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ON THE SIGN OF THE INTRINSIC HALL EFFECT IN CLEAN SUPERCONDUCTORS

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The Ohmic and Hall flux-flow conductivities are calculated at low temperatures for a general electronic spectrum in the normal state. The sign of the Hall effect is predicted to depend on the shape of the Fermi surface.

1. Introduction

The electric field in the mixed state is associated with a flux flow, $\mathbf{E} = [\mathbf{B}\times\mathbf{u}]/c$. If the vortex velocity $\mathbf{u}$ is not at the right angle to the transport supercurrent, a Hall voltage appears in addition to the dissipative component. The Hall effect shows a very unusual behavior: in some cases, its sign reverses on the transition from the normal to the superconducting state. There are reasons to believe that this behavior is intrinsic to the flux motion in type II superconductors (see [1] and references therein). This is corroborated by the fact that the sign reversal has been observed for conventional V and Nb superconductors [2, 3], as well.

Recently, the quasiclassical kinetic equations [4] for clean superconductors $\ell \gg \xi$ have been derived which can describe the Hall effect in the mixed state. The interest to the clean superconductors is also based on recent experiments in the superclean regime [5, 6] which show a large Hall angle and a high vortex viscosity at low temperatures.

In the present paper we use the new kinetic equations to calculate the Ohmic and Hall flux-flow conductivities at low temperatures, $\Delta^2/E_F \ll T \ll T_c$ employing the same physical ideas as in Refs. [7–9]. Being interested in the intrinsic aspects of the flux flow, we ignore the effects of pinning and consider a free flow of vortices. We consider a general electronic spectrum and investigate the sign of the Hall effect depending on the shape of the Fermi surface of the normal metal. The only assumption on the Fermi surface is that it is isotropic in the crystal $(a\,b)$ plane which is relevant to the high-T_c materials with the uniaxial anisotropy. We assume the most symmetric alignment of the magnetic field along the crystal c axis with the current flowing in the $(a\,b)$ plane. We consider isolated vortices in the limit of low fields $H \ll H_{c2}$.

2. Kinetic equations

The vortex velocity $\mathbf{u}$ and the average electric field are determined by the relaxation of perturbations produced by the moving vortices. One of the kinetic equations for clean superconductors derived in [4] is

$$\left(e\mathbf{v}_F\mathbf{E}g_- + \frac{1}{2}\left[f_-\frac{\partial\Delta^*}{\partial t} + f_-^+\frac{\partial\Delta}{\partial t}\right]\right)\frac{\partial f^{(0)}}{\partial\epsilon} + \mathbf{v}_F\nabla(f_2 g_-) -$$

$$-\left(\frac{1}{2}\left[(\hat{\nabla}\Delta)f_-^+ + (\hat{\nabla}\Delta^*)f_-\right] - \frac{e}{c}[\mathbf{v}_F\times\mathbf{H}]g_-\right)\frac{\partial f_1}{\partial\mathbf{p}} = J_1. \qquad (1)$$

The nonequilibrium corrections to the distribution functions, f_1 and f_2 are, respectively, odd and even in ϵ and p. The operators $\hat{\partial}/\partial t = \partial/\partial t \pm 2ie\varphi$, and $\hat{\nabla} = \nabla \mp (2ie/c)\mathbf{A}$ when acting on Δ (upper sign) or Δ^* (lower sign). The collision integral is $J_1 = J_1^{(1)}\{f_1\} + J_1^{(2)}\{f_2\}$; for simplicity we assume an isotropic scattering by impurities characterised by the mean free time τ. We put $g_\pm = \frac{1}{2}(g_\epsilon^R \pm g_\epsilon^A)$, etc., where $g^{R(A)}$ are the regular retarded (advanced) quasiclassical Green functions.

The new kinetic equations differ from the usual quasiclassical equations [10] by the terms with $\partial f/\partial \mathbf{p}$ multiplied by "forces" which include the Lorentz force $(e/c)[\mathbf{v}_F \times \mathbf{H}]$ and the forces due to spatial variations of the order parameter. The force terms are generalizations of the result of Ref.[11] derived for slow spatial variations of the order parameter.

