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NON-OHMIC HOPPING CONDUCTIVITY IN THE EXPONENTIAL BAND TAIL

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The variable range hopping conductivity in strong electric field E at temperature $T = 0$ is investigated. An exponential increase of density of states $g(\epsilon)$ with energy ϵ is assumed. It is shown that there is an energy level $\epsilon = \epsilon_E(E)$ ("the transport level"), along which an electron hops without rising or lowering in average. The dependence $\sigma(\epsilon)$ has the form: $-\ln\sigma \sim 1/E$.

The present paper discusses the hopping conductivity (σ) along localized states in strong electric field (E). At present this problem is being intensively investigated both theoretically and experimentally in application to disordered semiconductors [1-4]. Here the situation is considered when the density of unoccupied states $g(\epsilon)$ increases from the Fermi level $\epsilon = \epsilon_F$ with the increase of energy ϵ; the temperature $T = 0$.

In the field E the Fermi level ϵ_F inclines. An electron gets the opportunity to jump from under the Fermi level by making an activationless hop against the field (Figure). This hop and subsequent hops along unoccupied states give rise to electric current.

Such mechanism of conductivity at $T = 0$ with $g(\epsilon)$=const was considered by Shklovskii [5]. It turned out that

$$-\ln\sigma \sim E^{-1/4}. \tag{1}$$

The hops take place in energy range $\sim (e^3E^3/g)^{1/4}$ near $\epsilon = \epsilon_F$.

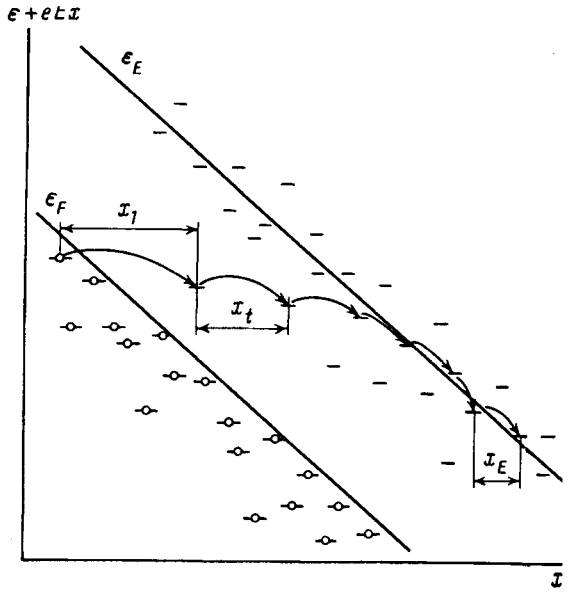
The one-dimensional situation will be examined at first. In this case a rather simple and complete investigation can be carried out. Let us suppose ϵ_F=0. An electron with energy ϵ (the charge(-e)) localized at the point $x = 0$ can make a hop to any unoccupied state with energy ϵ' and coordinate x, if $\epsilon' + eEx < e$. The concentration $N(\epsilon, x)$ of final states , accessible for an electron with initial energy $\epsilon > 0$ hopping from the point $x = 0$ to the point $x > 0$ (to the right) is

$$N(\epsilon, x) = \int_0^{\epsilon + eEx} g(\epsilon')\, d\epsilon'. \tag{2}$$

The typical hop length to the right $x_t(\epsilon)$ is determined from the condition

$$\int_0^{x_t} N(\epsilon, x)\, dx = 1 \tag{3}$$

(here and below $g(\epsilon)$ is the one dimensional density of states).



Scheme of electron transitions in disordered system of localized states in electric field

Let us determine the typical final state energy ε'_t. Hopping frequency ν is given by the expression

$$\nu = \nu_0 \exp\left(-\frac{2|x|}{a}\right), \tag{4}$$

where ν – characteristic frequency, a – state radius, $|x|$ – hop length. The frequency ν does not depend on ε and ε'. It means that an electron can get into any accessible state with equal probability. Following [6] ε'_t can be found from the condition, that the concentration of unoccupied states, laying below ε'_t, equals half of the accessible states concentration ("the typical hop approximation"):

$$\int_0^{\varepsilon'_t} g(\varepsilon)\, d\varepsilon = \frac{1}{2} N(\varepsilon, x_t). \tag{5}$$

The equations (3) and (5) determine ε'_t and x_t as functions of ε and E. We investigated the equations (3), (5) for the case when $g(\varepsilon)$ has the form

$$g(\varepsilon) = g_F \exp\left(-\frac{\varepsilon}{\varepsilon_0}\right) \tag{6}$$

(g_F – the density of states on the Fermi level, ε_0 – constant). Then the integrals in (3) and (5) are easily calculated. The results of this investigation are given below.

The basic result: the system (3), (5) has the solution $\varepsilon'_t = \varepsilon$ – the final energy is equal to the initial one. This solution takes place at some value $\varepsilon = \varepsilon_E$. In other words, there is an energy level ε_E – the transport level–along which the typical electron hops without lowering or rising. After excluding x_t from (3) and (5) the following equation is worked out for ε_E ($\alpha = eE/g_F\varepsilon_0^2$):

$$\exp\left(\frac{\varepsilon_E}{\varepsilon_0}\right) - 1 - \ln\left[2 - \exp\left(\frac{\varepsilon_E}{\varepsilon_0}\right)\right] = \alpha. \tag{7}$$

In week E there is a similar level ε_T, which position is determined by temperature [7]. Assuming the existence of effective temperature in strong E, authors [8] suggested by the analogy that there exists a transport level $\varepsilon_E(E)$. Our results show that such a level really exists. (About generalization on three-dimensional case see below).

