Pis'ma v ZhETF, vol. 80, iss. 7, pp. 563 – 568

Effects of spin-orbit interaction on superconductor-ferromagnet heterostructures: spontaneous electric and spin surface currents

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Submitted 24 August 2004
Resubmitted 8 September 2004

We find proximity-induced spontaneous spin and electric surface currents, at all temperatures below the superconducting T_c, in an isotropic s-wave superconductor deposited with a thin ferromagnetic metal layer with spin-orbit interaction. The currents are carried by Andreev surface states and generated as a joint effect of the spin-orbit interaction and the exchange field. The background spin current arises in the thin layer due to different local spin polarizations of electrons and holes, which have almost opposite velocities in each of the surface states. The spontaneous surface electric current in the superconductor originates in asymmetry of Andreev states with respect to sign reversal of the momentum component parallel to the surface. Conditions for electric and spin currents to show up in the system, significantly differ from each other.

PACS: 74.45.+c, 74.81.−g

Proximity effects in superconductor-ferromagnet heterostructures have attracted much attention for recent years. In contrast with the nonmagnetic case, magnetic surfaces and interfaces make spin-flip processes possible and suppress an s-wave superconducting order parameter, generating Andreev bound states in adjacent superconducting regions [1 – 5]. Spin structure of Andreev bound states near complex magnetic interfaces can be rather involved [5]. Triplet components of the order parameter in a singlet superconductor can be induced by ferromagnets under certain conditions [6]. Cooper pair wave functions exponentially decay into the bulk of ferromagnets, oscillating at the same time [7], and acquire a triplet component in the ferromagnetic region [8]. These proximity effects can lead, in particular, to specific properties of the Josephson current through magnetic interfaces, which have been intensively studied both theoretically and experimentally [7, 9 – 11, 1 – 3, 12 – 14, 4 – 6]. Also, proximity-induced nonmonotonic dependence of the superconducting critical temperature on the thickness of the ferromagnetic layer has been thoroughly studied for superconductor-ferromagnetic metal bilayers or heterostructures (see, for example, [10, 11, 15 – 18] and references therein).

Spontaneous surface currents represent other important example of possible proximity-induced effects. Spontaneous electric currents, taking place near surfaces or interfaces on the scale of the superconducting coherence length, produce a magnetic field and, hence, a counterflow of screening supercurrents on the scale of the penetration depth. The electric current can arise, for example, near nonmagnetic surfaces and interfaces of unconventional superconductors, whose states break time-reversal symmetry [19, 20]. In particular, the electric current, carried by Andreev states, appears near nonmagnetic surfaces and interfaces of chiral superconductors [21]. The current also occurs, if a surface-induced subdominating pairing shows up near surfaces of d-wave superconductors, breaking time-reversal symmetry of the superconducting state [22]. Other possible mechanism generating electric surface currents, is specifically based on a paramagnetic response of the zero-energy Andreev surface states to an applied magnetic field. This can take place at low temperatures at smooth (110) surfaces of d-wave superconductors [23, 24], as well as in a system with a thin ferromagnetic metallic layer deposited on a semi-infinite bulk isotropic s-wave superconductor [25]. In the latter case the energy of the surface states becomes zero only for several values of the layer thickness and, in the presence of particle-hole asymmetry, the spontaneous electric current is accompanied by a spontaneous surface spin current [25]. Dissipationless background spin currents, which take place in various systems in the equilibrium and do not lead to any spin accumulation, have been a subject of recent discussions and studies [26 – 28]. The spin currents can be generated, for example, by the spin-orbit interaction (in particular, via Rashba term) in two-dimensional metals. Measurements of background persistent spin currents are not carried out for now, although some suggestions for a direct detection of these currents have been proposed in the literature [26, 29].

