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AHARONOV–BOHM EFFECT IN LUTTINGER LIQUID

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In systems with the spin-charge separation, the period of the Aharonov–Bohm (AB) oscillation becomes half of the flux quantum. This effect is at least related to the fact that for the creation of the holons (spinons) are needed two electrons. The effect is illustrated on the example of the Hubbard Hamiltonian with the aid of the bosonization including topological numbers and exists also in the Luttinger liquid on two chains. The relation to a fractional $1/N$ AB effect, which can be associated with a modified Luttinger liquid, is discussed.

In practically all $1D$ strongly-correlated electron systems there exists the phenomenon known as "spin-charge" separation [1]. It was also recently argued [2] that the spin charge separation is not only inherent to $1D$, but also occurs in the two-dimensional systems related to HTSC. The degrees of freedom associated with the single electron are split into two independent spin and charge degrees of freedom associated with single particle gapless excitations: spinons and holons as in $1D$ Luttinger liquid [3].

We show [4] that the properties of strongly-correlated systems are associated with a new type of AB effect, namely, the period of the AB effect decreases and becomes half of the period of the AB oscillations for the free electrons [5]. This is valid for all systems where the spin-charge separation exists. The spinon, as well as the holon excitation, is created by two single electron operators associated with the spin and charge density fluctuations, which is also the reason why the period of the AB oscillation becomes halfes. With holons as well as with spinons two types of topological numbers are associated bound with some selection rules defined by the parity of the total number of electrons [3]. As a result all properties are parity dependent and the parity effect exists in the Luttinger liquid of spinful electrons. However, the period of the oscillation for the Hubbard ring in the limit of $U \to \infty$ decreases N_e times, where N_e is the number of electrons on the ring [6, 7]. The other important feature of this effect is the absence of the parity effect, which exists for free electrons [5, 8] as well as for interacting fermions [5, 9, 10]. The absence of the parity effect is also connected with the $1/N_e$ decrease of the AB period. The system in the strong-coupling regime can be described by a modified Luttinger liquid.

To illustrate the decrease of the AB period we consider the Hubbard Hamiltonian:

$$H = t\sum_{i,\sigma}(a^+_{i\sigma}a_{(i+1)\sigma} + \text{h.c.}) + U\sum_{i} n_{i\uparrow}n_{i\downarrow}, \qquad (1)$$

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where t and U are hopping integral and the constant of the on-site Coulomb interaction between electrons, respectively. First, we go to the continuum limit and then apply the bosonisation [11], where the Hamiltonian is [11]

$$H_e = it \sin k_F \sum_s \int_0^L dx(\Psi_{s-}(x)\partial_x\Psi_{s-}(x) - \Psi_{s+}(x)\partial_x\Psi_{s+}(x)) +$$
$$+U\int_0^L dx[: j_{0\uparrow} :: j_{0\downarrow} : +\Psi^+_{\uparrow+}(x)\Psi^+_{\uparrow-}(x)\Psi_{\downarrow+}(x)\Psi_{\downarrow-}(x) + \text{h.c.}], \tag{2}$$

where $\Psi_{s\pm}$ are left and right movers and $j_{0\uparrow} = \Psi^+_{\uparrow+}(x)\Psi_{\uparrow+}(x) + \Psi^+_{\uparrow-}(x)\Psi_{\uparrow-}(x)$. The analogous expression is written for the current $j_{0\downarrow}$ of down-spin fermions.

We take into account the periodical boundary (PB), and twisted boundary (TB) conditions, when the Hubbard ring is located in a transverse magnetic field. In both cases the fermion field $\Psi_{\beta\alpha}(x)$ can be represented as: $\Psi_{\beta\pm}(x) = \frac{1}{\sqrt{2\pi\alpha}}\exp(\pm i\sqrt{4\pi}\Phi_{\beta\pm}(x))$, where α is the cut parameter and the boson fields $\Phi_{\beta\pm}$ for PB conditions can be represented as: $\Phi_{\beta\pm}(x) = \Phi_\beta(x) \pm \int_{-\infty}^x \pi_\beta(x')dx'$. Here $\pi_\beta(x)$ is the conjugate variables to Φ_β. In terms of these fields the Hubbard Hamiltonian takes the form:

$$H = \int_0^L dx(\frac{t\sin k_F}{2}[\pi^2_\uparrow + (\partial_x\Phi_\uparrow)^2 + (\uparrow\rightarrow\downarrow)] +$$
$$+U[\frac{\partial_x\Phi_\uparrow\partial_x\Phi_\downarrow}{\pi} + \frac{1}{2\pi^2\alpha^2}\cos[\sqrt{4\pi}(\Phi_\uparrow - \Phi_\downarrow)]]. \tag{3}$$