Eq.(1) can be solved if the regular functions are given. For low temperatures $T \ll \Delta_\infty$, we can use the solution obtained in [12]. We remind here this result. We choose the coordinate z axis along the magnetic field and assume that the vortex has a positive circulation around the z axis which corresponds to the positive charge of carriers. Let $\mathbf{v}_\perp$ be the projection of the quasiparticle velocity $\mathbf{v}_F$ on the (x, y) plane. The vector $\mathbf{v}_\perp$ makes the angle α with the x axis. If ρ and ϕ are the distance and the azimuthal angle in the cylindrical frame, the impact parameter of a quasiparticle moving with $\mathbf{v}_\perp$ through the point (ρ, ϕ) is $b = \rho \sin(\phi - \alpha)$. The distance along the trajectory is $l = \rho\cos(\phi - \alpha)$, so that $\rho^2 = b^2 + l^2$. Within the leading approximation in ϵ/Δ_∞, one has

$$g_- = \frac{\pi v_\perp}{2C(v_\perp)} \exp[-K(\rho)]\delta[\epsilon - \epsilon(b)], \tag{2}$$

and

$$f_- e^{-i\phi} = -f_-^+ e^{i\phi} = \frac{i\pi v_\perp}{2C(v_\perp)} \cos(\phi - \alpha) e^{-K(\rho)}\delta[\epsilon - \epsilon(b)]. \tag{3}$$

Here

$$K(\rho) = \frac{2}{v_\perp}\int_0^\rho \Delta_0 \, d\rho'; \;\; C(v_\perp) = \int_0^\infty \exp[-K(\rho)]\, d\rho. \tag{4}$$

The bound state energy is [13]

$$\epsilon(b) = bC^{-1}(v_\perp)\int_0^\infty \frac{\Delta_0(\rho)}{\rho} \exp[-K(\rho)]\, d\rho. \tag{5}$$

The impact parameter $b = -n/p_\perp$ where n is the angular momentum of quasiparticles and $p_\perp$ is the momentum projection on the $(x\,y)$-plane. Therefore, the distance between the energy levels with angular momenta n and $n \pm 1$ is $\omega_0 = (1/p_\perp)(\partial\epsilon/\partial b)$.

Using the second kinetic equation (omitted for brevity), one can show that the function f_1 is constant on the quasiparticle trajectory [14]. Integrating Eq.(1) along the trajectory, we obtain

$$\left([\mathbf{v}_\perp \times \mathbf{u}]\hat{\mathbf{z}}\right)\frac{\partial f^{(0)}}{\partial \epsilon}\frac{\partial \epsilon(b)}{\partial b}\pi\delta[\epsilon - \epsilon(b)] + \left([\mathbf{v}_\perp \times \frac{\partial f_1}{\partial \mathbf{p}_\perp}]\hat{\mathbf{z}}\right)\frac{\partial \epsilon(b)}{\partial b}\pi\delta[\epsilon - \epsilon(b)] + \\ + \int_{-\infty}^{\infty} J_1^{(1)}\, dl = 0. \tag{6}$$

The collision integral here is

$$J_1^{(1)} = -\frac{1}{\tau}\left(f_1\langle g_-\rangle g_- + \frac{1}{2}\langle f_1 f_-^+\rangle f_- + \frac{1}{2}\langle f_1 f_-\rangle f_-^+\right), \tag{7}$$

where

$$\langle g \rangle = \frac{1}{\nu(0)} \int \frac{dS_F}{(2\pi)^3 v_F} g(\mathbf{p}_F, \mathbf{r}). \tag{8}$$

The integration is carried out over the Fermi surface, and $\nu(0)$ is the averaged density of states.

Eq.(6) can be solved only after certain simplifications of the collision integral. In Refs.[7, 14], the model proposed in [12] was used according to which the vortex core shrinks for low temperatures to $\xi_0 T/\Delta_\infty$. As the result, the dependence of $\epsilon(b)$ on $v_\perp$ reduces to a simple form, and one could calculate the averages in the collision integral for a spherical Fermi surface. However, this model is an oversimplification of the real order parameter behavior. Moreover, it does not help much, if the Fermi surface is not spherical.