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In the present paper we study spontaneous currents under conditions, when spin-orbit interaction takes place in a thin ferromagnetic metal layer in proximity to an isotropic s-wave superconductor. Joint effect of the spin-orbit interaction, described by the Rashba term, and the exchange field is shown to play an important role in generating spontaneous currents. We find that superconductor induces background spin currents in the ferromagnetic layer with the spin-orbit interaction (FSOL) for all temperatures below the superconducting T_c. This spin current is carried by Andreev surface states and takes finite values due to different local spin polarizations of electrons and holes, which have almost opposite velocities in each of these states. Maximal possible values of the background spin current density are the order of the Landau depairing current density. We find proximity-induced finite spin currents within the quasiclassical approach, when only linear terms in small parameters $\alpha p_f/\varepsilon_f$, h/ε_f are taken into account in describing the FSOL. Spontaneous background spin currents, arising in the two-dimensional electron systems with spin-orbit interaction without any proximity effects [27], contain higher powers of these parameters which are assumed small below. Further, a spontaneous electric surface current, carried by Andreev surface states, arises in the superconductor due to proximity to the FSOL. Respective structures of wave functions and spectra of the surface states are strongly influenced by the spin-orbit interaction and the exchange field and differ for quasiparticles with opposite momentum components $\mathbf{p}_{f\|}$ parallel to the surface. The spontaneous electric current arises as a result of this asymmetry of Andreev states with respect to $\mathbf{p}_{f\|} \to -\mathbf{p}_{f\|}$, to some extent analogously to the current induced by chiral surface or interface states. Conditions for electric and spin currents to show up in the system we study, significantly differ from each other. Thus, the spontaneous spin current in the FSOL arises even within the framework, when the surface electric current vanishes.

Consider an isotropic s-wave superconductor at $x > d$, deposited with a layer of thickness d made of a ferromagnetic metal. Let a macroscopic thickness of the layer be much less than the superconducting coherence length: $d \ll \xi_s$. Both the internal exchange field $\mathbf{h}$ and spin-orbit Rashba term $\mathbf{w}\boldsymbol{\sigma} = \alpha(\mathbf{n}\times\mathbf{p}_{||})\boldsymbol{\sigma}$ enter the Hamiltonian density of the FSOL: $\hat{\mathcal{H}}(x) = \hat{H}^{(0)} - (\mathbf{h}(x) + \mathbf{w}(x))\,\boldsymbol{\sigma}$. Here $\hat{H}^{(0)}$ describes the kinetic energy of free electrons, $\mathbf{n}$ is the unit vector along the surface normal and $\mathbf{p}_{||}$ the momentum component parallel to the surface. The exchange field is assumed always aligned along the z-axis. Both $\mathbf{h}(x)$ and $\mathbf{w}(x)$ are taken finite and spatially constant within the FSOL $0 < x < d$. The x-axis is taken directed into the depth of the superconductor and the system is confined by an impenetrable wall at $x = 0$.

We assume $\Delta \ll h, \alpha p_f \ll \varepsilon_f$ and describe the system in question by quasiclassical Eilenberger equations for Matsubara Green's function:

$$-iv_{f,x}\frac{\partial \check{g}}{\partial x} = \left[\left(i\varepsilon_n\hat{\tau}_z + \hat{\tau}_z\check{\Delta} + \mathbf{h}\check{\mathbf{s}} + \mathbf{w}\hat{\tau}_z\check{\mathbf{s}}\right), \check{g}\right], \qquad (1)$$

$$\check{g}^2 = -\pi^2. \qquad (2)$$

Here $\check{g}(x,\mathbf{p}_f,\varepsilon_n)$ takes 4×4 matrix form in the four-dimensional product space of particle-hole and spin variables. In the particle-hole space

$$\check{g}(\mathbf{p},\varepsilon_n,x) = \begin{pmatrix} \hat{g}(\mathbf{p},\varepsilon_n,x) & \hat{f}(\mathbf{p},\varepsilon_n,x) \\ \hat{\underline{f}}(\mathbf{p},\varepsilon_n,x) & \hat{\underline{g}}(\mathbf{p},\varepsilon_n,x) \end{pmatrix}, \qquad (3)$$

