Pis'ma v ZhETF, vol. 76, iss. 2, pp. 92 – 94

T-odd correlation in polarized neutron decay

V. V. Bytev, E. A. Kuraev[1)]

Joint Institute for Nuclear Research, 141980 Dubna, Russia

Submitted 10 June 2002

T-odd correlation $[\mathbf{p}_e, \mathbf{p}_\nu]\boldsymbol{\eta}$ appears in lowest order radiative correction to polarized neutron decay width, where $\boldsymbol{\eta}$ is polarization vector of neutron. It appears as the consequence of final state interaction and cannot be confused with real CP-violating of Standard Model. The value of asymmetry has an order of 10^{-5}. Final state interaction in the decay of unpolarized neutron caused the appearance of transversal polarization of recoil proton and electron. Explicit expression are given.

PACS: 13.40.–f, 13.88.+e, 14.60.Cd, 25.20.Lj

The nowadays interest to neutron β-decay is twofold. It provides an independent way to extract Cabbibo–Kobayashi–Maskawa (CKM) matrix element V_{ud} and is the laboratory for more precise measurement of CP(T) violating effects. The role of radiative corrections (RC) to the theoretically calculated width become essential. Papers, devoted to calculation of RC from late sixteens up to now [1] concern mainly about neutron lifetime and the behaviour of electron spectrum. RC as a source of CP-violating effects which is subject of present paper was first considered in 1967 by Callan and Treiman [2] for the case of β-decay of nuclei. Keeping in mind the experimental interest for T-odd correlation with the hope to discover a new physics the more information on decay rate distribution is needed. It's the motivation of our paper, where we consider the effect of non-zero imaginary part of 1-loop RC to matrix element. In Sec.1 we consider the first consequence of this RC in the CP(T)-odd correlation in differential decay rate.

In Sec.2 we present the degree of electron and proton polarization for the case of unpolarized neutron decay which is due to imaginary part of RC to matrix element.

In Discussion we dispute the approach used in experimental searching of T-odd effects.

Appendix is devoted to details of calculation.

1. CP(T) odd correlation. CP(T) violating correlation for neutron β-decay is of order 10^{-5} and can provide an essential background compared to tiny effects $\approx 10^{-8}$ related with phases of CKM matrix. The CP(T) odd correlation due to final state interaction is connected with the quantity $[\mathbf{p}_e, \mathbf{p}_\nu]\boldsymbol{\eta}$ in differential decay rate ($\mathbf{p}_e, \mathbf{p}_\nu$ are 3-momenta of electron and neutrino in neutron rest frame, $\boldsymbol{\eta}$ is neutron polarized vector) and is defined as

$$\frac{\Gamma_{\text{odd}}}{\Gamma_0} = \frac{\mathrm{Im}\sum M_0 M^{(1)\star}}{\sum |M_0|^2} = |\boldsymbol{\eta}_z| \cdot N \cdot F(\theta, E), \qquad (1)$$
$$N = \alpha m / 4M \approx 10^{-6}.$$

Here summation is performed over spin states of electron and proton and $|\eta_z|$ is projection of polarization vector of neutron on the transversal to production plane created by electron and proton momentum vectors. M_0 is the matrix element of neutron β-decay:

$$\begin{gathered} n(p_n, \boldsymbol{\eta}) \to p(p_p) + e(p_e) + \nu(p_\nu), \qquad (2) \\ M_n = M = E + E_\nu + E_P, \quad \mathbf{p}_p + \mathbf{p}_e + \mathbf{p}_\nu = 0, \\ M_0 = \frac{G}{\sqrt{2}} V_{ud} \overline{u}_p \gamma_\alpha (1 + \lambda\gamma_5) u_n \cdot \overline{u}_e \gamma_\alpha (1 + \gamma_5) u_\nu, \\ \lambda = a e^{-i\phi}, \quad a = g_A / g_V, \end{gathered}$$

and $d\Gamma_0$ is the differential decay rate [3]:

$$\begin{gathered} d\Gamma_0 = \frac{G^2}{4\pi^3} |V_{ud}|^2 \sqrt{(E^2 - m^2)} (\Delta - E)^2 \times \qquad (3) \\ \times E\rho (1 + 3a^2)(1 + \mathcal{O}(\frac{m}{M})) dE dc, \\ \rho = 1 + \mathcal{A} \frac{\mathbf{p}_e \mathbf{p}_\nu}{E E_\nu} + \boldsymbol{\eta} \left[A \frac{\mathbf{p}_e}{E} + B \frac{\mathbf{p}_\nu}{E_\nu} + D \frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E E_\nu} \right], \\ \Delta = M - Mp = 1.29 \text{ MeV}. \end{gathered}$$