On the ring the variables π_β and Φ_β are multivalued. It is, therefore, convenient to decompose them into the single valued variables and topological quantum numbers, related to the winding numbers on the ring:

$$\Phi_{\beta\pm}(x) = \Phi_\beta(x) \pm \int_{-\infty}^x \pi_\beta(x')dx' + (N_\beta \pm J_\beta)\frac{\sqrt{\pi}x}{2L}, \tag{4}$$

where the new variables $\pi_\beta(x)$ and Φ_β are single valued and N_β, J_β are topological numbers associated with the charge and current on the ring. These numbers are connected by the selection rules. These rules depend on the parity of the total number of electrons N_e. Imposing the periodical boundary conditions we get the following selection rules: $(-1)^{(N_\beta \pm J_\beta)} = (-1)^{(N_e-1)}$, which is a simple generalization of the selection rule for the Luttinger liquid of spinless fermions [10]. Implicitly, these selection rules dictate that if the number of electrons is odd, then the number N_β is even and the number J_β is odd, or the number N_β is odd and the number J_β is even. On the other hand if the number of electrons is even, then the number N_β is even and the number J_β is even, or the number N_β is odd and the number J_β is odd.

For the case of TB conditions one can introduce different flux values for up and down-spin electrons f_β, where the shift will have the form: $\Phi_{\beta\pm}(x) \Rightarrow$ $\Rightarrow \Phi_{\beta\pm}(x) \pm \sqrt{\pi} f_\beta x/L$. We separate the theory into two parts, introducing spin and charge fields $\varphi_s = (\Phi_\uparrow - \Phi_\downarrow)/2$ and $\varphi_c = (\Phi_\uparrow + \Phi_\downarrow)/2$, the fluxes of the electrical and magnetic field: $f_s = (f_\uparrow - f_\downarrow)/2$ and $f_c = (f_\uparrow + f_\downarrow)/2$ and the topological numbers: $N_s = N_\uparrow - N_\downarrow, J_s = J_\uparrow - J_\downarrow, N_c = N_\uparrow + N_\downarrow, J_c = J_\uparrow + J_\downarrow$. In terms of the topological numbers and the single valued variables π_β and Φ_β the Hamiltonian can be split into two parts $H = H_c + H_s$, where

$$H_c = A_c \int_0^L dx[\pi_c^2 + (\partial_x \varphi_c)^2] + \frac{A_c \pi}{16L}[(J_c + 4f_c)^2 + N_c^2], \tag{5}$$

$$H_s = A_s \int_0^L dx[\pi_s^2 + (\partial_x \varphi_s)^2] + \frac{A_s \pi}{16L}[(J_s + 4f_s)^2 + N_s^2] + $$
$$+ \frac{U}{2\pi^2 \alpha^2} \int_0^L dx \cos[\sqrt{16\pi}(\frac{\varphi_s}{A_s} + \frac{\sqrt{\pi} N_s x}{4A_s L})] \tag{6}$$

associated with the charge and spin degrees of freedom, respectively and $A^2_{c/s} = = t \sin k_F \pm U/\pi$. The choice of integer numbers J_c, N_c, J_s and N_s is dictated by the selection rules described above. For example, if the number of electrons N_e is odd, then this means that the numbers $N_\uparrow$ and $N_\downarrow$ have different parity, i.e. one of this numbers is odd the other is even, since $N_c = N_e = N_\uparrow + N_\downarrow$. This will mean that the numbers $J_\uparrow$ and $J_\downarrow$ have different parity, too and the number J_c is odd. The fact that the Hamiltonian for charge degrees of freedom is split into two parts and the number J_c consists of the sum of the two topological numbers $J_\uparrow$ and $J_\downarrow$ is the reason why the "holon" Hamiltonian has the flux period $f_T = \frac{1}{2}$ and not conventional $f_T = 1$.

In the case when N_e is even, the selection rules give that the numbers $N_\uparrow$ and $N_\downarrow$ have the same parity. This means that the AB effect is half-flux quantum periodic and the energy-flux dependence is described by parabolic segments with the minima located at the flux equal to integers and half-odd integers (see, eq.(5)). Thus, there occurs a new parity effect, where there is a difference in the behavior for the odd and the even numbers of electrons, i.e. there is a shift in the energy-flux dependence by a quarter of the elementary flux quantum. This is in contrast with the parity effect for spinless fermions [5, 8, 9], where the shift is by a half of the flux quantum. A similar situation occurs for an Aharonov–Cashier effect.

