Pis'ma v ZhETF, vol.60, iss.9, pp.613 - 618

ON ANOTHER VERSION OF THE TWISTOR-LIKE APPROACH TO SUPERPARTICLES

I.A.Bandos[1)], *A.Yu.Nurmagambetov*[1)], *D.P.Sorokin*[1)], *D.V.Volkov*[1)]

Kharkov Institute of Physics and Technology
310108 Kharkov, the Ukraine

Submitted 12 Oktober, 1994

Considered is a worldsheet supersymmetric generalization of the $D = 3$ Ferber–Shirafuji twistor–superparticle action.

Twistor–like doubly supersymmetric formulations of superparticles [1-6], superstrings [7-9] and supermembranes [10] have attracted great deal of attention, in particular, because of a hope to break through the long–standing problem of the covariant quantization of these theories.

In the twistor-like approach the infinite-reducible fermionic κ-symmetry [11, 12], which causes the problem of covariant quantization [13], is replaced by local worldsheet supersymmetry which is irreducible by definition, and the theory is formulated as a superfield theory in a worldsheet superspace imbedded into a target superspace. Thus, the model of this kind possesses doubly supersymmetry.

Earlier doubly supersymmetric dynamical systems (of more general physical contents) were considered by several groups of authors irrelative to the κ-symmetry problem [14].

Several versions of twistor-like doubly supersymmetric particles and heterotic strings have been constructed in $D = 3, 4$ and 6 dimensions of space–time, while in D=10 only one superfield formulation is known [6] and, unfortunately, the latter itself suffers the infinite reducibility problem arising for a new local symmetry [6] being crucial for the possibility of eliminating auxiliary degrees of freedom of the objects under consideration [2)].

The main motivation of the present paper is, from the one-hand side, to develop a version of the twistor-like formulation which would be free of the reducibility problem already at the superfield level, and, from the other hand, would look "twistor–like" as much as possible. The letter, as we hope, may allow one to better utilize the powerful twistor techniques for deeper implementation of the twistors into the structure of supersymmetric theories.

The superfield twistor-like models of $N = 1$ Brink – Schwarz superparticles in $D = 3, 4, 6$ and 10 considered so far are based on the doubly supersymmetric generalization of the following massless bosonic particle action [1]:

$$S = \int d\tau p_m(\dot{x}^m - \bar{\lambda}\gamma^m\lambda), \qquad (1)$$

[1)]e-mail: kfti@kfti.kharkov, ua

[2)]Note, however, that at the component level, when auxiliary fields were eliminated by gauge fixing and solving for relevant equations of motion, all remaining local symmetries are irreducible. This also takes place in a twistor-like Lorentz-harmonic formulation of super-p-branes [15, 16] developed in parallel to the superfield twistor approach.

where p_m is the particle momentum and λ^α is a commuting spinor variable ensuring the validity of the mass shell condition $p_m p^m = 0 = \dot{x}_m \dot{x}^m$ due to the Cartan–Penrose representation $\dot{x}^m = \bar{\lambda}\gamma^m\lambda$ of the light-like vectors in $D = 3, 4, 6$ and 10 space–time dimensions.

The straightforward doubly supersymmetric generalization of (14) is [6]

$$S = \int d\tau d^{D-2}\eta P_{mq}(D_q X^m - D_q\bar{\Theta}\gamma^m\Theta), \tag{2}$$

where the number $n = D - 2$ of the local worldline supersymmetries is equal to the number of the κ-symmetries in $D = 3, 4, 6$ and 10 ; $D_q = \frac{\partial}{\partial\eta^q} + i\eta_q\partial_\tau$ is an odd supercovariant derivative in a worldline superspace (τ, η^q) , $\{D_q, D_p\} = 2i\delta_{pq}\partial_\tau$ and (X^m, Θ^α) are worldline superfields which parametrize the "trajectory" of the superparticle in a target superspace. Bosonic spinor variables λ_q^α appear in (2) as superpartners of Grassmann coordinates $\theta^\alpha = \Theta^\alpha|_{\eta=0}$:

$$\lambda_q^\alpha = D_q\Theta^\alpha(\tau, \eta)|_{\eta=0} \ . \tag{3}$$

The analysis of the action (2) [1, 6] shows that it describes a superparticle classically equivalent to the massless $N = 1$ Brink–Schwarz superparticle in $D = 3, 4, 6$ and 10 .