Here $E_\nu = \Delta - E$ is energy of neutrino and E is electron energy, $c = \cos\theta$, θ is the angle between electron and neutrino3-momenta. The explicit form for quantities $\mathcal{A}, A, B, C, D$ will be given late. Besides we put here $E_P = M_p$ and we neglect the contribution of quantities of order electron to neutron masses ratio $m/M \ll 1$.

Among a set of Feynman diagrams describing 1-loop RC only one is relevant (as well as it has nonzero imaginary part) which corresponds to virtual photon exchange between electron and proton (final state interaction):

[1)]e-mail: kuraev@thsun1.jinr.ru

$$M^{(1)} = \frac{GV_{ud}}{\sqrt{2}} \frac{\alpha}{4\pi} \times$$
$$\times \int \frac{d^4k}{i\pi^2} \frac{\overline{u}_p \gamma_\mu (p + k + M_p) \gamma_\alpha (1 + \lambda\gamma_5) u_n}{(k^2 - \lambda^2)((p_p - k)^2 - M_p)} \times$$
$$\times \frac{\overline{u}_e \gamma_\mu (p_e - k + m_e) \gamma_\alpha (1 + \gamma_5) u_\nu}{((p_e - k)^2 - m_e^2)}. \qquad (4)$$

Using the formulae given in App. we obtain for T-odd correlation function (which was defined in (1))

$$F(\theta, E) = \frac{E}{\rho m} \left[(a^2 - 1)[3(\frac{E_\nu}{E})^2 - 4 - 6v\frac{E_\nu}{E}\cos\theta] + \right.$$
$$\left. + v^2(a-1)(13a+9) \right] \sin\theta, \qquad (5)$$

where $v = \sqrt{1 - (m/E)^2}$ is the electron velocity. This function is tabulated in Fig.1,2.

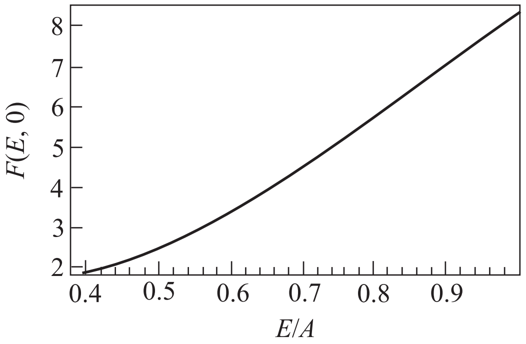

Fig.1. Dependence of $F(E, \theta)$ on E for $\theta = 0$ and $\eta_z = |\boldsymbol{\eta}|$

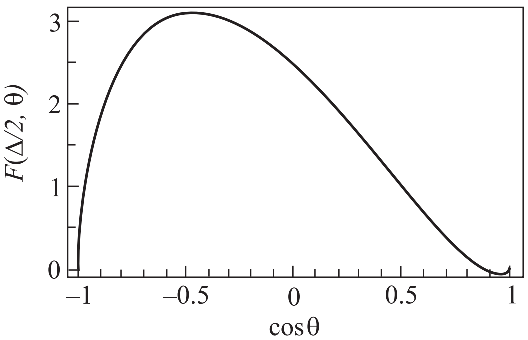

Fig.2. Dependence of $F(E, \theta)$ on $c = \cos\theta$ for $E = \Delta/2$ and $\eta_z = |\boldsymbol{\eta}|$

