ON FERMI CONDENSATE: NEAR THE SADDLE POINT AND WITHIN THE VORTEX CORE

G.E. Volovik

Low Temperature Laboratory, Helsinki University of Technology
02150 Espoo, Finland
Landau Institute for Theoretical Physics
117334, Moscow, Russia

Submitted May, 10, 1994

The situation when the Fermi surface is close to the saddle point of the electron spectrum is favourable for the formation of the fermionic condensate – a flat plateau in the quasiparticle energy at the Fermi level. This may explain the results of the angle-resolved photoemission experiments on YBaCuO and BiSrCaCuO in normal state, which revealed an "extended saddle point singularity" [1] or "flat band" [2] in vicinity of the Fermi surface. The Fermi condensate can also appear in the core of quantized vortices.

Recently it was found (in Hartree-Fock approximation) that at large enough interaction the Landau Fermi-liquid is unstable towards the Fermi-condensate [3-5]. In this new state the particles with the Fermi energy form the three-dimensional ($3D$) flat band instead of $2D$ Fermi-surface of conventional Fermi-liquid. While the residual interaction can in principle lift this degeneracy, nevertheless some topologically stable features can be preserved [6]. On the other hand recent photoemission experiments on high-T_c materials in a normal state revealed an existence of the flat band at the Fermi level [2]. This singular behavior was shown to occur in the vicinity of the saddle point of the electronic spectrum [1] and was interpreted as an extended saddle point singularity. In Sec.1 we show that the situation when the Fermi surface is close to the saddle point is the most favourable for the formation of the fermionic condensate, which is possibly manifested as a flat band in photoemission experiments. In Sec.2 an example of the $1D$ flat band at Fermi level is discussed, which occurs within the core of some quantized vortices. The existence of this $1D$ Fermi condensate is supported by the symmetry and topology of the vortex.

1. Fermi condensate in the vicinity of the saddle point

For simplicity, following the Ref. [4], we consider an extreme case of the contact interaction between the particles :

$$E = \sum_{\vec{k}} (\xi_{\vec{k}} n_{\vec{k}} + \frac{1}{2} U n_{\vec{k}}^2) \quad , \tag{1.1}$$

while the non-contact part is absorbed into the qusiparticle spectrum $\xi_{\vec{k}}$. Let us assume that the contact interaction U is positive and small and compare two different cases. (i) In conventional isotropic case the bare quasiparticle energy is $\xi_{\vec{k}} = v_F(k - k_F)$. (ii) Near the saddle point $\xi_{\vec{k}} = (k_x k_y/m) - \mu$, where the chemical potential μ is counted from the saddle point. Here we assumed the $2D$ character of the spectrum in the CuO_2 planes, i.e. $\xi_{\vec{k}}$ does not depend on the momentum k_z along the c-axis, and neglected the anisotropy of masses in x, y plane.

Minimization of the energy in Eq.(1.1) gives the Fermi condensate: The Fermi surface $\xi_{\vec{k}} = 0$, which takes place at $U = 0$, is smeared at $T = 0$ producing the flat plateau between two edge surfaces. Within the plateau the quasiparticle energy is exactly zero:

$$\varepsilon_{\vec{k}} = \frac{\delta E}{\delta n_{\vec{k}}} = \xi_{\vec{k}} + U n_{\vec{k}} = 0 \quad , \tag{1.2}$$

and the particle distribution, which follows from Eq.(1.2), is

$$n_{\vec{k}} = 1/2 - \frac{\xi_{\vec{k}}}{U} \quad . \tag{1.3}$$

(Here we changed the chemical potential by amount $U/2$ to have the same total number of particles as in the case of $U = 0$.) The plateau is limited by two surfaces $\xi_{\vec{k}} = U/2$ and $\xi_{\vec{k}} = -U/2$ at which $n_{\vec{k}}$ reaches the limiting values 0 and 1. In the conventional isotropic case the width of the flat band is small, $\delta k = k_2 - k_1 = U/v_F$. This width is now to be compared with the width σ of the quasiparticle interaction, which was assumed to be zero in Eq.(1.1). According to [4], the flat band exists only if σ is less than the critical value $\sigma^* \sim U/v_F$. If $\sigma > \sigma^*$ the Fermi liquid behavior is restored.