where all matrix elements are 2×2 matrices in spin space. Pauli-matrices in particle-hole space are $\hat{\tau}_j$, $\hat{\tau}_\pm = \hat{\tau}_x \pm i\hat{\tau}_y$, while in spin space $\hat{\sigma}_i$. The superconducting order parameter matrix is $\check{\Delta} = 1/2[\hat{\tau}_+\Delta - \hat{\tau}_-\Delta^*]i\hat{\sigma}_y$. The operator for quasiparticle spin is $(1/2)\check{\mathbf{s}}\hat{\tau}_z$, whereas the operator $\check{\mathbf{s}} = 1/2[(1+\hat{\tau}_z)\hat{\boldsymbol{\sigma}} - (1-\hat{\tau}_z)\hat{\sigma}_y\hat{\boldsymbol{\sigma}}\hat{\sigma}_y]$ enters the Zeeman term. The order parameter Δ is taken spatially constant throughout the superconducting half-space $x > d$. As this follows from recent results for two-dimensional superconductors with spin-orbit coupling [30], a possibility for proximity-induced inhomogeneous phase of the order parameter in the plane parallel to the interface should be studied for sufficiently thin superconducting layer in proximity to the FSOL. This two-dimensional inhomogeneous profile of the phase does not appear, however, for a massive superconducting sample.

Electric and spin current densities can be expressed via quasiclassical Green's function as follows

$$\mathbf{j} = N_f T\left\langle \mathbf{v}_f \sum_{\varepsilon_n} \mathrm{Sp}_2\hat{g}(\mathbf{p}_f,\varepsilon_n)\right\rangle_{S_f}, \qquad (4)$$

$$\mathbf{j}^s_i = \frac{N_f T}{2}\left\langle \mathbf{v}_f \sum_{\varepsilon_n} \mathrm{Sp}_2\hat{\sigma}_i\hat{g}(\mathbf{p}_f,\varepsilon_n)\right\rangle_{S_f}. \qquad (5)$$

Here N_f is the normal state density of states per spin direction, $<\ldots>_{S_f}$ means averaging over quasiparticle states at the Fermi surface. Spin current j^s_{il} carries i-th spin component along l-axis in coordinate space. One can introduce scalar g_0 and vector $\mathbf{g}$ components of the Green's function in spin space $\hat{g}(\mathbf{p}_f,\varepsilon_n) = g_0(\mathbf{p}_f,\varepsilon_n)\hat{\sigma}_0 + \mathbf{g}(\mathbf{p}_f,\varepsilon_n)\hat{\boldsymbol{\sigma}}$. As this is seen from Eqs. (4), (5), g_0 determines electric current, while spin current is associated with $\mathbf{g}$. We should emphasize, that (5) is an approximate equation, which is valid only within the quasiclassical accuracy. One can safely calculate with Eq. (5) the terms in the spin current of the order of $N_f v_f \alpha p_f$, $N_f v_f h$, $N_f v_f \Delta$, which can contain also any functions of $(\alpha p_f/h)$ and/or $\alpha p_f/\Delta$. However, terms with additional powers of small quasiclassical parameters $(\alpha p_f/\varepsilon_f)$, (Δ/ε_f) and (h/ε_f), lie beyond the accuracy of Eq. (5). These terms should be described with Gor'kov equations and the exact symmetrized operator for the spin current $\hat{J}_{ij} = (1/2)[(p_j/m)\hat{\sigma}_i + e_{xij}\alpha]$. The second term in the last expression is of the order $\alpha p_f/\varepsilon_f$ with respect to the first one. For this reason it contributes to the spin current beyond the quasiclassical accuracy and is not taken into account in Eq. (5).

The Green's function for the FSOL satisfies conventional boundary conditions on the impenetrable wall at $x = 0$: $\check{g}(0, \mathbf{p}_f, \varepsilon_n) = \check{g}(0, \tilde{\mathbf{p}}_f, \varepsilon_n)$, where $\mathbf{p}_f$ and $\tilde{\mathbf{p}}_f$ are the incoming and the outgoing quasiparticle momenta respectively. We match solutions of Eilenberger equations for the superconduting half-space and for the FSOL with the continuity conditions on a transparent interface at $x = d$. Substituting the final result for the Green's function into Eqs. (4), (5), we find no spontaneous electric current in the system and finite components j^s_{yz}, j^s_{zy} of spin current situated in the FSOL and flowing parallel to the surface.