Further, it can be shown that for any hop the final energy ε' is always located between ε and ε_E. This means that an electron with energy $\varepsilon < \varepsilon_E$ ($\varepsilon > \varepsilon_E$) by means of hopping gradually approaches the transport level $\varepsilon = \varepsilon_E$ from below (from above).

The conductivity mechanism can be considered now in the following way. An electron jumps from under the Fermi level into the region of unoccupied states $\varepsilon > 0$ (the field generation of electrons). This is the first, the longest and, consequently, the hardest hop. Then an electron makes a series of consecutively shortening hops, approaching the level $\varepsilon = \varepsilon_E$. Electron concentration n in the region $\varepsilon > 0$ increases. This increase is limited by electron-hole recombination. If electron life time is great enough electrons will accumulate at the level ε_E. Within our approximation there are no electrons in the region $\varepsilon > \varepsilon_E$.

When $\alpha \ll 1$ the hops take place in the small $(\varepsilon_0\sqrt{\alpha})$ vicinity of Fermi level (the one–dimensional analogy of the case investigated in [5]). We do not consider this case here. When $\alpha \gg 1$ it comes out ($x_E \equiv x_t(\varepsilon_E)$):

$$\varepsilon_E = \varepsilon_O \ln \alpha, \qquad \frac{eEx_E}{\varepsilon_0} \simeq \ln 2. \tag{8}$$

It is seen that ε_E – level is situated sufficiently far (at a distance of several ε_0) from the Fermi level.

It can be shown that the existence of the transport level is caused by the increasing profile of $g(\varepsilon)$ rather than by a specific form of (6) which is used here to obtain the analytical expressions.

Let us consider now the field generation of electrons. An electron with energy $\varepsilon < 0$ makes a hop to the right from the point $x = 0$ to another point $x > 0$ with $\varepsilon' > 0$ (Figure). The least distance at which an unoccupied state can exist is $(-\varepsilon/eE)$. Therefore the typical first hop length x_1 is determined from the equation (3) with $(-eE/\varepsilon)$ as a lower limit in the integral, and typical final energy ε_1' – from the equation (5), where ε_t' should be substitute for ε_1' and x_t – for x_1. When $\alpha \gg 1$ it comes out, that

$$\frac{eEx_1}{\varepsilon_0} = \ln(\alpha) - \frac{\varepsilon}{\varepsilon_0}, \qquad \frac{\varepsilon_1}{\varepsilon_0} = \ln\left(\frac{\alpha}{2}\right). \tag{9}$$

If $x_1(\varepsilon)$ is known, the field generation rate G(E) per length unit can be calculated as

$$G(E) = \int_{-\infty}^{0} g^*(\varepsilon)\nu[x_1(\varepsilon)]\, d\varepsilon$$

Here $g^*(\varepsilon)$–density of occupied states. (the specific form of $g^*(\varepsilon)$ turns out to be unsignificant).

To calculate electron linear concentration n in states with $\varepsilon > 0$ one needs to know the life time $\tau(\varepsilon)$. When τ is great enough most of electrons reaches the level $\varepsilon = \varepsilon_E$ and have time to make many hops along the level. We shall

restrict ourself here to this case and shall find n from the relation $n = G(E)\tau_E$ ($\tau \equiv \tau(\varepsilon_E)$). Using electron drift velocity $v_E = x\nu(\varepsilon_E) = \nu_0 x_E \exp(-2x_E/a)$ we obtain current $j = env_E$.

The time τ_E can depend on E. We suppose this dependence is not exponential. Then when $\alpha \gg 1$ it turns out, that

$$\ln\sigma \sim \left(-\frac{2\varepsilon_0}{eEa}\right) \ln(2\alpha). \tag{10}$$

We have observed the dependence $-\ln\sigma \sim E^{-1}$ in crystalline noncompensated silicon [9]. Conductivity was determined by hops along the impurity D^-–band tail states. (A detailed report on these results will be published later). It is believed at present [10] the D^-–band tail density of states of doped noncompensated crystalline semiconductor rapidly increases with the increase of ε, as was supposed above.

Certainly, it should keep in mind that the dependence $\ln\sigma \sim (-E^{-1})$ was observed in three dimensional samples. However, it is easy to show that for electrons with $\varepsilon = \varepsilon_E$ the hop frequency opposite the field is $\exp[\varepsilon_0(1 - \ln 2)/eEa]$ times greater than across E if $\alpha \gg 1$. (The typical hop lengths are compatible). When $\varepsilon < \varepsilon_E$ the difference is still great. Therefore when $\alpha \gg 1$ and $eEa/\varepsilon_0 \ll 1$ (the latter inequality take place in all experiments we know) the three-dimensional situation turns into one–dimensional one. Hence the main result of this work must be true for three-dimensional samples.

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