Now let us consider the case of the saddle point, where in the $\sigma = 0$ limit the Fermi condensate is concentrated in the region

$$\mu - \frac{1}{2}U < \frac{1}{m}k_x k_y < \mu + \frac{1}{2}U \quad , \tag{1.4}$$

and again consider the effect of finite σ. For $|\mu| \gg U/2$ the maximal width of the Fermi condensate is $\delta k \sim U(m/|\mu|)^{1/2}$. When μ approaches the saddle point this width increases and finally approaches the maximal value $\delta k \sim (mU)^{1/2}$ when $|\mu| \sim U$. This essential increase of the width of the Fermi condensate in the vicinity of the saddle point makes it less vulnerable to the effect of finite σ. Now the critical value of σ becomes much larger than in the isotropic case: $\sigma^* \sim (mU)^{1/2} \gg U/v_F$.

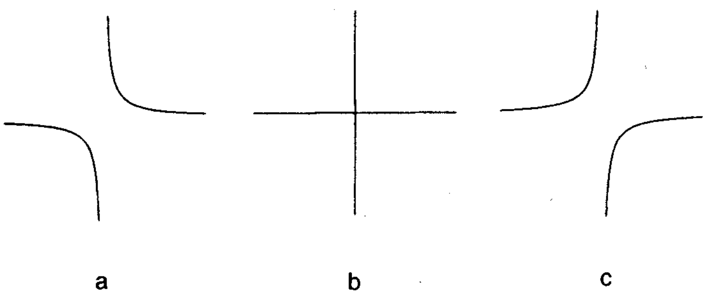

Fig.1 Reconstruction of the Fermi surface in conventional Lifshitz saddle-point transition: (a) before transition; (b) the Fermi surface at the moment of crossing the saddle point; (c) after transition

Now let us consider how the topology of the Lifshitz transition, which occurs when the chemical potential crosses the saddle point, changes when the Fermi condensate is taken into account. Without the Fermi condensate, i.e. at $\sigma > \sigma^*$, the transition takes place at one point $\mu = 0$, where the reconstruction of the Fermi surface takes place (Fig.1). If $\sigma < \sigma^*$ the spectrum can have four consequtive reconstructions. First at $\mu_1 \simeq mU^2/\sigma^2$ two Fermi condensates are formed in the vicinity of two Fermi surfaces (Fig.2a). Then at $\mu_2 = U/2$ these two condensates

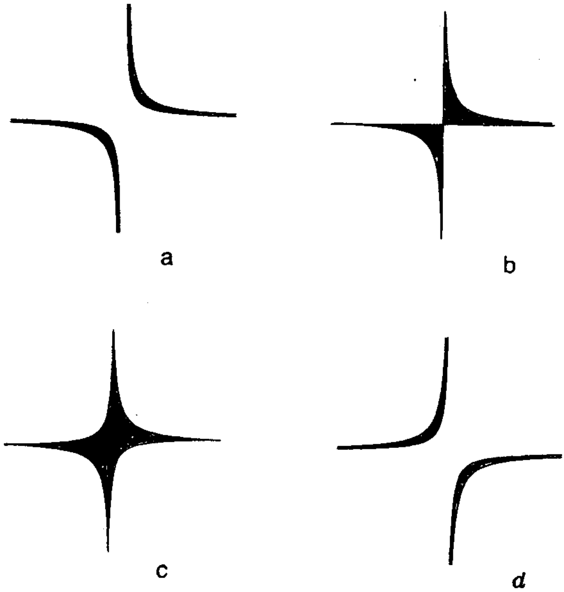

Fig.2 Intermediate states with the Fermi condensate: (a) two separated Fermi condensates (painted area) exist in the region $\mu_2 < \mu < \mu_1$ of the chemical potential; (b) condensates are merged at $\mu = \mu_2$ and (c) one condensate exists in the region $\mu_3 < \mu < \mu_2$, which (d) splits again in the region $\mu_4 < \mu < \mu_3$. At $\mu < \mu_4$ and $\mu > \mu_1$ there are pure Fermi surfaces on Fig.1b and 1a correspondigly

merge (Fig.2b). At $\mu_3 = -U/2$ the condensates again become separate (Fig.2d) and finally disappear at $\mu_4 \simeq -mU^2/\sigma^2$.

2. 1D Fermi condensate in the vortex core

Here we consider the $1D$ fermions localized in the core of vortex in superfluids and superconductors, first discussed in [7]. The energy $E(Q, k_z)$ of these fermions depends on the momentum k_z along the symmetry axis and on the quantum number Q, the eigen state of the generator of the "axial" symmetry of the vortex [8,9]. Q is similar to the angular momentum and can be either only integer or only half-integer, which depends on the type of the vortex and on the pairing state. The spectrum contains anomalous branches which as functions of discrete Q cross zero energy. In a large class of vortices the energy of such branch is odd in Q

$$E(Q, k_z) = Q\epsilon(k_z) \quad , \tag{2.1}$$

and changes sign together with Q . In conventional (s-wave) supercoductor Q is half-integer for the conventional vortex with single circulation quantum ($m = 1$) and the lowest excitation energy corresponds to $Q = 1/2$ and equals $(1/2)\epsilon(k_z = 0) \sim \Delta^2/E_F \ll \Delta$ [7].

