Письма в ЖЭТФ, том 59, вып.11, стр.754 - 759 ©1994 г. 10 июня

POSSIBILITY OF A POWER TEMPERATURE DEPENDENCE OF THE LOW-TEMPERATURE PENETRATION DEPTH IN AN S-WAVE SUPERCONDUCTOR

G. V. Klimovitch

Landau Institute for Theoretical Physics

117334, Moscow, Russia

Submitted 28 January, 1993

A number of experimental measurements of λ in HTSC have revealed a power temperature dependence of $(\lambda(T)-\lambda(0))$ at $T \ll T_c$, which is often considered as an unambiguous evidence of gap nodes, and ascribed to unconventional pairing. However, electron coupling to the low-lying excitations with energy $\lesssim T$ *always* gives rise to a power dependence of $\lambda(T)$ even for an S-wave superconductors without gap zeros. Negligible in conventional superconductors, this effect may be large enough to be observed experimentally in high-T_c compounds. Thus, power dependence of $(\lambda(T)-\lambda(0))$ does not exclude S-coupling. In particular, linear and quadratic temperature dependences of λ may occur. Besides analytical results, a calculational illustration is presented.

The temperature dependence of λ is usually associated with that of the superfluid density n_S: $\lambda^{-2} \sim n_S/m^*$ where m^* is an effective mass of the carriers. On the other hand, in case of electron coupling with low-energy excitations (say, with phonons) m^* also gets temperature-dependent. This may result in power dependence of $\lambda(T)$ even in an s-wave superconductor (see, e.g. [1]), where n_S changes only exponentially at low temperatures. To make this article complete we reiterate some results of [1]. Pure electron-phonon interaction results in a rather weak, $\sim T^5$ dependence [2] which is at least difficult to observe experimentally even in strong-coupling superconductors:

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{T}{\Delta}\left(\frac{T}{\omega_D}\right)^4 \tag{1}$$

where ω_D is the Debay frequency.

One reason for appearance of such a high power of T is that in this particular case thermal excitations — acoustic phonons — transfer small momentums $q \lesssim T/c_s$, and so their effect on transport properties, including London penetration depth, almost cancels. To obtain a more pronounced dependence, for example, quadratic or linear, we phenomenologically assume interaction of electrons with low energy modes which are quasilocal, so that the momentum transfer in scattering on such modes remains large ($\sim p_F$) as the energy transfer is approaching zero. Given the effective spectral density of the modes $g(E)$ (in the case of electron-phonon interaction $g(E)$ is usually referred to as Eliashberg function $\alpha^2F(E)$), we obtain

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{\int_0 g(E)N(E)dE}{\Delta}, \quad N(E) = \frac{1}{e^{E/T}-1} \tag{2}$$

so, for a simple form of g,

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{g(E\sim T)T}{\Delta}. \tag{3}$$

Such excitations might be related, for instance, with disorder [3]. Depending on low energy behavior of g(E), different types of $\Delta\lambda(T)=\lambda(T)-\lambda(0)$ functions may be obtained.

Origin of the power term in $\lambda(T)$

Within the scope of the strong-coupling theory of superconductivity the normal and anomalous parts of the electron self-energy are obtained from the well-known Eliashberg equations [4] ; for our purpose it suffices to consider the case of an isotropic clean one-band S-wave superconductor with the same coupling to the normal and anomalous electron self-energies:

$$[1-Z(i\epsilon_n)]i\epsilon_n = -i\pi T\sum_{\epsilon_m} A(\epsilon_n-\epsilon_m)\frac{\epsilon_m}{\sqrt{\epsilon_m^2+\Delta^2(i\epsilon_m)}}, \tag{4}$$

$$Z(i\epsilon_n)\Delta(i\epsilon_n) = \pi T\sum_{\epsilon_m} A(\epsilon_n-\epsilon_m)\frac{\Delta(i\epsilon_m)}{\sqrt{\epsilon_m^2+\Delta^2(i\epsilon_m)}}, \tag{5}$$

where the contribution of Coulomb interaction is omitted;
$\epsilon_n=(2n+1)\pi T$,

$$A(\omega)=2\int_0^{+\infty} dE\frac{Eg(E)}{E^2+\omega^2}.$$

If the peretration depth $\lambda(T)$ is calculated *without including corrections to the electromagnetic vertex*, than:

$$\lambda^{-2}(T)=\frac{8\pi}{3}\frac{e^2}{c^2}\nu_F v_F'^2\pi T\sum_{\epsilon_m}\frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2+\Delta^2(i\epsilon_m)]^{3/2}}, \tag{6}$$

where ν_F is the spectral density of electrons with a given spin projection in the normal state; we have to introduce the somewhat artificial parameter v_F'—would-be Fermi velocity of the noninteracting electrons; see the discussion of vertex corrections in the following Section.

