localized intensity burst, and not only to rotate the polarization direction.

It is generally assumed that the magnetic field applied to the transparent medium influences the state of polarization only, so that the azimuth of the latter is rotated at a rate

$$d\psi/dz = VH\cos\theta. \tag{1}$$

Here V is the Verdet constant, i.e. the constant of the Faraday effect [1], and θ is the angle between the wavevector $\mathbf{k}$ and magnetic field vector $\mathbf{H}$. Some transverse effects were discussed also; among them is ([2], section 101) the deviation of the group velocity vector $\mathbf{v}/v$ from $\mathbf{k}/k$ at an angle $\delta\theta(\mathbf{v},\mathbf{k}) = -\sigma k^{-1}VH\sin\theta$, where σ is the sign of polarization circularity: $\mathbf{E} \propto (\mathbf{e}_x + i\sigma\mathbf{e}_y)\exp(ikz - i\omega t)$. However in a typical case of moderate $HVL \sim 1$ (where L is the propagation distance) those effects are small. For example, if L corresponds to the Fresnel length $L = k{a_0}^2$ of a beam with a finite transverse size a_0, then the transverse shift of a beam $\delta x = L\delta\theta \approx (\sigma H a_0 V)a_0 \ll a_0$, since usually $HVa_0 \ll 1$.

In this letter we suggest an experiment where the longitudinal magnetic field leads to some kind of "rotation"

$$d\varphi_{\text{eff}}/dz \sim VH \tag{2}$$

of the azimuth φ_{eff} of the output intensity pattern. In this connection let's consider an axially symmetric optical wavegiude which possesses only three propagating modes $M_{ij}(x,y)$ in the scalar approximation (see e.g. [3])

$$M_{00} = f(r)e^{i\beta_0 z}, \quad M_{01} = xg(r)e^{i\beta_1 z}, \quad M_{10} = yg(r)e^{i\beta_1 z}, \tag{3}$$

where z is the axis of the fiber, and $x = r\cos\varphi$, $y = r\sin\varphi$. It is just the axial symmetry that guarantees the degeneracy of M_{01}-and M_{10}-modes, i.e. the coincedence of the propagation constants β_{01} and β_{10}. Therefore x and y may be combined into $r\exp(i\varphi)$ and $r\exp(-i\varphi)$. Vector nature of the (almost transverse) electromagnetic field leads to the splitting of those modes into the following six combinations (for $\sigma = \pm 1$), see [3],

$$\begin{aligned}
\mathbf{M}_{00\sigma} &= (\mathbf{e}_x + i\sigma\mathbf{e}_y)f(r)\exp(i\beta_0 z), \\
\mathbf{M}_{0+1+1} &= (\mathbf{e}_x + i\mathbf{e}_y)g(r)re^{i\varphi}\exp(i\beta_1 z + iDz), \\
\mathbf{M}_{0-1-1} &= (\mathbf{e}_x - i\mathbf{e}_y)g(r)re^{-i\varphi}\exp(i\beta_1 z + iDz), \\
\mathbf{M}_{0\text{h}} &= (\mathbf{e}_x\cos\varphi + \mathbf{e}_y\sin\varphi)g(r)r\exp(i\beta_1 z + ihz), \\
\mathbf{M}_{0\text{b}} &= (-\mathbf{e}_x\sin\varphi + \mathbf{e}_y\cos\varphi)g(r)r\exp(i\beta_1 z + ibz).
\end{aligned} \tag{4}$$

The degeneracy of the eigenvalues for $M_{00\pm1}$-modes follows again from the reflection $(x \to -x, y \to +y)$ symmetry of the waveguide which is made of isotropic material; the same is true for the M_{0+1+1}- and M_{0-1-1}-modes. The modes M_{0h} (hedgehog) and M_{0b} (bagel) are shown in the Fig.1. They are generally splitted, that is h generally is not equal to b. The latter two modes may be interpreted as the remnants of an ensemble of meredional rays with all possible values of azimuth φ_0; such a ray for a fixed φ_0 is shown in the Fig.2. At the same time all the other modes more or less correspond to the sagittal rays. Therefore the phase shift at the total internal reflection process is not accumulated for any particular direction of polarization, and the latter may be chosen circular. For more accurate interpretation of the vector mode structure in an optical waveguide see [3].