In general case the Eq.(2.1) does not hold but the main features are preserved. The existence of the branches, which cross zero as function of Q, is prescribed by the topological arguments[10] in a similar way as the existence of chiral fermions within the strings in relativistic theories[11]: the number of anomalous branches is defined by the winding number m of the vortex: $N = 2m$ (and $N = m$ for the ^{3}He-A$_1$, where the pair-correlated state contains only one spin component).

In some vortices Q is integer. This happens for example in $m = 2$ vortices in a conventional (s-wave) supercoductor, and in $m = 1$ vortices in some non-s-wave systems [12,9]. In this case Q can be zero, and if one assumes that the Eq.(2.1) does hold, one obtains an interesting result: all fermions with $Q = 0$ have zero energy, $E(0, k_z) = 0$, i.e. the absolutely flat $1D$ band with Fermi energy exists within the vortex. Such $1D$ Fermi condensate was found in the $m = 1$ vortex of some special type in superfluid ^{3}He-A [12]. Here we show that the existence of the flat band is not an artefact of the models used for calculation of the fermionic spectrum but is prescribed by the symmetry and topology of the vortex. The flat bands can exist in different p-wave and d-wave pair-correlated systems. Let us discuss the conditions at which the $1D$ Fermi condensate exists.

(i) The quantum number Q must be integer for the fermionic quasiparticles. Let is consider when this condition is satisfied. Q is the eigen value of the generator $\mathbf{Q}$ of the non-broken Abelian symmetry group of the axisymmetric vortices. This generator is different for s-wave superconductors, and the A, B and planar phases of ^{3}He, and also depends on the circulation number m of the vortex [8]:

$$\mathbf{Q}_s = \mathbf{L}_z - m\mathbf{I} \quad , \quad \mathbf{Q}_A = \mathbf{L}_z - (m + l_z)\mathbf{I} \quad , \quad \mathbf{Q}_B = \mathbf{Q}_{\rm planar} = \mathbf{L}_z + \mathbf{S}_z - m\mathbf{I} \quad . \tag{2.2}$$

Here $\mathbf{L}_z$ is the generator of the orbital rotations, which includes the internal (isotopic) rotation of the Cooper pair around the center of mass and external rotation related to the motion of the center of mass; $\mathbf{S}_z$ is the generator of spin rotations; $\mathbf{I}$ is the generator of "gauge rotations", which takes value $1/2$ for the Bogoliubov-Nambu particle, $-1/2$ for the hole, and 1 for the Cooper pair; l_z is z-component of the unit orbital vector $\hat{l}$ in bulk ^{3}He-A.

The vacuum state of the system with a given vortex has the quantum number $Q = 0$, while the excitations (collective modes of the vortex and fermionic quasiparticles) are described by half-integer or integer values of Q. Since fermionic quasiparticles have $S_z = \pm 1/2$ and $I = \pm 1/2$, their Q take values $k - (1/2)m$ in s-wave superconductors, $k - (1/2)(m + l_z)$ in ^{3}He-A and $k + (1/2)(1 - m)$ in ^{3}He-B and planar state, where k is integer. Therefore in s-wave superconductors Q is integer for fermions on vortices with even m ; in ^{3}He-B and planar state - for vortices with odd m. For ^{3}He-A Q is integer for vortices with even $m + l_z$, which is just the case in the $m = 1$ vortex considered in [12] since the $\hat{l}$ vector is oriented along the vortex axis. The same condition, $m + l_z$ is even, is satisfied for the $m = 1$ vortex in the d-wave superconductor with the gap function $\propto k_z(k_x + ik_y)$ which is believed to be the case in heavy fermionic UPt_3 (see Review [13]).

(ii) Some discrete symmetry should be satisfied. The symmetry group of the vacuum with the vortex line, in addition to the continuous axial symmetry Q, contains also the discrete symmetries. Theses are space inversion symmetry $\mathbf{P}$, and combined $\mathbf{TU}_2$ symmetry which corresponds to the overturn of the vortex axis with simultaneous time inversion: circulation does not change under this combined operation. One more important symmetry is related to the Bogoliubov fermions: this is the symmetry under operation $\mathbf{C}$ of transformation of Bogoliubov particle into Bogoliubov hole. Transformation of the quasiparticle spectrum under these three operations are (see [9])

$$\mathbf{C}E(Q, k_z) = -E(-Q, -k_z) \ , \ \mathbf{P}E(Q, k_z) = E(Q, -k_z) \ , \ \mathbf{TU}_2E(Q, k_z) = E(Q, k_z) \ . \tag{2.3}$$