As we have already mentioned, in $D = 4, 6$ and 10 the action (2) possesses a local symmetry [6] under the following transformations of the Lagrange multiplier P_{mq} :

$$\delta P_{mq} = D_p\bar{\Xi}_{qpr}\gamma_m D_r\Theta, \tag{4}$$

with $\bar{\Xi}_{qpr}^\alpha$ being symmetric and traceless with respect to the indices (p, q, r). This symmetry is infinite reducible since P_{mq} is inert under the transformations (4) with

$$\bar{\Xi}_{qpr}^\alpha = D_s\bar{\Xi}_{qprs}^\alpha \tag{5}$$

where $\bar{\Xi}_{qprs}^\alpha$ is again symmetric and traceless, and (5) is trivial if $\bar{\Xi}_{qprs}^\alpha = D_s\bar{\Xi}_{qprst}^\alpha$ and so on and so far.

The reducibility of the transformations (4) is akin to the reducibility of the gauge symmetries of the antisymmetric gauge fields. It is just the problem of reducible symmetries in these theories that stimulated further development of the quantization procedure which was consistently fulfilled for finite reducible symmetries [17] to which the gauge transformations of the antisymmetric bosonic tensor fields belong to. However, the general receipt for dealing with the infinite reducible symmetries is still unknown (see [18] and refs. therein). Thus, one has to avoid this problem one way or another . In the case under consideration one may try to find another form of the twistor-like superfield action for the superparticle.

To this end, let us choose, as a starting point, the form of the twistor particle action considered by Ferber [19] and Shirafuji [20]

$$S = \int d\tau\bar{\lambda}\gamma_m\lambda\dot{x}^m, \tag{6}$$

For simplicity, we shall consider the case of $N = 1$, $D = 3$ superparticle.

To generalize (6) to the doubly supersymmetric case one could naively try (using (3)) to write down an action in the following form

$$S = \int d\tau d\eta D\Theta_\alpha D\Theta_\beta DX^{\alpha\beta}, \tag{7}$$

where $X^{\alpha\beta} \equiv X^m \gamma_m{}^{\alpha\beta}$.

However, action (7) does not describe $N = 1$, $D = 3$ Brink–Schwarz superparticle, but a model with an odd physical contents. The reason is that (7) is invariant under the following transformations

$$\delta\Theta^\alpha = \epsilon_1^\alpha, \qquad \delta X^{\alpha\beta} = \Theta^\alpha \epsilon_2^\beta + \Theta^\beta \epsilon_2^\alpha,$$

so that the target space is not the usual superspace, but one with additional θ–translations.

Note that action (7) is part of a so called spinning superparticle model considered several years ago [14].

To construct a doubly supersymmetric action for describing an $N = 1$ Brink–Schwarz superparticle we have to keep only one target space supersymmetry. The right action turns out to be as follows

$$S = \int d\tau d\eta \Lambda_\alpha \Lambda_\beta (DX^{\alpha\beta} - iD\Theta^\alpha \Theta^\beta - iD\Theta^\beta \Theta^\alpha), \tag{8}$$

where $\Lambda_\alpha(\tau, \eta)$ is a commuting spinor superfield.

In addition to $N = 1$ target space supersymmetry, and $n = 1$ local worldline supersymmetry

$$\delta\eta = \frac{i}{2} D\Xi(\tau, \eta), \qquad \delta\tau = \Xi + \frac{1}{2}\eta D\Xi, \qquad \delta D = -\frac{1}{2}\partial\Xi D , \tag{9}$$

action (8) is invariant under bosonic transformations

$$\delta X^{\alpha\beta} = b(\tau, \eta)\Lambda^\alpha \Lambda^\beta, \qquad \delta\Theta^\alpha = 0 = \delta\Lambda^\alpha \tag{10}$$

and under a superfield irreducible counterpart of the conventional fermionic κ-symmetry

$$\delta\Theta^\alpha = \kappa(\tau, \eta)\Lambda^\alpha, \qquad \delta X^{\alpha\beta} = 2i\delta\Theta^{\{\alpha}\Theta^{\beta\}}, \qquad \delta\Lambda^\alpha = 0, \tag{11}$$

which resembles the fermionic symmetry of component twistor-like actions for super-p-branes [1, 16] (the braces $\{...\}$ denote symmetrization of the indices).

The algebra of the transformations (10), (11) is closed.