One can show that the whole spin current is carried by Andreev surface states taking place in the system. We find two dispersive branches of Andreev surface states, whose energies depend on momentum component parallel to the surface:

$$\varepsilon_{1,2} = \mp \mathrm{sgn}\left[\sin\left(\frac{\Phi}{2}\right)\right] \Delta \cos\left(\frac{\Phi}{2}\right) \quad . \tag{6}$$

Here

$$\cos\Phi = \cos^2\frac{\varphi}{2}\cos\frac{\Theta_+ + \Theta_-}{2} + \sin^2\frac{\varphi}{2}\cos\frac{\Theta_+ - \Theta_-}{2}, \tag{7}$$

$$\Theta_\pm = \frac{4|\mathbf{h}\pm\mathbf{w}|d}{|v_{f,x}|}, \quad \cos\varphi = \mathbf{e}_+\mathbf{e}_-, \quad \mathbf{e}_\pm = \frac{(\mathbf{h}\pm\mathbf{w})}{|\mathbf{h}\pm\mathbf{w}|}. \tag{8}$$

In the absence of spin-orbit interaction spectra of Andreev states, descibed by Eqs. (6)–(8), reduce to the results for spin-discriminated Andreev states at a ferromagnetic surface [1, 4].

Andreev surface states carry no spin current in a singlet superconductor, since particles and holes, occupying the state, have identical spatially constant local spin polarization and opposite velocities. However, the wave function of Andreev surface states does not vanish in the FSOL and has a qualitatively different spin structure there, as compared with the superconducting region. One can extract pole-like terms from the whole expression for the electron retarded Green's function $\hat{g}^R(x, \mathbf{p}_f, \varepsilon)$ near bound state energies $\varepsilon_{1,2}$. We determine the spin structure of electrons in the Andreev states in terms of eigenvectors of these pole-like terms in spin space $\binom{\alpha}{\beta}(x, \mathbf{p}_f, \varepsilon)$. The unit vector $\mathbf{P}^e$, describing electron spin polarization, can be found from the equation $\mathbf{P}^e\hat{\boldsymbol{\sigma}}\binom{\alpha}{\beta}(x, \mathbf{p}_f, \varepsilon) = \binom{\alpha}{\beta}(x, \mathbf{p}_f, \varepsilon)$. As a result, we obtain the following spatially dependent spin polarization for electrons in Andreev surface states (6) at $0 < x < d$:

$$\begin{aligned}\mathbf{P}^e(\mathbf{p}_f, \varepsilon_{1,2}) = \mp\frac{1}{\sin\Phi}\Bigg[&(\mathbf{e}_+\times\mathbf{e}_-)\sin\frac{\Theta_+ x}{2d}\sin\frac{\Theta_-}{2} - \\ -&(\mathbf{e}_+\times(\mathbf{e}_+\times\mathbf{e}_-))\sin\frac{\Theta_-}{2}\left(\cos\frac{\Theta_+ x}{2d} - \cos\frac{\Theta_+}{2}\right) + \\ +&\left(\mathbf{e}_-\cos\frac{\Theta_+}{2}\sin\frac{\Theta_-}{2} + \mathbf{e}_+\cos\frac{\Theta_-}{2}\sin\frac{\Theta_+}{2}\right)\Bigg].\end{aligned} \tag{9}$$

Local spin polarization of electrons, occupying Andreev states, is spatially constant inside the superconductor and takes there the same value as follows from Eq. (9) at $x = d$. Parallel and normal to the surface components of spin polarizations, taken for incoming and outgoing electrons in one and the same Andreev state, are related with each other as $\mathbf{P}^e_\parallel(\tilde{\mathbf{p}}_f, \varepsilon_{1,2}) = \mathbf{P}^e_\parallel(\mathbf{p}_f, \varepsilon_{1,2})$, $\mathbf{P}^e_\perp(\tilde{\mathbf{p}}_f, \varepsilon_{1,2}) = -\mathbf{P}^e_\perp(\mathbf{p}_f, \varepsilon_{1,2})$. Also, since $\varepsilon_1 = -\varepsilon_2$, we find from Eq. (9), that $\mathbf{P}^e(\mathbf{p}_f, -\varepsilon_m) = -\mathbf{P}^e(\mathbf{p}_f, \varepsilon_m)$.