To avoid integrals of unknown functions, we use here another approximation which is equivalent to a renormalization of τ: instead of the last two equal terms in the true collision integral Eq.(7), we take the first term twice. In the moderately clean regime, the both models give the same Ohmic conductivity. We have now

$$\int_{-\infty}^{\infty} J_1^{(1)}\, dl = -\frac{\pi v_\perp \ln(\Delta_\infty/T)}{\tau C(v_\perp)} \langle \frac{v_\perp}{C(v_F)(\partial\epsilon/\partial b)} \rangle \delta[\epsilon - \epsilon(b)] f_1. \tag{9}$$

The ligarithmic divergence in the integral for small ρ is cut off at distances $\rho \sim b \sim \xi(T/\Delta_\infty)$.

For the Fermi surface isotropič in the $(x\, y)$ plane, one has

$$\left([\mathbf{v}_\perp \times \frac{\partial f_1}{\partial \mathbf{p}_\perp}]\hat{\mathbf{z}}\right) = \pm \frac{v_\perp}{p_\perp} \frac{\partial f_1}{\partial \alpha}. \tag{10}$$

Here the sign $+$ is for quasielectrons while the sign $-$ is for quasiholes, since the directions of $\mathbf{v}_\perp$ and $\mathbf{p}_\perp$ are either same or opposite for these two types of quasiparticles, respectively.

The kinetic equation (6) gives

$$f_1 = -\frac{\partial f^{(0)}}{\partial \epsilon} \left(\frac{p_\perp \gamma_H}{v_\perp} (\mathbf{u}\, \mathbf{v}_\perp) + \frac{p_\perp \gamma_O}{v_\perp} ([\mathbf{u} \times \mathbf{v}_\perp]\hat{\mathbf{z}}) \right), \tag{11}$$

where

$$\gamma_H = \pm \frac{(\omega_0 \tau_{eff})^2}{1 + (\omega_0 \tau_{eff})^2}; \quad \gamma_O = \frac{\omega_0 \tau_{eff}}{1 + (\omega_0 \tau_{eff})^2}. \tag{12}$$

The "effective scattering time" is $\tau_{eff} = \tau \beta(v_\perp)$, where

$$\beta(v_\perp) = \frac{C(v_\perp)}{\ln(\Delta_\infty/T) \langle v_\perp \left[C(v_\perp)(\partial\epsilon/\partial b) \right]^{-1} \rangle}, \tag{13}$$

is of the order of unity (apart from a possible logarithm).

The transport current can be calculated using the general scheme described, for example, in Ref. [10]. With the distribution function of Eq. (11) we have

$$\mathbf{j}_{tr} = e \int \frac{dS_F}{(2\pi)^3 v_F} v_\perp p_\perp \left(\gamma_H \mathbf{u} + \gamma_O [\hat{\mathbf{z}} \times \mathbf{u}] \right). \tag{14}$$

Since $\mathbf{u} = c[\mathbf{E} \times \hat{z}]/B$, the flux-flow conductivities become

$$\sigma_f^{(O)} = \frac{2\,|e|}{(2\pi)^3}\left[\int_{q.e.} S(p_z)\,|\gamma_O|\,dp_z + \int_{q.h.} S(p_z)\,|\gamma_O|\,dp_z\right]\frac{c}{B}; \tag{15}$$

$$\sigma_f^{(H)} = \frac{2e}{(2\pi)^3}\left[\int_{q.e.} S(p_z)\,|\gamma_H|\,dp_z - \int_{q.h.} S(p_z)\,|\gamma_H|\,dp_z\right]\frac{c}{B}. \tag{16}$$

The modulus of the charge appears in Eq. (15) since the circulation of the vortex has been chosen along the positive z axis (along the magnetic field) for positive charge of carriers. In the opposite case, both γ_O and $\partial\epsilon/\partial b$ change their signs together with the vortex circulation. Eqs.(15) and (16) are generalizations of the result of Ref. [7] to the case of nonparabolic spectrum of quasiparticles.