To proceed with the calculation of the current of the Hubbard ring we transfer our problem into the Lagrangian formalism. We drop the irrelevant spin degrees of the freedom and consider only the holon Lagrangian L_c and the action S_c. In the Lagrangian formalism our fields φ_c will depend on space and time variables $\varphi_c = \varphi_c(x,t)$ and will satisfy PB conditions for both variables. The multivalued field $\varphi_c(x,t)$ can be split into single valued field $\tilde{\varphi}_c(x,t)$ and terms related to the winding numbers n and m with the aid of the relation: $\varphi_c(x,t) = \tilde{\varphi}_c(x,t) + \sqrt{\pi} xn/(2L) + \sqrt{\pi} tm/(2L)$. Therefore, in the Lagrangian for the charge degrees of the freedom

$$L_c = -\int_0^L dx[\dot{\varphi}_c^2/(4A_c) + A_c(\partial_x \varphi_c)^2] + i\frac{\sqrt{\pi}}{4L}(J_0 + 4f)\int_0^L \dot{\varphi} dx, \tag{7}$$

the single valued field $\tilde{\varphi}_c(x,t)$ can be separated. The contribution of the orbital motion into the partition function Z of the ring can be calculated with the aid of the continual integral over the single valued field $\tilde{\varphi}_c(x,t)$ and sums over the winding numbers n and m [12, 13]:

$$Z_c = \int D\tilde{\varphi}_c \sum_{n,m} \exp[-S_c(\tilde{\varphi}_c, J_0, n, m)] \tag{8}$$

where the action has the form:

$$S_c = \int_0^L \int_0^\beta dxdt[\dot{\varphi}_c^2/(4A_c) + A_c(\partial_x\varphi_c)^2 - i\sqrt{\pi}\frac{\dot{\varphi}_c(J_0+4f)}{4L}] \tag{9}$$

After the summation over the winding numbers the partition function Z_c takes the form: $Z_c = Z_0\Theta_3(z_J, q_J)\Theta_3(z_m, q_m)$ where $\Theta_3(x, y)$ is the theta function and Z_0 is the partition function, associated with the single valued field $\tilde{\varphi}_c$;

$$z_J = \frac{(J_0+4f)\pi}{16}, \quad q_J = \exp(-\frac{\pi L}{16A_c\beta}), \quad z_m = 0, \quad q_m = \exp(-\frac{\pi A_c\beta}{4L}).$$

One can derive the low and high temperature asymptotic of the free energy $F = -T\log Z_c$. In the case of the low temperature limit $\beta \to \infty$ we have

$$\Delta F = \frac{\pi A_c(J_0+4f)^2}{16L} \tag{10}$$

which is a flux dependent term of eq.(5), where J_0 is even or odd, which corresponds to even or odd number of particles on the ring respectively. In the case when $\pi L/16A_c\beta >> 1$ or $\beta \to 0$, the contribution of the orbital motion is:

$$\Delta F = -2T\exp(-\frac{\pi LT}{16A_c})\cos(\frac{\pi(J_0+4f)}{8}). \tag{11}$$

Recently, after several beautiful experimental works [14] an enormous theoretical attention has been devoted to the problem of the persistent current (see, Refs.[7, 15] and references there). The problem was stimulated by the discrepancy between the amplitude of the current estimated theoretically and experimental observations. The experiments indicate that this amplitude is about several orders larger than the theories predict.

The persistent current at zero temperature is equal to

$$J_p = -\partial F/\partial f = -2\pi\frac{V_F}{L}(f + \frac{J_0}{4}), \tag{12}$$

where $-\frac{1}{4} - \frac{J}{4} \leq f \leq -\frac{J}{4} + \frac{1}{4}$ and $V_F = A_c$ which increases with U. This means the enhancement of the current with electron-electron interaction. At high temperatures this enhancement even larger:

$$J_p = -\pi T\exp(-\frac{\pi LT}{16V_F})\sin[\frac{\pi}{2}(f + \frac{J_0}{4})]. \tag{13}$$

Because of the exponential prefactor the current is strongly reduced with the temperature but increases exponentially with U. The characteristic temperature, where the current is still visible is about $T_c \sim V_F/L$, which is nothing but the inter-level distance of the size quantization. The described enhancement is not consistent with arguments of the Ref.[16] but agrees with numerical simulations [17].