Spin polarization $\mathbf{P}^h$ for holes, occupying Andreev states, can be derived from Eq. (9). The quantity $\mathbf{P}^h$ satisfies the equation $-\mathbf{P}^h i\hat{\sigma}_y\hat{\boldsymbol{\sigma}}i\hat{\sigma}_y\binom{\alpha_h}{\beta_h}(x, \mathbf{p}_f, \varepsilon) = \binom{\alpha_h}{\beta_h}(x, \mathbf{p}_f, \varepsilon)$, which contains the spin operator for holes $-(1/2)i\hat{\sigma}_y\hat{\boldsymbol{\sigma}}i\hat{\sigma}_y$. Here $\binom{\alpha_h}{\beta_h}(x, \mathbf{p}_f, \varepsilon)$ is the eigenvector of the pole-like term in the Green's function $\hat{\tilde{g}}^R(x, \mathbf{p}_f, \varepsilon)$ near ε_1 or ε_2. As this follows from the general relation $\hat{\tilde{g}}^R(x, \mathbf{p}_f, \varepsilon) = \hat{g}^{A^*}(x, -\mathbf{p}_f, -\varepsilon)$, the eigenstates for holes and for electrons are associated with each other as $\binom{\alpha_h}{\beta_h}(x, \mathbf{p}_f, \varepsilon) = \binom{\alpha^*}{\beta^*}(x, -\mathbf{p}_f, -\varepsilon)$. Hence, the spin polarization for holes in the state $\binom{\alpha_h}{\beta_h}(x, \mathbf{p}_f, \varepsilon)$ coincides with that for electrons in the state $\binom{-\beta^*}{\alpha^*}(x, -\mathbf{p}_f, -\varepsilon)$. Further, as this follows from the equation for $\mathbf{P}^e$ and the relation between $\mathbf{P}^e(\mathbf{p}_f, \varepsilon_m)$ and $\mathbf{P}^e(\tilde{\mathbf{p}}_f, -\varepsilon_m)$, the quantity $\mathbf{P}^h(\mathbf{p}_f, \varepsilon_m)$ coincides with electron spin polarization in the state $\binom{\alpha^*}{-\beta^*}(x, p_{fx}, -\mathbf{p}_{f\parallel}, \varepsilon_m)$. Comparing electron spin polarizations of states $\binom{\alpha^*}{-\beta^*}(x, p_{fx}, -\mathbf{p}_{f\parallel}, \varepsilon_m)$ and $\binom{\alpha}{\beta}(x, p_{fx}, -\mathbf{p}_{f\parallel}, \varepsilon_m)$, we find finally that spin polarization for holes can be found from Eq. (9) as

$\mathbf{P}^h_{\parallel}(\mathbf{p}_f, \varepsilon_{1,2}) = \mathbf{P}^e_{\parallel}(p_{fx}, -\mathbf{p}_{f\parallel}, \varepsilon_{1,2})$, $\mathbf{P}^h_{\perp}(\mathbf{p}_f, \varepsilon_{1,2}) = -\mathbf{P}^e_{\perp}(p_{fx}, -\mathbf{p}_{f\parallel}, \varepsilon_{1,2})$.

The first term in Eq. (9) describes $\mathbf{P}^e_{\perp}$ component of the spin polarization, while the second and third terms determine $\mathbf{P}^e_{\parallel}$. Under the transformation $\mathbf{p}_{f\parallel} \to -\mathbf{p}_{f\parallel}$ one finds $\mathbf{e}_{\pm} \to \mathbf{e}_{\mp}$ and $\Theta_{\pm} \to \Theta_{\mp}$. The first two terms in the square brackets in Eq. (9) are responsible for a spatially dependent difference between electron and hole local spin polarizations, taking place in Andreev states in FSOL as the joint effect of Zeeman and spin-orbit couplings. Indeed, for vanishing h or α vectors $\mathbf{e}_{\pm}$ become parallel to each other. Then the first and the second terms in Eq. (9) vanish, resulting in identical spin polarizations of electrons and holes. Also, at $x = d$ this follows from Eq. (9) $\mathbf{P}^e(\mathbf{p}_f, \varepsilon_{1,2}) = \mathbf{P}^h(\mathbf{p}_f, \varepsilon_{1,2})$.

