

Large-scale non-locality in “doubly special relativity” with an energy-dependent speed of light

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There are two major alternatives for violating the (usual) Lorentz invariance at large (Planckian) energies or momenta – either not all inertial frames (in the Planck regime) are equivalent (e.g., there is an effectively preferred frame) or the transformations from one frame to another are (non-linearly) deformed (“doubly special relativity”). We demonstrate that the natural (and reasonable) assumption of an energy-dependent speed of light in the latter method goes along with violations of locality/separability (and even translational invariance) on macroscopic scales.

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1. Introduction. The observation that there is no invariant energy (and length, etc.) scale in special relativity on the one hand and the expected physical significance of the Planck scale on the other hand has motivated (see, e.g., [1–5] and [6, 7]) the suggestion that the (usual) Lorentz invariance might be broken at large (Planckian) energies. It has also been suggested (see, e.g., [1–7], and [8–10]) that several yet unexplained observations – such as the indications for ultra-high energy cosmic rays (UHECR) with energies above the Greisen–Zatsepin–Kuzmin (GZK) cut-off [11, 12] of order $\mathcal{O}(10^{19}$ eV) induced by the interaction with the cosmic microwave background – could be interpreted as empirical evidence for deviations from the Lorentz invariance at high energies. Furthermore, variable speed-of-light (VSL) cosmologies (see, e.g., [13, 14]), which also require breaking the local Poincaré symmetry, have been considered as alternative solutions of the cosmological problems which lead to the idea of inflation. In particular in the first case, one needs to explain the apparent large gap between the energy range of those phenomena $\mathcal{O}(10^{19}$ eV) and the Planck scale $\mathcal{O}(10^{28}$ eV) – but we are not going to discuss these phenomenological issues here. (For phenomenological constraints on Lorentz violation see, e.g., [15, 16] and references therein, cf. [17]).

There are two major alternatives for breaking the Lorentz invariance: either not all inertial frames (in the Planck regime) are equivalent (e.g., there is an effectively preferred frame) or the transformations from one frame to another are different (deformed). In the following we shall consider the second possibility in more detail

and discuss some consequences which arise thereof. For the sake of simplicity (and since the masses of all known “elementary” particles are small compared to Planck scale) we shall consider massless particles, such as photons, only. In addition, we shall work in 1+1 dimensions (unless otherwise noted).

The main idea of the “doubly special relativity” (DSR, see, e.g., [1–7, 18]) is to replace the usual linear Lorentz transformation \mathcal{L} by the following non-linear representation $F \circ \mathcal{L} \circ F^{-1}$, i.e.

$$\begin{aligned} \begin{pmatrix} E \\ p \end{pmatrix} &\rightarrow \begin{pmatrix} E' \\ p' \end{pmatrix} = F \circ \mathcal{L} \circ F^{-1} \begin{pmatrix} E \\ p \end{pmatrix} = \\ &= F \left[\begin{pmatrix} 1 & v \\ v/c^2 & 1 \end{pmatrix} F^{-1} \begin{pmatrix} E \\ p \end{pmatrix} \right], \end{aligned} \quad (1)$$

with some non-linear function $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$

$$\begin{pmatrix} E \\ p \end{pmatrix} = F \begin{pmatrix} \mathfrak{E} \\ \mathfrak{p} \end{pmatrix}, \quad (2)$$

which reduces to the identity for small energies

$$F \begin{pmatrix} E \ll M_{\text{Planck}} c^2 \\ p \ll M_{\text{Planck}} c \end{pmatrix} = \begin{pmatrix} E \\ p \end{pmatrix}. \quad (3)$$

Note that the group structure of the deformed transformations in Eq. (1) is the same as that of the ordinary Lorentz group. This appears quite reasonable as the only suitable six-parameter extension (cf. [6, 7, 18]) of the group $SO(3)$ of spatial rotations (which we want to

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retain) seems to be the Lorentz group itself – especially since we want to reproduce the usual Lorentz transformations at small energies. It should also be mentioned here that this approach relies on the particle picture – there is no (unique and well-defined) field-theoretic formulation at this stage.