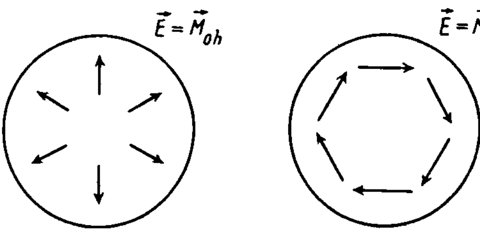

Рис.1

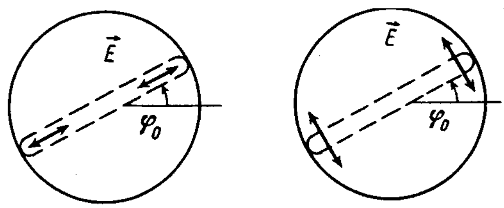

Рис.2

Fig.1. Electric field of a hedgehog mode $\mathbf{E} = \mathbf{M}_{0h}(x, y)$ and bagel mode $\mathbf{E} = \mathbf{M}_{0b}(x, y)$

Fig.2. Interpretation of splitting $\beta_h - \beta_b$ in terms of meredional rays (dashed line) with different polarizations, which acquire slightly different phase shifts at total internal reflection on the core-cladding boundary

Now the application of the longitudinal magnetic field $\mathbf{H} = H\mathbf{e}_z$ does not break the axial symmetry; however it breaks the reflection symmetry $(x \to -x, y \to +y)$. As it is well known (see e.g. crystal optics theory in [1,2]), the σ-circularly polarized modes acquire an addition $\delta\beta = -VH\sigma$ to the propagation constant, while the linearly polarized modes keep their β-values intact. So we suggest the following scheme of experiment. Let us put two $\mathbf{e}_x$-transmitting polarizers both at the input and at the output of our fiber, and let's illuminate its input by a more or less arbitrary field containing some combination of M_{00}, M_{01}, M_{10} modes:

$$\mathbf{E}_{in} = \mathbf{E}(r, \varphi, z = 0) = \mathbf{e}_x \left(C_0 M_{00} + C_x M_{01} + C_y M_{10} \right). \tag{5}$$

The output intensity pattern $I_{\text{out}}(r, \varphi) = |(\mathbf{E}(r, \varphi, z) \cdot \mathbf{e}_x)|^2$ will exhibit some distortion under the application of a moderate $(VHz \sim 1)$ magnetic field.

The main statement of the present letter is that some part of that distortion may be interpreted as a "ray rotation" $\delta\varphi \propto H$. Direct calculation of the intensity profile I_{out} with account of the propagation laws gives very large number of H-dependent terms even in this oversimplified model (three modes only, $\mathbf{e}_x \to \mathbf{e}_x$). It is just their abundance which constituted the main difficulty in the painstaking process of elucidation of the necessary terms, namely the ones which may be qualitatively interpreted as a result of the "pattern rotation". Namely, there are terms

$$\delta I_{\text{out}}(r, \varphi) = \frac{1}{8} r^2 g^2 \text{Re} \left[(C_x - iC_y)(C_x^* - iC_y^*) e^{2i(\varphi - VHz)} \right] +$$

$$+ \frac{1}{2} r f g \cos(VHz) \text{Re} \left[C_0^* e^{i(\beta_1 + D - \beta_0)z} \left[(C_x - iC_y) e^{i(\varphi - VHz)} + (C_x + iC_y) e^{-i(\varphi - VHz)} \right] \right] +$$

$$+\frac{1}{8}r^2g^2\mathrm{Re}\left[(C_x - iC_y)(C_x^* e^{-ihz} - iC_y^* e^{-ibz})e^{iDz}e^{2i(\varphi - VHz/2)}\right] + \cdots.$$

Important consequences are that 1) there is a kind of "rotation" of the output speckle-pattern at an angle $\delta\varphi_{eff} \sim VHz$, 2) the sign of that rotation is changed under the switch of the magnetic field H $\rightarrow$ −H, 3) the sign of the "ray rotation" coinsides with that for the "polarization rotation" due to the Faraday effect in the same medium. Similar results were obtained for the scheme in which input and output polarizers are circular with *the same* sign of circularity. Besides that, the effect also takes place in a really multimode waveguide, but the analytical expressions look unpleasant in that case.

Similarly to Optical Magnus Effect [4], [5], the magnetic "rotation of the ray" is based on the spin-orbit interaction of a photon in an inhomogeneous medium, i. e. on the coupling between propagation and polarization. However, contrary to the Magnus case, here the participation of the hedgehog and bagel modes is essential for the existence of the effect of "ray rotation". We wonder if there are any magnetic effects of that kind in homogeneous medium. (An effect like Magnus one in homegeneous medium was recently suggested [6] and observed [7]).

To conclude, we have predicted the rotation $\delta\varphi \sim VHz$ of the speckle-pattern of intensity at the output of the multimode fiber due to a longitudinal external magnetic field H. Thus we hope to fulfill the claim by M. Faraday "to magnetize the ray".

Just recently the effect was observed in a physical experiment [8].

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