Different spin polarizations and almost opposite velocities of electrons and holes, occupying Andreev surface states Eq. (6), result in a net spin current in the FSOL. The local spin current density carried by two Andreev states can be represented as $j^s_{i,\parallel} = j^{s,1}_{i,\parallel} + j^{s,2}_{i,\parallel}$, where

$$j^{s,m}_{i,\parallel} = \frac{1}{2}\left\langle \mathbf{v}_{f,\parallel} W_m \left[P^e_i(\mathbf{p}_f, \varepsilon_m) - P^h_i(\mathbf{p}_f, \varepsilon_m)\right] \times \times n_f(\varepsilon_m)\right\rangle_{S_f}. \tag{10}$$

Here $W_m = \frac{1}{2}\pi\Delta N_f \left|\sin\frac{\Phi}{2}\right|$ is the weight of the delta-peak in the local density of states, taken in the FSOL for m-th Andreev state ($m = 1, 2$), and $n_f(\varepsilon)$ is the Fermi distribution function for quasiparticles. Substitution of the represented results into Eq. (10) gives exactly the spin current density, which follows from Eq. (5) and respective solutions of the Eilenberger equations for the Green's function.

As this follows from Eq. (10) after integration over the Fermi surface, only parallel to the surface components $j^s_{y,z}(x)$ and $j^s_{z,y}(x)$ of the spin current remain finite. The spin current $j^s_{x,\parallel}$, carrying along the surface perpendicular to the surface spin component, vanishes in accordance with the relation $\mathbf{P}^e_{\perp}(\tilde{\mathbf{p}}_f, \varepsilon_{1,2}) = -\mathbf{P}^e_{\perp}(\mathbf{p}_f, \varepsilon_{1,2})$, since the contributions from incoming and outgoing electrons, as well as holes, cancel each other. The proximity-induced background spin current we have found does not lead to any spin accumulation. Since spin does not conserve due to the presence of the spin-orbit coupling, the local conservation equation for the spin current contains "external" sources: $\sum_l \partial j^s_{il}/\partial x_l = -2N_f T \sum_{\varepsilon_n} \langle [(\mathbf{h} + \mathbf{w}) \times \mathbf{g}]_i \rangle_{S_f}$. One can show that these sources, taking place for each separate quasiparticle trajectory, cancel each other in the averaging over the Fermi surface. Eventually, the proximity-induced background spin current in the problem in question satisfies the continuity equation $\sum_l \partial j^s_{il}/\partial x_l = 0$.

In the limit of small Zeeman coupling $h \ll \alpha p_f$, we find the following simple estimations for the spin current in the thin layer with spin-orbit interaction $d \ll v_f/(\alpha p_f)$:

$$j^s_{\alpha\beta} = -A_{\alpha\beta}\left(\frac{h}{\alpha p_f}\right)^2 \left(\frac{d\alpha p_f}{v_f}\right) j_{cL}\,. \tag{11}$$

Here $\alpha, \beta = y, z$ and $\alpha \neq \beta$, $A_{\alpha\beta} > 0$ is a constant the order of unity, $j_{cL} = n_s\Delta/p_f$ is the Landau depairing current density. At low temperatures $j_{cL} \sim N_f v_f \Delta$.

In the opposite limit $h \gg \alpha p_f$, when the exchange field in the FSOL significantly exceeds spin-orbit coupling, estimations for the two components of the spin current give different results:

$$j^s_{yz} = B_{yz}\left(\frac{\alpha p_f}{h}\right)\left(\frac{dh}{v_f}\right) j_{cL}\,, \tag{12}$$

$$j^s_{zy} = -B_{zy}\left(\frac{\alpha p_f}{h}\right)^3 \left(\frac{dh}{v_f}\right) j_{cL}\,. \tag{13}$$

Here B_{yz}, B_{zy} are constants of the order of unity. For $h \sim \alpha p_f \sim v_f/d$ spontaneous spin current densities reach the maximal value of the order of j_{cL}.