2. Field-theoretic example. Unfortunately, there is no unique prescription (so far) for translating the behavior in momentum space (E, p) into position space (t, x) , which is required for formulating a corresponding field theory. There is not even consistency in the literature regarding the velocity of propagation of Planckian particles: In Ref. [19], it is argued that the speed of light does not depend on the energy (i.e., that all massless particles have the same velocity c) in all DSR theories. Ref. [20], on the other hand, arrives at the (natural) result that the propagation speed is given by the group velocity $v_g = dE/dp$. In Ref. [6], however, the phase velocity $v_p = E/p$ is used instead (in some limit). This depends on whether (and how) one modifies the commutators such as $[x, p] = i\hbar$ and hence the identifications $ip \leftrightarrow \hbar\partial/\partial x$, etc., or not (see, e.g., [21, 22]).

However, let us consider one possible example for a field-theoretic formulation motivated by an analogy to condensed matter systems. The propagation of sound waves is governed by a dispersion relation which is linear at low energies and shows deviations (sub- or super-sonic) at high energies (cf. also [23–25]). Although there certainly exists a preferred frame in such systems, one might (formally) perform the same steps as described in the previous Section and parameterize the non-linear dispersion relation $E(p)$ with a (non-unique) function F as in Eq. (2) by $\mathfrak{E}^2 = c^2 p^2$ (but here c denotes the speed of sound). In this (somewhat artificial) way the usual (linear) Lorentz transformation \mathcal{L} in the (\mathfrak{E}, p) -space can be used to define transformations from one frame to another. In order to Lorentz transform the field $\phi(t, x)$ (e.g., a wave-packet), one first does a Fourier transform \mathcal{F} assuming $ip \leftrightarrow \hbar\partial/\partial x$ etc., then applies the non-linear Lorentz transformation and finally transforms back:

$$\begin{aligned}\phi(t, x) &\rightarrow \tilde{\phi}(E, p) = \mathcal{F}\phi, \\ \tilde{\phi}(E, p) &\rightarrow \tilde{\phi}(E', p'), \\ \phi'(t, x) &= \mathcal{F}^\dagger \tilde{\phi}(E', p').\end{aligned}\quad (4)$$

Since the function F and its inverse F^{-1} as well as the dispersion relation $E(p)$ are non-polynomial in general, the above procedure is clearly non-local in position space (t, x) , see also Sections 4 and 5 below.

3. Energy of composite systems. As mentioned in the Introduction, one of the main motivations for deforming the (usual) Lorentz boosts is to require that

not only the speed of light (at low energies) but also the Planck scale is invariant²⁾ under the modified transformations (hence the notion “doubly special relativity”). Since the usual linear Lorentz boosts \mathcal{L} do not possess any fixed points (in E, p) except zero and infinity and Eq. (3) connects $E = p = 0$ to $\mathfrak{E} = p = 0$ the Planck scale must be mapped by the function F^{-1} to infinity in order to be invariant. As we shall see below, this property has rather dramatic consequences for composite systems.

Demanding that energy-momentum conservation in one frame has to be equivalent to energy-momentum conservation in all frames implies the following non-linear composition law³⁾

$$\begin{pmatrix} E_{\text{total}} \\ p_{\text{total}} \end{pmatrix} = F_{(N)} \left[\sum_{i=1}^N F^{-1} \begin{pmatrix} E_i \\ p_i \end{pmatrix} \right], \quad (5)$$

i.e., one has to add the \mathfrak{E}_i and p_i , see, e.g., [7, 18]. The subscript (N) indicates that the function $F_{(N)}$ could be modified, i.e., differ from F .

Let us first discuss the implications of using the same [18] function as in the one-particle case $F_{(N)} = F$: Since F maps infinity to the Planck scale, the total energy can never exceed the Planck energy – which a weird result and raises serious questions concerning the physical significance of such an energy concept, see, e.g., [7]. Moreover, a Galilei-type argument points out another contradiction if the velocity of propagation depends on the energy (see also the next Section): If two or more particles have equal energies E and hence velocities v then the speed of the composite system obviously should be the same. However, according to Eq. (5) with $F_{(N)} = F$ the total energy E_{total} of the composite system is closer to the limiting Planck energy and hence the derived velocity would be different!