It may be shown that at $T \ll T_c$ equations (4), 5, 6 may be rewritten with exponential accuracy ($e^{-\Delta/T}$) in the following way:

$$[1-Z(i\epsilon_n)]\epsilon_n = -\frac{1}{2}\int_{-\infty}^{+\infty} d\epsilon' A(\epsilon_n-\epsilon')\frac{\epsilon'}{\sqrt{\epsilon'^2+\Delta^2(i\epsilon')}} -$$

$$-2\pi\int_0^{+\infty} dEg(E)N(E)\mathrm{Re}\left\{\frac{\epsilon_n+iE}{\sqrt{(\epsilon_n+iE)^2+\Delta^2(i\epsilon_n-E)}}\right\}, \tag{7}$$

$$\Delta(i\epsilon_n)Z(i\epsilon_n) = \frac{1}{2}\int_{-\infty}^{+\infty} d\epsilon' A(\epsilon_n-\epsilon')\frac{\Delta(i\epsilon')}{\sqrt{\epsilon'^2+\Delta^2(i\epsilon')}} +$$

$$+2\pi\int_0^{+\infty} dEg(E)N(E)Re\left\{\frac{\Delta(i\epsilon_n-E)}{\sqrt{(\epsilon_n+iE)^2+\Delta^2(i\epsilon_n-E)}}\right\}, \tag{8}$$

$$\lambda^{-2}(T) = \frac{8}{3}\frac{\nu_F v_F'^2}{c^2}\int_0^{+\infty}\frac{\Delta^2(i\epsilon)}{Z(i\epsilon)[\epsilon^2+\Delta^2(i\epsilon)]^{3/2}}d\epsilon, \tag{9}$$

where $N(E) = [e^{E/T}-1]^{-1}$; $\epsilon_n > 0$, $\Delta = \Delta^R$, $Z = Z^R$ are obtained by analytical continuation of $Z(i\epsilon_n)$, $\Delta(i\epsilon_n)$. The idea of derivation of equations (7), (8), (9) was given in [1] It uses the exponential smallness within the gap of the differences between retarded and advanced $Z(\epsilon)$, $\Delta(\epsilon)$ [5]: $(\Delta^R - \Delta^A)$, $(Z^R - Z^A) \sim e^{(|\epsilon|-\Delta)/T}$ at $|\epsilon| < \Delta$

From (7), (8) we obtain:

$$Z(i\varepsilon, T) = Z(i\varepsilon, 0) + 2\pi\frac{\int_0^{+\infty} dEg(E)N(E)}{\sqrt{\varepsilon^2+\Delta^2(i\varepsilon,0)}} + O(T^3 g(T)) \sim g(T)\frac{T}{\Delta}, \tag{10}$$

$$(\Delta(i\epsilon,T)-\Delta(i\epsilon,0)) \sim \frac{\int_0^{+\infty} dEg(E)N(E)E^2}{\Delta^2} \sim \frac{O(g(T)T^3)}{\Delta^2}.$$

Additional smallness ($\sim T^2/\Delta^2$) of change in Δ is a consequence of Anderson theorem, as power terms arise from interaction with low-lying excitations [6]. Generally speaking, such a cancellation in temperature dependence of Δ is model-dependent. It is due to identical coupling of normal and anomalous electron self-energies to the low-energy modes, which is not the case if, for instance, $g(E \lesssim T)$ contains contribution from spin fluctuations , or if Δ is anisotropic [6] and $g(E \lesssim T)$ corresponds to large momentums $\sim p_F$.

In any case, if corrections to the electromagnetic vertex [1] may be neglected in calculations of $\lambda(T)$, then from (9), (10) we get estimate (3). Such correction should be considered if the momentum of thermal bosons q_T is small in comparison with p_F. Then additional cancellations occur:

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{g(E\sim T)T}{\Delta}\left(\frac{q_T}{p_F}\right)^2 \tag{11}$$

(in much the same fashion as they do in the normal state transport properties) For phonons ($g(E) \sim E^2/\omega_D^2$ $q_T/p_F \sim T/\omega_D$) it is the vertex corrections that change the dependence of $\lambda(T)$ from $\sim T^3$ to $\sim T^5$

Another peculiarity of long-wave excitations is that in calculations of *their* effect on the electron self-energy parts the Migdal's theorem gets incorrect for modes with momentum $q \lesssim \Delta/v_F$ As long-wave soft modes slightly affect the penetration depth, hereafter we restrict ourselves to the case of quasilocal thermal modes, then for qualitative estimate (2) holds true. Let us formulate some direct consequences of (2): Firstly, if $g(E) \sim a(E/E_c)^n$ then