The equations of motion derived from (8) are

$$\Pi^{\alpha\beta}\Lambda_\beta \equiv (DX^{\alpha\beta} - 2iD\Theta^{\{\alpha}\Theta^{\beta\}})\Lambda_\beta = 0, \tag{12}$$

$$\Lambda_\beta D\Theta^\beta = 0, \tag{13}$$

$$\Lambda_{\{\alpha} D\Lambda_{\beta\}} = 0 . \tag{14}$$

The general solutions to (12) and (13) are, respectively,

$$\Pi^{\alpha\beta} = \Psi(\tau, \eta)\Lambda^\alpha \Lambda^\beta, \tag{15}$$

$$D\Theta^\alpha = a(\tau, \eta)\Lambda^\alpha . \tag{16}$$

At the same time, from (14) it follows that

$$D\Lambda^\alpha = 0 . \tag{17}$$

On the mass shell (15) – (17) the fermionic superfield Ψ and the bosonic superfield a transform under (11), (10) and (9) as follows:

$$\delta\Psi = Db - \frac{1}{2}\partial_\tau \Xi \Psi - 2ia\kappa, \qquad \delta a = D\kappa - \frac{1}{2}\partial_\tau \Xi a. \tag{18}$$

Hence, one can fix a gauge

$$\Psi = 0, \qquad a = 1, \tag{19}$$

at which (15), (16) are reduced to

$$\Pi^{\alpha\beta} = 0, \tag{20}$$

$$D\Theta^\alpha = \Lambda^\alpha, \tag{21}$$

This gauge [3)] is conserved under the κ-symmetry reduced to the worldline supersymmetry

$$D\kappa - \frac{1}{2}\partial_\tau \Xi = D(\kappa + \frac{i}{2}D\Xi) = 0. \tag{22}$$

As a result the twistor superfield Λ^α is expressed in terms of $D\Theta^\alpha$ and does not carry independent degrees of freedom, and in the gauge (19) the equations for $X^{\alpha\beta}$ and Θ^α coincide with those in the conventional twistor-like formulation (2) [1, 6].

Thus we conclude that the doubly supersymmetric action (8) is classically equivalent to (2) and describes the massless $N=1$ superparticle.

The relationship between the two actions can be understood using the following reasoning. It was proved in [5] that for $n=1$ action (2) is classically equivalent to

$$S = \int d\tau d\eta (P_{\alpha\beta}\Pi^{\alpha\beta} - \frac{1}{2}EP_{\alpha\beta}P^{\alpha\beta}) \tag{23}$$

due to the existence of the following counterparts of the symmetry transformations (11), (10) [5]

$$\delta X^{\alpha\beta} = \tilde{b}P^{\alpha\beta}, \qquad \delta E = D\tilde{b}, \qquad \Theta^\alpha = 0, \tag{24}$$

$$\delta X^{\alpha\beta} = 2i\delta\Theta^{\{\alpha}\Theta^{\beta\}}, \qquad \delta E = -2i\kappa_\alpha D\Theta^\alpha, \qquad \delta\Theta^\alpha = \kappa_\beta P^{\beta\alpha}, \tag{25}$$

which allow one to put the Grassmann superfield $E(\tau,\eta)$ equal to zero globally on the worldline superspace [4)]

At the same time the variation of (23) with respect to $E(\tau,\eta)$ leads to the equation

$$P^{\alpha\beta}P_{\alpha\beta} = 0, \tag{26}$$

which can be solved as

$$P_{\alpha\beta} = \Lambda_\alpha\Lambda_\beta\,, \tag{27}$$

with Λ_α being an arbitrary bosonic spinor superfield. Substituting (27) back into (23) we get just the action (8).

---

3) Note, that the gauge choice $a=0$ in Eq. (19) is inadmissible since then from (15), (16) it would follow that $\frac{d}{d\tau}X^m|_{\eta=0} = 0$, which, in general, is incompatible with boundary conditions $X^m(\tau_1)|_{\eta=0} = x_1$, $X^m(\tau_2)|_{\eta=0} = x_2$.

4) Note that in contrast to (11) the transformations of eq. (25) correspond to an infinite reducible κ-symmetry [11, 12, 13].

In conclusion we have constructed a version of the twistor-like formulation of the massless $N=1$, $D=3$ superparticle based on eq. (8) with all symmetries of the model being irreducible. Action (8) looks very much like a worldline superfield generalization of the supertwistor action proposed by Ferber [19].

One can even rewrite (8) in a complete supertwistor form a la Shirafuji [20] by introducing the second bosonic spinor component and the Grassmann component of the supertwistor [19]:

$$M^{\alpha} = X^{\alpha\beta}\Lambda_{\beta}, \qquad \Upsilon = \Theta^{\alpha}\Lambda_{\alpha} . \tag{28}$$

Then, with taking into account constraint (28), action (8) takes the form

$$S = \int d\tau d\eta (\Lambda_{\alpha} DM^{\alpha} - D\Lambda_{\alpha} M^{\alpha} - 2i\Upsilon D\Upsilon) \cdot$$

Note that the transformations (11) resemble an extra hidden local worldline supersymmetry which relates Θ^{α} and Λ^{α}, and it would be of interest to understand the nature and the role of this symmetry in more detail.