Alternatively, it has been suggested [7] that one constructs $F_{(N)}$ via replacing M_{Planck} by NM_{Planck} in the explicit expression for F . In that case one has to know how many (elementary) particles the system is composed of – which seems to be rather artificial, see, e.g., [18]. For example – in addition to the arguments presented in [18] – if only one particle has the Planck energy (or very nearly so), adding one photon with an arbitrarily small energy to the system increases the total energy by $M_{\text{Planck}}c^2$ – which is also an odd feature.

²⁾In this sense the idea of DSR can be compared to the transition from the Galilei transformations (no invariant velocity) to special relativity, cf. [1, 2].

³⁾The main argument is basically the same as for the usual linear Lorentz transformations where (under certain assumptions, such as commutativity) only a linear composition law is invariant, see, e.g., [18].

In view of the above considerations, one might question the physical significance of the quantities E and p in comparison with \mathcal{E} and \mathcal{p} . The laws of energy-momentum conservation assume a much simpler form in terms of \mathcal{E} and \mathcal{p} [26]. Apparently the only justification for considering E and p instead of \mathcal{E} and \mathcal{p} could be that the former quantities, E and p , are related to the space-time behavior and determine the (energy-dependent) velocity of propagation, etc., whereas the latter, \mathcal{E} and \mathcal{p} , are not.

4. Energy-dependent speed of light. Since the dispersion relation $E(p)$ must assume the same form in all frames, it can be derived from the usual invariant $\mathcal{E}^2 = c^2\mathcal{p}^2$ (remember $m = 0$) of the linear Lorentz transformations \mathcal{L} . Owing to the non-linear function F the dispersion relation can involve a rather complicated dependence $E(p)$ with a possibly changing speed of light (cf. the VSL cosmologies). As indicated above, a varying velocity of propagation seems to be the only possible way for the quantities E and p to acquire more physical significance than \mathcal{E} and \mathcal{p} .

Obviously, the particle picture – which the whole approach is based on – and the concept of a velocity of propagation derived therefrom do only make sense if we are able to localize the particle under consideration with a (space/time) uncertainty much smaller than the length of the particle’s world-line. For example, we may derive the velocity of a Planckian particle by determining its position within a few Planck lengths and following its propagation over a macroscopic time duration and distance. Here macroscopic means much larger than the Planck length/time (we want to retain the usual space-time translation symmetry and the concept of internal motion).

As motivated in the previous Section, the inverse function F^{-1} diverges at the Planck scale and hence cannot be written as a polynomial (polynomials are regular everywhere). In general one would expect $E(p)$ to be singular at the Planck scale too – also displaying a non-polynomial behavior – and therefore non-local effects to arise. At a first glance, one might argue that these non-localities occur in the Planck regime only and are therefore not problematic. However, as we shall demonstrate now, these non-local effects arise on a *macroscopic* scale – provided that the particles under consideration can travel a distance much larger than the Planck length (see the arguments above).

Let us consider the two limiting cases – for higher and higher energies, the speed of light goes to zero (sub-luminal dispersion) or to infinity (super-luminal). In the first case, the particle basically stops moving and just sits there. Now, if we can localize this highly Planck-

ian particle within a few Planck lengths for a finite time duration (i.e., much longer than the Planck time) this clearly singles out a preferred frame, since we are supposed to know how Lorentz boosts act on macroscopic (i.e., sub-Planckian) scales!

In order to further study this apparent contradiction, let us consider a concrete example. Here one encounters a problem since, as mentioned in Section 2, the velocity of propagation is not uniquely determined. In the following we assume that the speed of the particle is given by the group velocity dE/dp (cf. Section 2 and [20]) and choose a dispersion relation which is linear in some interval $p \in [p_1, p_2]$ – though not with the usual proportionality factor c – say,

$$F \begin{pmatrix} E \\ p_1 \leq p \leq p_2 \end{pmatrix} = \begin{pmatrix} E/2 \\ p \end{pmatrix}. \quad (6)$$

If we assume a very small boost velocity $v \ll c$ (Galilei limit), the Lorentz transformation in Eq. (1) acts as

$$\begin{aligned} \begin{pmatrix} E \\ p \end{pmatrix} &\rightarrow \begin{pmatrix} E' \\ p' \end{pmatrix} = \begin{pmatrix} E + vp/2 \\ p + 2vE/c^2 \end{pmatrix} = \\ &= \begin{pmatrix} 1 & v/2 \\ 2v/c^2 & 1 \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix}, \end{aligned} \quad (7)$$

i.e., with v and c being replaced by $v/2$ and $c/2$.