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{aT^{n+1}}{E_c^n\Delta}.$$

Secondly, if there is a gap E_0 in the spectrum of the coupling excitation, which is smaller than Δ_0, than the dependence of $(\lambda(T)-\lambda(0))$ is exponential $e^{-E_0/T}$ only at $T \lesssim E_0$. In particular, given a low-energy peak in g: $g = aE_0\delta(E-E_0)$, the dependence of λ changes from exponential to linear aT/Δ at $T \sim E_0$:

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{aE_0}{\Delta}\frac{1}{e^{E_0/T}-1}$$

Figure 1 demonstrates strong effects of low-energy part of $g(E)$ on $\lambda(T)$. λ^{-2} is taken in dimensionless units:

$$\lambda^{-2}(T) = \pi T \sum_{\epsilon_m} \frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2 + \Delta^2(i\epsilon_m)]^{3/2}}.$$

The dashed curve is $\lambda^{-2}(T)$ computed for the spectral density without a low-

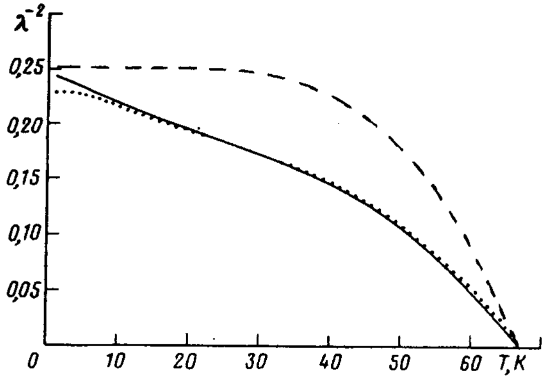

Figure how $\lambda(T)$ in an S-wave superconductor is affected by electron interaction with low-lying bosonic excitations (solid and dotted curves). Dashed curve is $\lambda(T)$ computed in the absence of such soft excitations. Gap in the electronic spectrum remains large: $\Delta_0 \sim 200K$

energy part: $g(E) = 2E_0\delta(E-E_0)$, $E_0 = 200$K. The other two curves show $\lambda^{-2}(T)$ when a low-energy spectral density is added: $g(E) = 2E_0\delta(E-E_0) + 2E_1\delta(E-E_1)$; $E_1 = 15,\ 5K$ for dotted, solid curves respectively. As a result, linear dependence of $\lambda^{-2}(T)$ begins at $T \sim E_1$. Our results are easily generalized on another physically possible situation when there is coupling with low-energy nonbosonic excitations. As an example, some effects of disorder may be described in terms of electron interaction with two-level systems [3]. The associated *effective* spectral density $g(E)$ is temperature dependent at thermal energies; in the simplest case [8]

$$g_T(E) = g_0(E)\tanh\left(\frac{E}{2T}\right) \tag{12}$$

Then

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{1}{\Delta}\int_0 \left[g_T(E)\coth\left(\frac{E}{2T}\right) - g_0(E)\right] dE + O\left(g(E\sim T)\frac{T^2}{\Delta^2}\right) \tag{13}$$

Thus the particular case (12) of coupling with two-level centers is exceptional as the major term in (13) vanishes. Specifically, given a constant low-energy density of centers $g_0(E)$, we obtain a quadratic temperature dependence of the London penetration depth λ for interaction with two-level centers given by (12), and a linear one in a more general model (see also [7]). In any event, if the power term in $\lambda(T)$ mainly results from effects of disorder, it should strongly depend on the specimen, which may be one of the reasons for discrepancy in experimental data (see, e.g., [11]–[15]).

The effect of vertex corrections on the penetration depth in the *whole* temperature range $(0, T_c)$

Near T_c the correction result in substitution of the real Fermi velocity of the interacting system v_F for v'_F in (6):

$$\lambda^{-2}(T) = \frac{8\pi}{3}\frac{e^2}{c^2}\nu_F v_F^2 \pi T \sum_{\epsilon_m} \frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2 + \Delta^2(i\epsilon_m)]^{3/2}},$$

$$v_F = -\frac{dG^{-1}}{dp}(\epsilon = 0, p = p_F) = \frac{v'_F}{1-\gamma_1},$$

here γ_1 is the (dimensionless) first spherical harmonic of the vertex function Γ^k [9]. At lower temperatures the renormalization of λ changes [10]. In particular, there appears a 'ladder contribution' [1] of the electron-boson interaction to the electromagnetic vertex, which can not be found analytically in the general case. Therefore, we will restrict our consideration to two illustrative examples: If the gap function is small as compared to the characteristic boson frequency ω_0 then the BCS-like approximation works: $\Delta(i\epsilon) \approx \text{const}$, $Z(i\epsilon) \approx 1 + \lambda_{ph}$, $\epsilon \ll \omega_0$. The zero-temperature penetration depth is [10]

$$\lambda^{-2}(0) = \frac{2}{3}\frac{\nu_F v_F^2}{c^2}\frac{1-\gamma_1}{1 + (\lambda_{ph} - \lambda_{ph}^1)(1-\gamma_1)},$$

where λ_{ph}, λ_{ph}^1 are the (dimensionless) zeroth and first harmonics of electron-boson interaction.

