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Charge qubit rotations in a double-dot nanostructure

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Quantum operations with a charge solid-state qubit whose logical states are formed by two spatially separated localized states of an electron in the double-dot structure are studied theoretically. We show that it is possible to perform various one-qubit rotations making use of the microwave pulses tuned to the resonances between the localized states and the excited state delocalized over the nanostructure. Explicit analytic expression for the time-dependent electron state vector is derived, and the appropriate pulse parameters are determined.

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During past decade, quantum computing (QC) or, more generally, quantum information processing, attracted much attention [1]. The reasons for that interest are (i) the existence of quantum algorithms [2] that could perform calculations exponentially faster as compared with classical ones (QC software), and (ii) the rapid development of technology and material science [3] that allowed to realize some prototypes of QC devices (QC hardware). The key elements of the QC hardware are the quantum bit or *qubit*, which is a generic two-level system, and a register of such qubits. The register allows to store the quantum information, which is processed by means of unitary transformations (*quantum gates*) through an external control.

Quite generally, any two-level system that has sufficiently long-lived states and allows for efficient readout can be used for QC. A lot of proposals for qubit realization have been made, see, e. g., Refs. [1, 2, 4 – 8] For practical applications one has to look for a physical system that could serve as a base for a *scalable* QC. It is commonly believed that the problem of scalability can be effectively solved with the solid-state systems. In this work, we focus on the nanostructure consisted of two quantum dots (QDs) and containing an excess electron in the superposition of the orbital states localized in different QDs [9, 6]. Those states form the logical states of the charge qubit. We show that various one-qubit rotations, including the phase gate, the NOT operation, and the Hadamard transformation can be performed on such a qubit by means of one (if QDs are identical) or, generally, two (if QDs differ from each other) microwave pulses tuned to the resonances with one of the excited states delocalized over the nanostructure.

Let us consider the nanostructure composed of two QDs (L and R) and containing an excess electron in the conduction band. Provided that the distance between the QDs is sufficiently large, the wave functions $\langle\mathbf{r}|L\rangle$ and $\langle\mathbf{r}|R\rangle$ of the lowest size-quantized QD states $|L\rangle$ and $|R\rangle$ with the energies ε_L and ε_R, respectively, are localized in the corresponding QDs, and the overlap $\langle L|R\rangle$ is negligibly small. We assume that there is at least one excited state $|TR\rangle$ in the nanostructure whose energy ε_{TR} lies just below the top of the potential barrier separating the QDs (see Figure), so that the wave function $\langle\mathbf{r}|TR\rangle$ is delocalized over both QDs [10]. Previously [9, 10] we have shown that in the case of the symmetric double-dot nanostructure, $\varepsilon_L = \varepsilon_R$, the resonant electromagnetic pulse with the frequency $\omega = \varepsilon_{TR} - \varepsilon_{L,R}$ (hereafter $\hbar = 1$) can be used to achieve the complete population transfer between the states $|L\rangle$ and $|R\rangle$. In such a process, an excited state $|TR\rangle$ plays the role of the "transport" state: it assists the electron transfer between the QDs but remains unpopulated after the pulse is off. Here we show that in the double-dot nanostructure various types of the temporal evolution of a *superpositional* state $\alpha|L\rangle + \beta|R\rangle$ can be realized in both $\varepsilon_L = \varepsilon_R$ and $\varepsilon_L \neq \varepsilon_R$ cases through the proper choice of the pulse parameters.

We subject the nanostructure to the external electromagnetic pulse of the form

$$\mathbf{E}(t) = \Big[\mathbf{E}_0\cos(\omega_0 t + \varphi_0) + \mathbf{E}_1\cos(\omega_1 t + \varphi_1)\Big] \times \\ \times \Big[\theta(t) - \theta(t - T)\Big], \qquad (1)$$

where T is the pulse duration, i. e., in fact, the operation time. We assume that the frequency $\omega_{0,1}$ is close to the resonant frequency $\omega_{L,R} = \varepsilon_{TR} - \varepsilon_{L,R}$ for the electron transition $|L, R\rangle \rightleftharpoons |TR\rangle$, so that $|\delta_{0,1}| << \omega_{0,1}$, where