Since the dispersion relation $E^2 = c^2p^2/4$ arising from Eq. (6) is linear (between p_1 and p_2), the Lorentz transformation in position space (t, x) ought to be the same as in momentum space (E, p) – no matter whether we consider a particle with $p_1 \leq p \leq p_2$ or a wave-packet (cf. Section 2) with support in the interval $[p_2, p_3]$. In this way the presence of the Planckian particle with $p_1 \leq p \leq p_2$ traveling over a long distance enforces a Lorentz boost with $v/2$ and $c/2$ instead of v and c – at macroscopic (sub-Planckian) scales!

Evidently, the same phenomenon occurs for a superluminal (speed of light goes to infinity) dispersion relation. Any Planckian particle with a sub- or superluminal velocity of propagation either introduces a preferred frame or necessitates the modification of the Lorentz transformation on its travel time and distance – i.e., on macroscopic (sub-Planckian) scales – which demonstrates the occurrence of large-scale non-locality [27].

5. Loss of coincidence. The fact that the presence of a Planckian particle affects the Lorentz transformations has further bizarre consequences. If we go to 3+1 dimensions, the position-space representation of the deformed Lorentz transformation described in Eq. (4) of Section 2 acts as

$$\begin{aligned} \phi'(t, \mathbf{r}) &= \int dE' d^3p' dt' d^3r' \phi(t', \mathbf{r}') \times \\ &\times \frac{1}{(2\pi)^4} e^{i[tE' - \mathbf{r} \cdot \mathbf{p}' - t'E(E', \mathbf{p}') + \mathbf{r}' \cdot \mathbf{p}(E', \mathbf{p}')] } = \\ &= \int dt' d^3r' \mathfrak{G}(t, t', \mathbf{r}, \mathbf{r}') \phi(t', \mathbf{r}'). \end{aligned} \quad (8)$$

The non-linearity in $E(E', \mathbf{p}')$ and $\mathbf{p}(E', \mathbf{p}')$ results in a very strange behavior under space-time translations. For the sake of illustration, we again (as in the previous Section) consider a function F which is linear both, for low momenta and in some interval $p \in [p_1, p_2]$

$$F \begin{pmatrix} E \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \left\{ \begin{array}{l} E : \mathbf{p}^2 \ll p_1^2 \\ E/2 : p_1^2 > \mathbf{p}^2 > p_2^2 \end{array} \right. \\ \mathbf{p} \end{pmatrix}. \quad (9)$$

Now let us follow the evolution of two wave-packets – one $\phi'_{\text{low}}(t, \mathbf{r})$ is decomposed of sub-Planckian energies $\mathbf{p}^2 \ll p_1^2$ and the other one $\phi'_{\text{Planck}}(t, \mathbf{r})$ contains momenta in the interval $p \in [p_1, p_2]$ only. In this situation, the transformation in Eq. (8) can be calculated easily, and in the Galilei limit $v \ll c$, we obtain, cf. Eq. (7)

$$\begin{aligned} \phi'_{\text{low}}(t, \mathbf{r}) &= \phi_{\text{low}}(t' + \mathbf{r}' \cdot \mathbf{v}/c^2, \mathbf{r}' + \mathbf{v}t'), \\ \phi'_{\text{Planck}}(t, \mathbf{r}) &= \phi_{\text{Planck}}(t' + 2\mathbf{r}' \cdot \mathbf{v}/c^2, \mathbf{r}' + \mathbf{v}t'/2). \end{aligned} \quad (10)$$

Note that the relativistic corrections to the time coordinates in the first arguments on the right-hand side are different due to the non-linearity. Consequently, if we change the origin of our spatial coordinate system, we introduce a relative time shift

$$\mathbf{r}' \rightarrow \mathbf{r}' + \mathbf{a} \rightsquigarrow \Delta t = \mathbf{a} \cdot \mathbf{v}/c^2, \quad (11)$$

between the two wave-packets. Ergo, if the velocities of the two wave-packets and the boost direction \mathbf{v} are linearly independent and the two wave-packets hit each other (i.e., coincide within their width at some space-time region) in one coordinate system, they may miss each other (one wave-packet comes too late) in another coordinate representation!