2. Polarization degree of final particles. Another consequence of nonzero imaginary part of $M^{(1)}$ is the appearance of electron and proton polarization for the case of unpolarized neutron decay, which has the next form:

$$< \boldsymbol{\eta}_p >= N \frac{E \sin\theta}{m\rho(1 + 3a^2)} \mathbf{n} \times$$
$$\times \left[(a^2 - 1)[3(\frac{E_\nu}{E})^2 + 9v^2 + 6v\frac{E_\nu}{E}\cos\theta] + 4(a-1)^2 \right]; \quad (6)$$
$$< \boldsymbol{\eta}_e >= \frac{\alpha \sin\theta (1 - a^2) m}{\rho E (1 + 3a^2)} \mathbf{n}$$

where unit vector $\mathbf{n}$ has the direction along vector $[\mathbf{p}_e \times \mathbf{p}_\nu]$. The magnitude of induced proton spin $< \boldsymbol{\eta}_p >$ is small enough ($\approx 10^{-5}$), whereas $< \boldsymbol{\eta}_e >$ is of order anomalous magnetic moment of electron (see Fig.3,4).

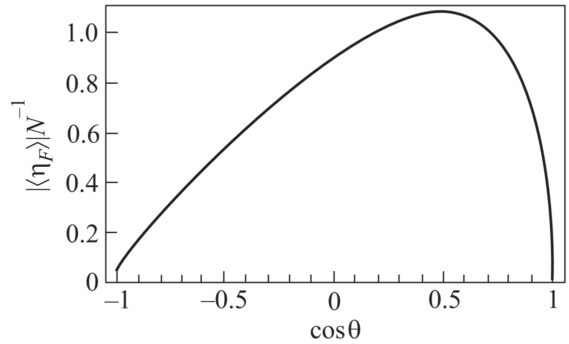

Fig.3. Dependence of electron polarization on $c = \cos\theta$ for $E = \Delta/2$

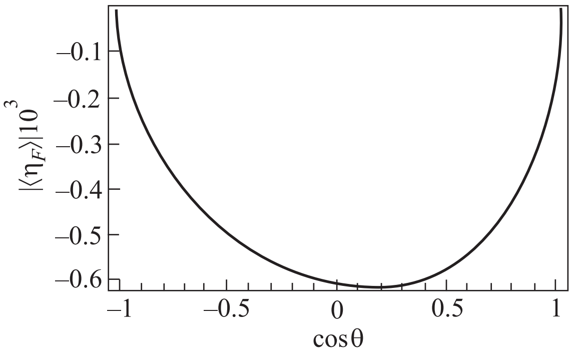

Fig.4. Dependence of proton polarization on $c = \cos\theta$ for $E = \Delta/2$

We note that as well as T-odd correlation as well as the induced final electron and proton polarizations turns out to zero at $a = 1$.

3. Discussion. Coefficients $\mathcal{A}, A, B, C, D$ entering Born differential decay rate (1)

$$\mathcal{A} = \frac{1 - a^2}{1 + 3a^2}, \quad A = -2\frac{a^2 + a\cos\phi}{1 + 3a^2}, \qquad (7)$$
$$B = 2\frac{a^2 - a\cos\phi}{1 + 3a^2}, \quad D = 2\frac{a\sin\phi}{1 + 3a^2},$$

are related with the magnitude and phase of ratio of axial vector to vector formfactors of the nuclei. The results of recent experiments (see [4, 5]) are

$$a = |\lambda| = 1.2670 \pm 0.0035, \quad \phi = 180.07 \pm 0.18, \qquad (8)$$

and the coefficient D [6]:

$$D = (-0.6 \pm 1.2 \pm 0.5) \cdot 10^{-3}, \qquad (9)$$

is consistent with zero on the modern level of accuracy. Coefficient D is sensitive to possible scalar and tensor type forces in weak interaction [7] whose contribution as well on the level of $3 \cdot 10^{-5}$. Further information can be found in paper [6].

Our hope that on the next level of accuracy of experiment our formulae will be relevant.

Our main result is presented in formulae (5) for differential decay rate as a function of energy and emitting angle of electron, which is illustrated in Fig.1,2 as distribution on emission angle at typical electron energy ($E \approx \Delta/2$) and the distribution on electron energy at fixed emission angle.

Another result is the formulae for induced averaged proton and electron spin polarization vectors directed normally to production plane (see (6)).

We are grateful to A. Ali and G. Bunatyan for informative discussion. We are also grateful to V.N. Samoilov, (NCPI, JINR) and Haisenberg-Landau fond for support.