3. Discussion

In the superclean limit, $\ell \gg \xi_0 E_F/\Delta_\infty$ or $\omega_0\tau \gg 1$, the Hall factor $|\gamma_H| = 1$, while the dissipative part γ_O vanishes as $(\omega_0\tau_{eff})^{-1}$. The Hall angle defined by $\tan\theta_H = \sigma_f^{(H)}/\sigma_f^{(O)} \sim \omega_0\tau$ becomes equal to $\pi/2$. The Hall conductivity is $\sigma_f^{(H)} = (N_e - N_h)ec/B$, as for the normal metal in high magnetic fields. The transport current is $\mathbf{j}_{tr} = (N_e - N_h)e\mathbf{u}$, where $N = 2V/(2\pi)^3$, and V_e and V_h are the volumes of the electron-like and hole-like parts of the Fermi surface, respectively.

The electron-phonon relaxation can also play a role if the impurity content is low enough. This mechanism can become more effective in the superclean regime since, for high-T_c superconductors, the electron-phonon relaxation is quite substantial. It is possible that the increase in the Hall angle towards the superclean limit observed in [6] is due to the increase in τ_{ph} with the lowering temperature.

In the moderately clean case, $\gamma_O = \omega_0\tau_{eff} \ll 1$. Within the model of Ref. [12] for the vortex core structure, the Ohmic conductivity is $\sigma_f^{(O)} = \nu(0)\,|e|\cdot\tau\Delta_\infty^2\ln(\Delta_\infty/T)c/B$ and reproduces the results of Refs. [7, 14].

In the limit $\omega_0\tau \ll 1$, one has $|\gamma_H| = (\omega_0\tau_{eff})^2$. The Hall angle is small $\theta_H \sim \omega_0\tau$. The sign of the flux-flow Hall conductivity Eq.(16) can be compared with the normal-state Hall conductivity at low fields $\omega_c\tau \ll 1$. For the model with an isotropic scattering by impurities one has [15]:

$$\sigma_n^{(H)} = \frac{He^3\tau^2}{(2\pi)^2 c}\left[\int_{q.e.} v_\perp^2\,dp_z - \int_{q.h.} v_\perp^2\,dp_z\right]. \tag{17}$$

We see that the signs of Hall conductivities in the normal and in the mixed superconducting states may be different since the functions under the integrals in Eqs.(16) and (17) have different dependencies on the momentum. Therefore, the sign of the Hall effect can change after the transition from the normal to the superconducting state. The sign reversal depends on the shape of the Fermi surface; it is absent for a simple parabolic spectrum $\epsilon_p = p^2/2m^*$ when the flux-flow and the normal-state Hall conductivities have the same sign [7]. This conclusion differs from the models of Refs. [16] and [17] which predict the Hall effect in the mixed state of the same sign as in the normal state. The latter theories treat the quasiparticles in the vortex core as being the same as in the normal state. However, the localized quasiparticles have different energy spectrum and their behavior thus differs from that in the normal state.

The sign of the Hall conductivity of clean superconductors at low temperatures can be compared with the result of Refs.[18, 19] for gapless superconductors in the dirty limit $\ell \ll \xi_0$. For the latter case, the sign of the Hall effect depends on the average energy derivative of the density of states at the Fermi surface which, in principle, may be different from those of Eq.(16) or Eq.(17). Therefore, the sign reversal depends on the purity of the sample as well as on the shape of the Fermi surface, and the transition between presence or absence of the sign reversal for the same doping level may occur in the range $\ell/\xi_0 \sim 1$.

In conclusion, we find that (1) in the superclean limit, $\ell \gg \xi E_F/\Delta$, the Hall conductivity at low temperatures is the same as for high fields in the normal state. (2) In the moderately clean case, $\xi \ll \ell \ll \xi E_F/\Delta$, the sign of the Hall effect in the mixed state at low temperatures may differ from that in the normal state depending on the detailed structure of the Fermi surface. (3) The sign in a clean superconductor can be different from that of the dirty compound in gapless regime. This indicates that the sign reversal can depend on the impurity content in the range $\ell \sim \xi$. Therefore, the sign reversal is very sensitive to the Fermi supface structure, to the doping level, and to the impurity content [1, 20].

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