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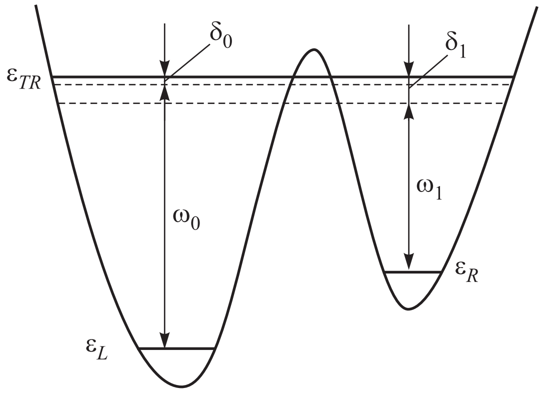
$\delta_{0,1} = \omega_{L,R} - \omega_{0,1}$ is the corresponding detuning (see Figure). In the resonant approximation (see Refs. [9] and [10] for details), only the states $|L\rangle$, $|R\rangle$, and $|TR\rangle$



Schematics of energy levels and resonant frequencies for the double-dot nanostructure driven by the microwave pulse, see text for details.

participate the electron evolution, so that the problem reduces to a three-level model, and the state vector $|\Psi(t)\rangle$ can be written as

$$|\Psi(t)\rangle = \sum_{k=L,R,TR} C_k(t)e^{-i\varepsilon_k t}|k\rangle \ , \tag{2}$$

and the Hamiltonian reads

$$H(t) = \sum_{k=L,R,TR} \varepsilon_k a_k^+ a_k + \\ + \frac{1}{2}\left(\lambda_L a_{TR}^+ a_L e^{-i\omega_0 t} + \lambda_R a_{TR}^+ a_R e^{-i\omega_1 t} + \text{h.c.}\right), \tag{3}$$

where a_k^+ (a_k) is the operator of creation (annihilation) of an electron in the state $|k\rangle$, $\lambda_{L,R} = \mathbf{E}_{0,1}\mathbf{d}_{L,R}e^{-i\varphi_{0,1}}$, and $\mathbf{d}_{L,R} = -e\langle TR|\mathbf{r}|L,R\rangle$ is the dipole matrix element for the transitions $|L,R\rangle \rightleftharpoons |TR\rangle$.

To solve the non-stationary Schrödinger equation

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H(t)|\Psi(t)\rangle \tag{4}$$

with the Hamiltonian (3) and the initial conditions

$$|\Psi(0)\rangle = \alpha|L\rangle + \beta|R\rangle, \tag{5}$$

we make use of the unitary transformation

$$|\Psi(t)\rangle = U(t)|\tilde{\Psi}(t)\rangle \tag{6}$$

with

$$U(t) = \exp\left(i\omega_0 t a_L^+ a_L + i\omega_1 t a_R^+ a_R\right). \tag{7}$$

Substituting Eq. (6) into Eq. (4), we obtain the Schrödinger equation for $|\tilde{\Psi}(t)\rangle$:

$$i\frac{\partial|\tilde{\Psi}(t)\rangle}{\partial t} = \tilde{H}|\tilde{\Psi}(t)\rangle, \tag{8}$$

with the Hamiltonian $\tilde{H}$ in the basis $\{|L\rangle, |R\rangle, |TR\rangle\}$ being

$$\tilde{H} = U^+(t)H(t)U(t) - iU^+(t)\frac{\partial U(t)}{\partial t} = \\ = \begin{pmatrix} \varepsilon_{TR} - \delta_0 & 0 & \frac{\lambda_L^*}{2} \\ 0 & \varepsilon_{TR} - \delta_1 & \frac{\lambda_R^*}{2} \\ \frac{\lambda_L}{2} & \frac{\lambda_R}{2} & \varepsilon_{TR} \end{pmatrix}. \tag{9}$$

Since the Hamiltonian $\tilde{H}$ is time-independent, the general solution of Eq. (8) for $0 \le t \le T$ is

$$\tilde{\Psi}(t) = \sum_{k=1}^{3} A_k e^{-i\tilde{\varepsilon}_k t}|\tilde{k}\rangle, \tag{10}$$

where $|\tilde{k}\rangle$ and $\tilde{\varepsilon}_k$ are, respectively, the eigenstates and eigenenergies of the stationary Schrödinger equation

$$\tilde{H}|\tilde{k}\rangle = \tilde{\varepsilon}_k|\tilde{k}\rangle. \tag{11}$$

They can be found from the cubic equation for eigenvalues of the 3×3 matrix (9).