The symmetry $\mathbf{C}$ is satisfied for any vortex state, while the $\mathbf{P}$ and $\mathbf{TU}_2$ symmetries are often spontaneously broken in the vortex core [8]. We are interested in only such vortices in which either the symmetry $\mathbf{P}$ or $\mathbf{PTU}_2$ is conserved which results in equation $E(Q, k_z) = E(Q, -k_z)$. Then applying the symmetry $\mathbf{C}$ one obtains

$$E(Q, k_z) = -E(-Q, k_z) \quad . \tag{2.4}$$

This means that for each branch $E(Q, k_z)$ one can find the branch with opposite Q and E: $E_p(Q, k_z) = -E_q(-Q, k_z)$.

(iii) Chirality of fermions on vortices. In the ^{3}He-A$_1$ with one spin population, the $m = 1$ vortex has only one low-energy branch ($N = 1$). Therefore the Eq.(2.4) inevitably gives the flat band with $Q = 0$: $E_\uparrow(0, k_z) = -E_\uparrow(0, k_z) = 0$. The existence of only one branch which crosses zero as a function of Q is the result of the fact that the fermions on the low-energy branch are chiral: they have positive E for positive Q and negative E for negative Q. The chirality is the direct consequence of the topology and takes place only if $m \neq 0$.

Now let us check that the flat band occurs for $m = 1$ ^{3}He-A vortices, discussed in [12]. Conditions (i) and (ii) are fulfilled, because $m + l_z$ is either 2 or 0 for their vortex and the P symmetry is conserved. As for the condition (iii), there are two chiral branches ($N = 2$), therefore the equation $E_p(Q, k_z) = -E_q(-Q, k_z)$ does not automatically produce the flat band. However these chiral branches are degenerate over spin, $E_\uparrow(Q, k_z) = E_\downarrow(Q, k_z)$, if one neglects a tiny spin-orbital coupling. Then it follows from Eq.(2.4) that $E_\uparrow(0, k_z) = E_\downarrow(0, k_z) = 0$ for all k_z. That is why the flat zero mode calculated in [12] in a simple model, will survive any perturbation if they do not violate the vortex symmetry. The same $1D$ fermionic condensate should exist in UPt_3 vortices if the identification of the order parameter in this heavy-fermion superconductor is correct.

In conclusion, two physical systems where the Fermi condensate is most probable to occur were discussed: $3D$ condensate can arise in the metals if the Fermi surface is close to the saddle point of the electron spectrum, and $1D$ condensate should arise within the cores of vortices in superfluids and supercoductors if some, not very restrictive, symmetries are satisfied.

I am grateful to V.A. Khodel, A.V. Balatsky and T.Sh. Missirpashaev for fruitful discussions. This work was supported through the ROTA co-operation plan of the Finnish Academy and the Russian Academy of Sciences and by the Russian Foundation for Fundamental Sciences, Grants No. 93-02-02687 and 94-02-03121.

---

1. J.C.Campuzano, K.Gofron, H.Ding et al, J. Low Temp. Phys. **95**, 245 (1994); A.A. Abrikosov, J.C. Campuzano and K. Gofron, Physica C **214**, 73 (1993)
2. D.S.Dessau, Z.-X.Shen, D.M.King, et al, Phys. Rev. Lett. **71**, 2781 (1993).
3. V.A.Khodel and V.R.Shaginyan, JETP Lett. **51**, 553 (1990).
4. P.Nozieres, J.Phys. I France **2**, 443 (1992).
5. D.V.Khveshchenko, R.Hlubina, and T.M. Rice, Phys. Rev. **B 48**, 10766 (1993).
6. G.E.Volovik, Pis'ma ZETF **53**, 208 (1991); [JETP Lett. **53**, 222 (1991)].
7. C.Caroli, P.G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964)
8. M.M.Salomaa and G.E. Volovik, Rev. Mod. Phys. **59**, 533 (1987).
9. G.E.Volovik, J. Phys.: Cond. Matter **3**, 357 (1991)
10. G.E.Volovik, Pis'ma ZETF **57**, 233 (1993); [JETP Lett. **57**, 244 (1993)].
11. R.Jackiw and R. Rossi, Nucl. Phys. **B 190**, 681 (1980); M. Hindmarsh, Physica **B 178**, 47 (1992).
12. N.B.Kopnin and M.M. Salomaa, Phys. Rev. **B 44**, 9667 (1991)
13. J.A. Sauls, J. Low Temp. Phys. **95**, 153 (1994).