Action (8) admits a generalization to D=4 and 6 superparticles and, possibly, heterotic strings within the line of the twistor–like formulation developed in [1, 4, 5, 8], where the notion of a doubly Grassmann analiticity (for D=4) and a doubly harmonic analiticity (for D=6) have been explored. But the generalization to the case of D=10 twistor–like objects with irreducible local symmetries seems to be more subtle. Work on this subject is in progress.

Work supported in part by the International Science Foundation under the Grant RY No 9000, and by Ukrainian State Committee on Sciece and Technology under contract No 2.3/664.

The authors are thankful to A. Pashnev for discussion.

---

1. D.Sorokin, V.Tkach and D.V.Volkov, Mod. Phys. Lett. **A4**, 901 (1989).
2. D.V.Volkov and A.Zheltukhin, Sov. Phys. JETP Lett. **48**, 61 (1988); Letters in Math. Phys. **17**, 141 (1989); Nucl. Phys. **B335**, 723 (1990).
3. D.Sorokin, V.Tkach, D.V.Volkov and A.Zheltukhin, Phys. Lett. **B216**, 302 (1989).
4. D.P.Sorokin, Fortshr.Phys. **38**, 923 (1990); F.Delduc and E.Sokatchev, Class. Quantum Grav. **9**, 361 (1992).
5. A.Pashnev and D.Sorokin, Class. Quantum Grav. **10**, 625 (1993).
6. A.Galperin and E.Sokatchev Phys. Rev. **D46**, 714 (1992).
7. N.Berkovits, Phys. Lett. **B241**, 497 (1990); M.Tonin, Phys. Lett. **B266**, 312 (1991), Int. J. Mod. Phys **A7**, 613 (1992); S.Aoyama, P.Pasti and M.Tonin, Phys. Lett. **B283**, 213 (1992); F.Delduc, A.Galperin, P.Howe and E.Sokatchev, Phys. Rev. **D47**, 587 (1992).
8. E.Ivanov and A.Kapustnikov, Phys. Lett. **B267**, 175 (1991); F.Delduc, E.Ivanov, and E.Sokatchev, Nucl. Phys. **B384**, 334 (1992).
9. V. Chikalov and A. Pashnev, Mod. Phys. Lett. **A8**, 285 (1993); Preprint ICTP, 1993; A.Galperin and E.Sokatchev, Phys. Rev. **D48**, 4810 (1993).
10. P.Pasti and M.Tonin, Nucl. Phys. **B418**, 337 (1994); E. Bergshoeff and E. Sezgin, Nucl. Phys. **B422**, 329 (1994).
11. J. De Azcarraga and J.Lukierski, Phys. Lett. **B113**, 170 (1982).
12. W.Siegel, Phys. Lett. **B128**, 397 (1983).
13. M.Green, J.Schwarz, and E.Witten, Superstring Theory, CUP, 1987
14. S.J.Gates Jr. and H. Nishino, Class. Quantum Grav. **3**, 391 (1986); J.Kowalski-Glikman, Phys. Lett. **B180**, 358 (1986); R.Brooks, F.Muhammed, and S.J.Gates Jr., Class. Quantum Grav. **3**, 745 (1986); R.Brooks, Phys. Lett. **B186**, 313 (1987); J.Kowalski-Glikman, J.W. van Holten, S.Aoyama and J. Lukierski, Phys. Lett. **B201**, 487 (1987); A.Kavalov and R.L.Mkrtchyan. Spinning superparticles. Preprint Yer. PhI 1068(31)-88, Yerevan, 1988 (unpublished).

15. I.A.Bandos, Sov. J. Nucl. Phys. **51**, 906 (1990); JETP. Lett. **52**, 205 (1990).
16. I.A.Bandos and A.A.Zheltukhin , JETP. Lett. **54**, 421 (1991); **55**, 81 (1992); Phys. Lett. **B288**, 77 (1992); Int. J. Mod. Phys. **A8**, 1081 (1993); Sov. J. Nucl. Phys. **56**, n1, 198 (1993). Preprint IC/92/422 ICTP, Trieste, 1992. Preprint DFPD/94/TH/35, Padova, 1994, **hep-th/9405113**.
17. I.A.Batalin and G.A.Vilkovisky, Phys. Rev. **D28**, 2567 (1983).
18. E.Bergshoeff, R.Kallosh and A. van Proeyen, Phys. Lett. **B251**, 128 (1990); A.Micovic, M.Rõcek, W.Siegel et. al., Phys. Lett. **B235**, 106 (1990).
19. A.Ferber, Nucl. Phys. **B132**, 55 (1977).
20. T.Shirafuji, Progr. Theor. Phys. **70**, 18 (1983).