The general features of the qubit evolution in a three-level model have been discussed in Ref. [11] by the example of the Josephson phase qubit. Here we consider an important particular case of exact resonances, $\delta_0 = \delta_1 = 0$, that allows for a simple analytical solution. In this case one has

$$\tilde{\varepsilon}_1 = \varepsilon_{TR} - 2\Omega,\ \tilde{\varepsilon}_2 = \varepsilon_{TR},\ \tilde{\varepsilon}_1 = \varepsilon_{TR} + 2\Omega, \tag{12}$$

$$|\tilde{1}\rangle = \frac{1}{\sqrt{2}}\left[-\frac{\lambda_L^*}{4\Omega}|L\rangle - \frac{\lambda_R^*}{4\Omega}|R\rangle + |TR\rangle\right], \\ |\tilde{2}\rangle = \frac{1}{4\Omega}\left[\lambda_R|L\rangle - \lambda_L|R\rangle\right], \\ |\tilde{3}\rangle = \frac{1}{\sqrt{2}}\left[\frac{\lambda_L^*}{4\Omega}|L\rangle + \frac{\lambda_R^*}{4\Omega}|R\rangle + |TR\rangle\right], \tag{13}$$

where

$$\Omega = \frac{\sqrt{|\lambda_L|^2 + |\lambda_R|^2}}{4} \tag{14}$$

is the Rabi frequency for the system under consideration.

Taking into account that $|\tilde{\Psi}(0)\rangle = |\Psi(0)\rangle$, it is straightforward to find the coefficients A_k in Eq. (10)

from the initial condition (5). Then we have from Eqs. (6) and (7) the explicit expression for the state vector $|\Psi(t)\rangle$:

$$|\Psi(t)\rangle = e^{-i\varepsilon_L t}\left[\alpha - \frac{\lambda_L^*(\alpha\lambda_L+\beta\lambda_R)}{8\Omega^2}\sin^2(\Omega t)\right]|L\rangle + \\ + e^{-i\varepsilon_R t}\left[\beta - \frac{\lambda_R^*(\alpha\lambda_L+\beta\lambda_R)}{8\Omega^2}\sin^2(\Omega t)\right]|R\rangle - \\ - ie^{-i\varepsilon_{TR} t}\frac{\alpha\lambda_L+\beta\lambda_R}{4\Omega}\sin(2\Omega t)|TR\rangle. \quad (15)$$

One can see from Eq. (15) that at operation times $T_n = \pi n/2\Omega$ $(n = 1, 2, \dots)$ the state vector is completely localized in the logical qubit subspace $\{|L\rangle, |R\rangle\}$. In this sense, the qubit evolution appears to be stroboscopic with respect to the logical subspace.

Now let us demonstrate how various qubit rotations can be performed through the proper choice of the pulse parameters. At $T = \pi k/\Omega$ $(k = 1, 2, \dots)$ the relative phase shift operation is realized provided $\varepsilon_L \neq \varepsilon_R$,

$$|\Psi(T)\rangle = e^{-i\varepsilon_L T}\left[\alpha|L\rangle + \beta e^{-i(\varepsilon_R-\varepsilon_L)T}|R\rangle\right]. \quad (16)$$