Of course, this breaking of translational (i.e., Poincaré) invariance – again on large scales – has been demonstrated using the special field-theoretic representation described in Section 2; and one could argue that the above effect is an artifact of the special construction in Section 2 and that in a different representation, this problem can be avoided. However, in order to prove this assertion, one has to provide another explicit field-theoretic example and to study its consequences. It seems that one faces similar difficulties – breaking of translational invariance $x_\mu \rightarrow x_\mu + a_\mu$ (see, e.g.,

[28]) and deviations from the usual behavior on large scales $x_\mu \gg L_{\text{Planck}}$ – when introducing non-commuting coordinates via

$$[x^\mu, x^\nu] = \Lambda^{\mu\nu\rho} x_\rho \quad (12)$$

(instead of $[x^\mu, x^\nu] = \zeta g^{\mu\nu}$, for instance) as it is done for example in [21, 22] in relation to DSR theories.

We would also like to stress that the counter-argument presented in the previous Section is independent of any field-theoretic representation (and would therefore not go away).

6. Summary. Apart from the weird properties of composite systems discussed in Section 3, the theory of “doubly special relativity” goes along with violations of locality and separability if the speed of light depends on the energy, since the presence of a single Planckian particle can modify the action of the Lorentz transformation at macroscopic scales⁴⁾ (i.e., much larger than the Planck length, see Sections 4 and 5). On the other hand, if the speed of light does not depend on the energy (e.g., the dispersion relation is $E^2 = c^2 p^2$), then there is no discernible reason to assign E more physical significance than \mathcal{E} , see Section 3. Although one should bear in mind that the whole approach purely relies on the particle picture (not a field theory), one would expect that the energy – defined as the generator of the time-translation symmetry – is an additive quantity for independent systems (which brings us back to the question of locality and separability).

In search of alternatives one could imagine that – even though the (still to be found) underlying theory (including quantum gravity) might not possess a preferred frame – the physical state of the system describing the actual gravitational field, etc., indeed does introduce an effectively preferred frame with respect to the interaction with Planck-scale photons, for example, that propagate within the gravitational field.

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⁴⁾Note, however, that this result does not prove that DSR is conceptually inconsistent or in conflict with experiments or observations since we have not observed Planckian particles (at least not knowingly).

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After finishing the work on our manuscript, we found that several other authors (based on different approaches and assumptions) have also pointed out strange consequences of DSR and/or concluded that DSR is either inconsistent with our present understanding of physics or trivial (i.e., indistinguishable from ordinary special relativity), see [29–32].

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26. At least for $F_{(N)} = F$ or for $N_{\text{initial}} = N_{\text{final}}$. In these cases Eq. (5) implies $\sum_i \mathcal{E}_i^{\text{initial}} = \sum_i \mathcal{E}_i^{\text{final}}$ as well as $\sum_i p_i^{\text{initial}} = \sum_i p_i^{\text{final}}$. In view of this observation, it is hard to see how DSR can solve the threshold anomalies – such as the UHECR problem – which are basically based on energy conservation arguments, because one should be able to choose initial and final configurations with the same number of particles; and at low energies, we have $\mathcal{E}_i = E_i$. (Unless, of course, one argues that the cross section changes significantly at high energies, for example due to an energy-dependent speed of light.)
27. Consider the following *gedanken* experiment: Let us assume that we can localize the (sub-luminal) Planckian photon with almost zero velocity within 1 cm for a few seconds. Now we build a box made of ordinary (sub-Planckian) material around the Planckian photon, which is nearly at rest. Another (inertial) observer, however, who walks by at 1 m/s, also sees the Planckian photon standing still (remember: no preferred frame). So either this observer sees the Planckian photon (eventually) outside the box or, even more drastically, the (sub-Planckian) box following him/her. In both cases, locality is violated. Imagine, for example, that the Planckian photon interacts with some ordinary matter inside the box; and, without non-locality, we know how ordinary matter and space-time behave on large scales.
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