Background spin current density, arising without any proximity effects in the two-dimensional metal with Rashba spin-orbit interaction [27], takes the form $(\alpha p_f/\varepsilon_f)^3 \varepsilon_f N_f v_f/6$. It is of the third-order in parameter $\alpha p_f/\varepsilon_f$, which is presumably a small quasiclassical parameter. These spin currents are carried by all the occupied states at a given temperature [27, 28], in contrast with the currents induced by a proximity to the superconductor. For this reason respective reference quantity $\varepsilon_f N_f v_f$ contains a large parameter ε_f/Δ as compared with j_{cL}. The quantity j_{cL}, characterizing spontaneous spin current densities calculated above, exceeds the result [27] under the condition $\Delta > \alpha p_f\,(\alpha p_f/\varepsilon_f)^2$.

We return now to the problem of spontaneous surface electric current. Each separate Andreev surface state, taken for given $\mathbf{p}_{\parallel}$, carries finite surface electric current. There is no net electric current under the conditions considered above, since electric currents carried by Andreev surface states Eq. (6)–(8) with $\mathbf{p}_{\parallel}$ and $-\mathbf{p}_{\parallel}$ cancel each other. This is associated with the symmetry of scalar component g_0 of the quasiclassical Green's function with respect to the sign reversal of the momentum parallel to the surface. Spin current takes finite values since vector component $\mathbf{g}$ of the Green's function does not possess the symmetry. However, the symmetry of g_0 turns out to be approximate, taking place only under the conditions $\alpha p_f, h \ll \varepsilon_f$, within the quasiclassical approximation applied to the FSOL. For this reason we find below finite spontaneous surface electric current, assuming $\Delta \ll \alpha p_f, h \lesssim \varepsilon_f$ and applying the $\check{S}$-matrix approach for describing the FSOL. Then the Eilenberger equations should be solved only for the superconducting region, whereas effects of the FSOL are taken into account via respective boundary conditions.

A surface with the FSOL is characterized by the normal-state scattering $\check{S}$-matrix, contained reflection amplitudes for quasiparticles. The $\check{S}$-matrix can be represented as $\check{S} = \hat{S}(1+\hat{\tau}_z)/2 + \hat{\underline{S}}(1-\hat{\tau}_z)/2$, where $\hat{\underline{S}}(\mathbf{p}_{f\|}) = \hat{S}^{tr}(-\mathbf{p}_{f\|})$ and

$$\hat{S} = \begin{pmatrix} r_{\uparrow\uparrow} & r_{\uparrow\downarrow} \\ r_{\downarrow\uparrow} & r_{\downarrow\downarrow} \end{pmatrix} = \frac{1}{2}\left[r_\uparrow + r_\downarrow + (r_\uparrow - r_\downarrow)\frac{\mathbf{h}+\mathbf{w}}{|\mathbf{h}+\mathbf{w}|}\boldsymbol{\sigma}\right]. \tag{14}$$

Here $r_{\uparrow,\downarrow} = e^{i\Theta_{\uparrow,\downarrow}}$ and, assuming spatially constant $\mathbf{h}$ and α in the FSOL,

$$\Theta_{\uparrow,\downarrow} = \pi + 2\mathrm{arctan}\left[\frac{|p_{fx}|}{p_{fx\uparrow,\downarrow}}\tan(p_{fx\uparrow,\downarrow}d)\right] - 2|p_{fx}|d, \tag{15}$$

where Fermi momenta in the normal metal $\mathbf{p}_f$ and in the FSOL $\mathbf{p}_{f\uparrow,\downarrow}$ satisfy the relation $p^2_{fx\uparrow,\downarrow} = p^2_{fx} \pm 2m|\mathbf{h} + \mathbf{w}(\mathbf{p}_{f\|})|$.

Making use of explicit expression for the $\check{S}$-matrix (14) and following the quasiclassical approach with Riccati amplitudes in describing the superconductor [31, 1], we obtain the quasiclassical Green's function. In particular, we obtain spectra of Andreev surface states, which take the following form now

$$\varepsilon_{1,2} = \mathrm{sgn}\left[\sin\left(\frac{X\mp\Phi}{2}\right)\right]\Delta\cos\left(\frac{X\mp\Phi}{2}\right). \tag{16}$$