In the strong-coupling limit $U \to \infty$ the problem can be diagonalized with the aid of the Bethe ansatz the spectrum obtained originally by the author [6] has the form:

$$K_n = \frac{2\pi I_n}{L} + \frac{2\pi}{L}\frac{\sum J_\alpha}{N} + \frac{2\pi f}{L}. \tag{14}$$

The (half) integers I_n and J_α are holon and spinon quantum numbers, respectively. If we introduce the notations $\phi = \sum J_\alpha/N$ then this equation looks like the spectrum of spinless fermions in the flux $f + \phi$. In the continuum limit for this spectrum one can write an effective Hamiltonian of spinless fermions:

$$H = \int_0^L [\psi^+(x)(K_{f\phi}^2 - k_F^2)\psi(x)]dx, \tag{15}$$

where $k_F = \pi N/L$, and $K_{f\phi} = K + \frac{2\pi\phi}{L} + \frac{2\pi f}{L}$ and K is a momentum operator.

For the comparison of the weak and strong-coupling cases we represent the holon Hamiltonian (15) in the bosonized form. With the aid of Loss result[10], the Hamiltonian of the charge degrees of freedom takes the form

$$H = V_F \int_0^L [\pi^2 + (\partial_x\varphi)^2]dx + \frac{V_F\pi}{L}[N^2 + (J + 2\phi + 2f)^2]. \tag{16}$$

In comparison with eq.(5), there appears the fictitious flux ϕ, having fractional values $\phi = p/N$. Without external magnetic field the selection rules have the form: $(-1)^{N+J} = (-1)^{N_e+1+\phi}$, where the value ϕ can be equal to 0 or 1. The latter means that the topological numbers N and J, which are in the Luttinger liquid coupled, now become decoupled. The latter came from the fact that the parity of N_e plays no role, since we can change the value ϕ from 0 to 1 and the value J by 2 without change of the energy. This indicates the violation of the conventional Luttinger liquid where the topological numbers N and J are coupled by the parity of N_e.

The parity effect appears at a finite U. Then the solution has a structure similar to eq.(14) plus the energy of the spin-wave excitations. Therefore, for an odd number of particles in the bosonized form the Hamiltonian is

$$H = V_F \int_0^L [\pi^2 + (\partial_x\varphi)^2]dx + \frac{V_F\pi}{L}[N^2 + (J + 2\phi + 2f)^2] + \frac{V_F^2}{LU} \mid \sin 2\pi\phi \mid . \tag{17}$$

One sees that in addition to N and J the topological quantum numbers, there appears a new term, which is an internal energy of the field ϕ and the energy of the spin-wave excitations. Now the value of the field ϕ can not take any rational number, i.e. the finite U lifts the degeneracy and the parity effect appears.

For an even number of particles we must change in eq.(17) the $\sin 2\pi\phi$ to $\cos 2\pi\phi$. At zero external magnetic flux the selection rules take the conventional form $(-1)^{N+J} = (-1)^{N_e-1}$, which dictates that $\phi = 0$. This corresponds to the maximum of the spin-wave excitation spectrum, i.e. with the external magnetic flux f there appears the spin-wave excitations (nonzero ϕ, compensating the f), which again indicates a violation of the conventional Luttinger liquid properties, where the field ϕ does not exist. This question needs special attention.

Thus, in weak coupling the magnetization is a half flux periodical function. The amplitude of the oscillations increases with U. Whenever the spin and charge degrees of freedom are separated and composite particles (they are the "holons and spinons" in the considered case) are created, half-flux periodical oscillations of the AB type occur (see, for comparison, Ref. [18]). Therefore, the period of the AB oscillations in any strongly correlated systems will always decrease.

This effect does not exist for interacting spinless fermions on a single channel ring [5, 19]. But the effect arises when the ring consists of two or many chains

[20]. The reason for the effect is similar to that for the Hubbard ring. One can prove exactly that the AB effect will have here the period of the half-flux quantum for any interactions which are no larger than the Fermi energy. When the interaction is comparable with the Fermi energy the continuum approach is not applicable and there can occur the fractional $1/N_e$ AB effect [6] or a fractional M/N_e AB effect [21]. Thus, if in the real HTSC materials in a normal state there occurs spin-charge separation, the AB effect must have a half-flux quantum period in the units of the elementary flux quantum. To observe such predictions in HTSC might be a good challenge for experimentalists.

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