At $T = \pi(2k-1)/2\Omega$ $(k = 1, 2, \dots)$ one has

$$|\Psi(T)\rangle = \\ = e^{-i\varepsilon_L T}\left[\alpha\frac{|\lambda_R|^2-|\lambda_L|^2}{|\lambda_R|^2+|\lambda_L|^2} - \beta\frac{2\lambda_L^*\lambda_R}{|\lambda_R|^2+|\lambda_L|^2}\right]|L\rangle + \\ + e^{-i\varepsilon_R T}\left[\beta\frac{|\lambda_L|^2-|\lambda_R|^2}{|\lambda_R|^2+|\lambda_L|^2} - \alpha\frac{2\lambda_L\lambda_R^*}{|\lambda_R|^2+|\lambda_L|^2}\right]|R\rangle, \quad (17)$$

so that the NOT gate,

$$|\Psi(T)\rangle = \pm e^{-i(\varepsilon_L+\varepsilon_R)T/2}\left[\beta|L\rangle + \alpha|R\rangle\right], \quad (18)$$

is implemented if $|\lambda_L| = |\lambda_R|$ and $\varphi_1 - \varphi_0 = \pi n + (\varepsilon_R - \varepsilon_L)T/2$ (n is an integer). Note that the NOT gate can be realized in both asymmetric $\varepsilon_L \neq \varepsilon_R$ and symmetric $\varepsilon_L = \varepsilon_R$ nanostructures. In the latter case there is no need in the second component of $\mathbf{E}(t)$, see Eq. (1). Next, it follows from Eq. (17) that the Hadamard gate,

$$|\Psi(T)\rangle = \pm e^{-i\varepsilon_L T}\left[\frac{\alpha+\beta}{\sqrt{2}}|L\rangle + \frac{\alpha-\beta}{\sqrt{2}}|R\rangle\right], \quad (19)$$

is realized if $(\varepsilon_R - \varepsilon_L)T = 2\pi n$ ($n \neq 0$ is an integer), $\varphi_1 - \varphi_0 = \pi m$ (m is an integer), and $\lambda_L/\lambda_R = 1 \pm \sqrt{2}$.

We note that the values of ε_L and ε_R can be varied independently by applying the voltage biases to the surface gates, while the values of λ_L and λ_R can be adjusted through the appropriate changes in the electric field amplitudes. So, various rotations of a charge qubit in the QD nanostructure can be implemented through appropriate choice of the pulse parameters (frequencies, phases, intensity, duration) and proper nanostructure engineering. The unavoidable small deviations of the pulse frequencies from the ideal resonance conditions and the departures of $\lambda_{L,R}$ from their optimal values can be accounted for in a way similar to that used in Ref. [10] for the resonant electron transfer between the QDs.

The characteristic operation times are $T \sim 1/(\varepsilon_R - \varepsilon_L) \sim 1$ ps for $\varepsilon_R - \varepsilon_L \sim 1\,$meV. Since $T \sim 1/\Omega \sim 1/|\lambda_{L,R}| \sim 1/eaE_0$, where $a \sim 10\,$nm is the QD size, such values of T correspond to the electric field strength $E_0 \sim 10^3\,$V/cm. The short operation times allow to minimize the unwanted decoherence effects. Indeed, the lowest bounds for the decoherence times due to the Nyquist-Johnson noise from the gates and 1/f noise from the background charge fluctuations are [8] $\tau \sim 1$ μs and $\tau \sim 1$ ns, respectively, so that the corresponding error rates [12] $D(T) = 1 - \exp(-T/\tau)$ do not exceed 10^{-3}. As for the phonon-induced decoherence, in the case that the absolute value of the difference $\varepsilon_R - \varepsilon_L$ greatly exceeds the energy of electron tunneling between the QDs (i.e., in the case of well separated QDs), the main contribution to decoherence comes from dephasing processes, and the error rate in common semiconductors is $D(t) = 10^{-4} \div 10^{-3}$ [12]. Such values of $D(t)$ are close to the fault-tolerant threshold for quantum computation [13]. The detailed analysis of the decoherence effects is, however, beyond the scope of this paper.

To summarize, we have shown that making use of the microwave pulses allows one to implement various operations on a charge qubit encoded in two spatially separated states of an electron in the double-dot nanostructure. The extremely short operation times ~ 1 ps make it possible to minimize the decoherence effects below the level sufficient for at least the proof-of-principle experiments and demonstration of the feasibility of the scheme discussed. Although we restricted ourselves to the rectangular pulses, our treatment can be generalized to other pulse shapes [14]. The results obtained can be applied also to the Josephson three-level gates [11, 15, 16]. Finally, it does not seem unrealistic to organize the coupling of QDs-based charge qubits for conditional quantum operations.

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