Here $X(\mathbf{p}_{f\|}) = \frac{1}{2}(\Theta_\uparrow(\mathbf{p}_{f\|}) + \Theta_\downarrow(\mathbf{p}_{f\|}) - \Theta_\uparrow(-\mathbf{p}_{f\|}) - \Theta_\downarrow(-\mathbf{p}_{f\|}))$ and $\Phi(\mathbf{p}_{f\|})$ is defind in Eq. (7), where one should use the generalized definition for $\Theta_\pm(\mathbf{p}_{f\|})$: $\Theta_\pm(\mathbf{p}_{f\|}) = \Theta_\uparrow(\pm\mathbf{p}_{f\|}) - \Theta_\downarrow(\pm\mathbf{p}_{f\|})$. For a small parameter $|\mathbf{h}+\mathbf{w}|/\varepsilon_f \ll 1$ the quantity $X(\mathbf{p}_{f\|})$ vanishes in the first approximation, while the definition for $\Theta_\pm$ reduces to that given in Eq. (8).

In general, energies $\varepsilon_{1,2}(\mathbf{p}_{f\|})$ in Eq. (16) are situated asymmetrically with respect to the Fermi level for a given $\mathbf{p}_{f\|}$. Since $X(\mathbf{p}_{f\|})$ and $\Phi(\mathbf{p}_{f\|})$ are odd and even functions of $\mathbf{p}_{f\|}$ respectively, each energy branch $\varepsilon_{1,2}(\mathbf{p}_{f\|})$ in Eq. (16), as well as the Andreev spectra as a whole, is neither odd nor even with respect to the transformation $\mathbf{p}_{f\|} \to -\mathbf{p}_{f\|}$: $\varepsilon_{1,2}(-\mathbf{p}_{f\|}) = -\varepsilon_{2,1}(\mathbf{p}_{f\|})$. As a result of the asymmetry, the spontaneous electric current density $j_y(x)$, flowing along the surface perpendicular to the exchange field in the superconducting region, arises near the surface with the FSOL. The spontaneous surface current density at the interface $x = d$ takes comparatively simple form in the case of small spin-orbit coupling $\alpha p_f \ll (\varepsilon_f \pm h)$:

$$j_y(d) = = \frac{\pi e N_f \Delta}{2}\left\langle v_{fy}\left(\frac{\Delta}{2T}\sin^2\frac{\Theta_0}{2}\cosh^{-2}\frac{\Delta\cos\Theta_0}{2T} - \cos\frac{\Theta_0}{2}\tanh\frac{\Delta\cos\Theta_0}{2T}\right)X(\mathbf{p}_{f\|})\right\rangle_{S_f}. \tag{17}$$

Here Θ_0, taken for zero spin-orbit coupling, is defined as $\Theta_0 = \Theta_+|_{\alpha=0} = \Theta_-|_{\alpha=0}$. The expression for $X(\mathbf{p}_{f\|})$ in Eq. (17) should be taken linear in small parameters $\alpha p_f/(\varepsilon_f \pm h)$. Then $X(\mathbf{p}_{f\|}) \propto w_z = \alpha p_{fy}$ and averaging over the Fermi surface in Eq. (17) gives nonzero result for j_y, while j_z vanishes.

We notice, that an expression for the Josephson critical current in S-F-S junctions with small momentum dependent transparencies $D(\mathbf{p}_{f\|})$ [3, 4] can be obtained from Eq. (17) by replacing $X(\mathbf{p}_{f\|})v_{fy} \to -2D(\mathbf{p}_{f\|})|v_{fx}|$. This is not surprising, since both the spontaneous surface current and the Josephson current are actually the two components of total supercurrent carried by the same Andreev interface states, which reduce to surface states in the tunneling limit. In the particular case $h \sim \alpha p_f \ll \varepsilon_f$ the spontaneous surface electric current $j_y \propto \alpha p_f h/\varepsilon_f^2$ is of the second order in a small parameter $(h/\varepsilon_f) \sim (\alpha p_f/\varepsilon_f)$. Since these small second-order terms are disregarded within the quasiclassical approach to describing the FSOL, solutions of Eq. (1) found above show no spontaneous electric surface current, in contrast with the spin currents in the FSOL.

This work was supported by the Russian Foundation for Basic Research under grant # 02-02-16643, scientific programs of Russian Ministry of Science and Education and Russian Academy of Sciences. I.V.B. acknowledges the support of the Dynasty Foundation and thanks the Forschungszentrum Jülich for financial support within the framework of the Landau Program.

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