Appendix

For the one-spin correlation we need the imaginary part of vector and tensor one-loop Feynman integrals:

$$I_\mu; I_{\mu\nu} = \int \frac{d^4k}{i\pi^2} \frac{k_\mu; k_\mu k_\nu}{k^2((p_e-k)^2 - m_e^2 + i0)} \times \\ \times \frac{1}{((k+p_p)^2 - M_p^2 + i0)}. \qquad (10)$$

Using the Feynman denominators joining procedure we introduce two Feynman parameters:

$$x((k-p_e)^2 - m_e^2 + i0) + \\ + (1-x)((k+p_p)^2 - M_p^2 + i0) = k^2 - 2kp_x + i0, \\ y(k^2 - 2kp_x) + (1-y)k^2 = (k - yp_x)^2 - y^2 p_x^2 + i0, \\ p_x = xp_e - (1-x)p_p. \qquad (11)$$

Imaginary part arises from the region where $p_x^2 < 0$. Putting it in the form

$$p_x^2 = m_e^2 x + M_p^2(1-x) - sx(1-x) = \\ = s(x - x_-)(x_+ - x), \qquad (12) \\ x_\pm = \frac{M_p^2 + p_e p_p \pm R}{s},$$

$$R = \sqrt{(p_e p_p)^2 - M_p^2 m_e^2}, \\ s = (p_e + p_p)^2.$$

Further we use

$$\mathrm{Im} \int_0^1 \frac{f(x)dx}{p_x^2 - i0} = \frac{\pi}{2R}[f(x_-) + f(x_+)]. \qquad (13)$$

Using these formulae and the standard algebraical technique we calculate

$$\mathrm{Im}[I_\mu] = A_e p_{e\mu} + B_p p_\mu; \\ \mathrm{Im}[I_{\mu\nu}] = E g_{\mu\nu} + F p_{e\mu} p_{e\nu} + \qquad (14) \\ + G p_{p\mu} p_{p\nu} + H(p_{p\mu} p_{e\nu} + p_{p\nu} p_{e\mu}),$$

with

$$A_e = -\frac{\pi}{2R}(x_+ + x_-); B_p = \frac{\pi}{2R}(2 - x_+ - x_-), \\ E = \frac{1}{4}\frac{\pi}{2R}[(m^2 - M^2)(x_+ + x_-) - \\ - s(x_+ + x_- - 2(x_-^2 + x_+^2))], \qquad (15) \\ F = -\frac{1}{2}\frac{\pi}{2R}[x_-^2 + x_+^2], \; G = -\frac{1}{2}\frac{\pi}{2R}[\overline{x}_-^2 + \overline{x}_+^2], \\ H = \frac{1}{2}\frac{\pi}{2R}[x_- \overline{x}_- + x_+ \overline{x}_+], \; \overline{x} = 1 - x.$$

1. F. Gluck and K. Toth, Phys. Rev. **D46**, 2090 (1992); Y. Yokko, S. Suzuki, and M. Morita, Prog. of Theor. Phys. **50**, 1894 (1973).
2. C.G. Callan and S.B. Treiman, Phys. Rev **162**, 1494 (1967); J.D. Jackson, S.B. Treiman, and H.W. Wyld, Jr., Nucl. Phys. **4**, 206 (1957).
3. L.B. Okun, *Leptons and Quarks*, Nauka, Moscow, 1990; S.M. Bilenky, R.M. Ryndin, Ya.A. Smorodinsky, and Ho Tso-shiu, JETP **37**, 1758 (1959).
4. C. Caso et al., Eur. Phys. J. **C3**, 1 (1998) and (1999) off-year partial update for 2000 edition availabel on the PDC WWW pages (URL: http://pgb.lbl.gov/).
5. B.G. Erosolimski, Uspekhi Fizicheskih Nauk **116**, 145 (1975); B.G. Erosolimski et al.¡ JETF Lett. **20**, 345 (1974).
6. L. Lising et al., arXiv:nucl-ex/0006001 v.2 19 Jul 2000; T. Soldner et al., Neutron spectroscopy, Nuclear Structure, related topics ISINN-9, Dubna-2001, p. 49.
7. C.G. Carnoy, J. Deutsch, and P. Quinn, Nucl. Phys. **A568**, 256 (1994).