Thus, as temperature varies from T_c to zero, the vertex contribution to $\lambda^{-2}(T)$ changes by the factor

$$\frac{(1-\gamma_1)(1+\lambda_{ph})}{1 + (\lambda_{ph} - \lambda_{ph}^1)(1-\gamma_1)},$$

amounting to $(1-\gamma_1)$ in the weak-coupling limit $\lambda_{ph} \ll 1$ Another example, if the first harmonic of electron-boson interaction may be neglected (i.e. no 'bosonic' ladder corrections to the electromagnetic vertex are considered), we get

$$\lambda^{-2}(T) = \frac{8\pi}{3}\frac{e^2}{c^2}\nu_F v_F'^2 \frac{1-\gamma_1}{1-\gamma_1(1-\Pi(T))},\ \Pi(T) = \pi T \sum_{\epsilon_m} \frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2 + \Delta^2(i\epsilon_m)]^{3/2}}$$

so, the renormalization of λ due to vertex corrections changes by

$$\frac{1-\gamma_1}{1-\gamma_1(1-\Pi(T))}$$

as the temperature drops from T_c. These examples show once again that in quantitative calculations of $\lambda(T)$ the corrections to the electromagnetic vertex should be included, as they multiply the penetration depth by the *temperature-dependent* factor, and so they change, e.g.,

$$\left(\frac{\lambda(0)}{\lambda(T)}\right)^2 \text{ vs } \left(\frac{T}{T_c}\right)$$

It is shown that nodes in the gap are not the necessary prerequisite for a power low-temperature dependence of the penetration depth. Given considerable coupling of electrons to soft ($E \ll T_c$) short-wave modes in an s-superconductor, the power term in $\lambda(T)$ is strong enough to be detected in experiments.

It is unclear whether such an s-wave scenario realizes in HTSC, but the question may in principle be resolved by the experiment.

We also demonstrate the importance of vertex corrections in calculations of the penetration depth.

The author gratefully acknowledges help from Prof. Eliashberg and appreciate financial support from the Landau Institute and KFA-Jülich.

---

1. G.M.Eliashberg, G.V.Klimovitch, and A.V.Rylyakov, J. Supercond. **4**, 393 (1991).
2. At very low temperatures ($T \sim T_c\omega_D/E_F$) the dependence smoothly changes to $\sim T^6$.
3. P.W.Anderson, B.I.Halperin, and C.Varma, Phil. Mag. **25**, 1, (1972).
4. G.M.Eliashberg, Zh. Eksp. Teor. Fiz. 38, 966 (1960); Sov. Phys. JETP 11, 696 (1960).
5. G.M.Eliashberg, Zh. Eksp. Teor. Phys. 39, 1437 (1960) (Sov. Phys. JETP 12, 1000 (1961)).
6. A.J.Millis, Subir Sachdev, and C.M.Varma, Phys. Rev. B **37**, 4975 (1988).
7. It is even possible to contrive *a certain* coupling with three-level systems which would lead to *nonmonotonous* $\lambda(T)$. But such a coupling seems unreasonable from the physical point of view.
8. G.M.Eliashberg, Pis'ma Zh. Eksp. Teor. Phys. **45**, 28 (1987) (Sov. phys. JETP Lett. 45 35 (1987)).
9. A.A.Abrikosov, L.P.Gorkov, I.E.Dzyaloshinsky, *Methods of Quantum Field Theory in Statistical Physics*, ed. by R.A.Silverman (Dover Publications, New York, 1975), p.175.
10. G.M.Eliashberg, private communications.
11. W.N.Hardy, D.A.Bonn, D.C.Morgan, *et al.*, Phys. Rev. Lett. **70**, 3999 (1993), and references therein.
12. J.E.Sonier, R.F.Kielf, J.H.Brewer, *et al.*, Phys. Rev. Lett. **72**, 744 (1994).
13. N.Klein, N.Tellmann, H.Schulz, *et al.*, Phys. Rev. Lett. **71**, 3355 (1993), and references therein.
14. Zhengxiang Ma, R.C.Taber, L.W.Lombardo, *et al.*, Phys. Rev. Lett. **71**, 781 (1993).
15. Dong Ho Wu, Jian Mao, S.N. Mao, *et al.*, Phys. Rev. Lett. **70**, 85 (1993).