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Excitations in quantum hall ferromagnet with strong Coulomb interaction

S. V. Iordanski, A. Kashuba

L. D. Landau Institute for Theoretical Physics RAS, 117334 Moscow, Russia

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Quantum Hall ferromagnet is considered at integer fillings ν provided typical Coulomb interaction energy E_c is large compared to the cyclotron energy ω_H. Low energy collective modes consist of magnetoplasmon exciton and gapeless spin exciton. All charged excitations have a gap. The activation energy gap for a pair of charged topological excitations – skyrmion and antiskyrmion – is small $\Delta = \nu\omega_H$. Electric charge of skyrmion is multiple $q = e\nu Q$, where Q is the integer topological charge.

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Commonly used theoretical approach for 2D electron quantum Hall state is to consider the limit of extremely large cyclotron energy ω_H compared to typical Coulomb interaction energy E_c when electron wave functions can be projected onto the states of few lowest Landau levels [1 – 3]. The simplest case corresponds to a non-degenerate Fermi gas with all states within the lowest Landau levels being filled. In particular one finds a ferromagnetic ordering due to the exchange Coulomb interaction. In the limit $r_s = E_c/\omega_H \to 0$ it is possible to find exactly energies and wave functions of electron and hole excitations as well as various collective excitations with charge and spin distortions of the ground state [1 – 3]. This model features also topological charged excitations – skyrmions – that renders the activation energy two fold lower then that for an electron – hole pair excitation [4]. But in real experiments the condition of small r_s is violated and for Si heterostructures and organic MOSFETs based on molecular crystals this violation is quite severe with $r_s \sim 10$ [5, 6]. Newly developed AlAs heterostructures also fall in the class of strongly interacting 2DEG. Some predictions of the standard theoretical model are not consistent with experiments even in the simplest case of integer filling ν. The most apparent discrepancy concerns the activation gap for charged excitations which is found to be substantially smaller than the predicted exchange Coulomb energy [6] and linearly depends on the strength of magnetic field. In this letter we consider quite opposite case of theoretical model with Coulomb interaction being large compared to the cyclotron energy: $r_s = E_c/\omega_H \gg 1$. In spite of calculational difficulties it is possible to make some predictions concerning the lowest energy of various collective excitations and charged topological excitations skyrmions in this limit.

Obviously at strong Coulomb interaction the ground state does not coincide with the Hartree-Fock ground state where electrons completely occupy several lowest Landau levels (Ll). Actually one has to take into account virtual transitions to a number of higher Ll's and therefore the projection of electron wave function onto few lowest Ll states is invalid. It is difficult to construct analytically the ground state wave function or to find corresponding correlation functions. The ground state of an ideal electron gas with integer Ll fillings is nondegenerate and therefore it is possible to use a perturbation theory in powers of interaction and to assume in the spirit of Landau Fermi liquid theory that the exact summation of perturbation series will preserve the ideal Fermi gas classification of one particle excitations. There are quasielectron and quasihole excitations with different energies $\varepsilon_e(s)$ and $\varepsilon_h(s)$ where the index s counts discrete energy levels of charged quasiparticles in magnetic field. In Ref.[7] an excitation gap $\Delta_{eh} = \min_{ss'}(\varepsilon_e(s) - \varepsilon_h(s')) = 0.1\ E_c$ was established numerically. These levels must be degenerate in continous index p that specifically dependes on the gauge because of the existence of magnetic translations commuting with the Hamiltonian but not commuting between themselves. One particle Green function matrix $G_{nn'}(p,\omega)$ is not diagonal in inter Ll indices n and n' but it is diagonal in intra Ll index p. ω is the time Fourrier frequency. We assume that one particle Green function has simple poles at these energies:

$$G(p,s,\omega) \approx \frac{A_s}{\omega - \varepsilon_e(s) - i\delta},$$
$$G(p,s,\omega) \approx \frac{B_s}{\omega - \varepsilon_h(s) + i\delta}. \qquad (1)$$

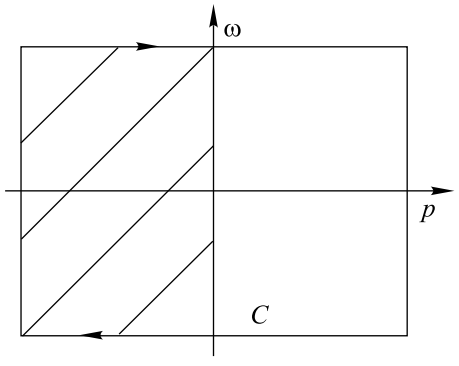
Here the index s denotes quasiparticle eigenstates diagonalizing the Green function $G_{nn'}(p,\omega) =$

$= \sum_s \phi_n(s) G(p,s,\omega) \phi^*_{n'}(s)$ where $\phi_n(s)$ is the one particle quasiexcitation wave functions.

It is possible to relate the position of chemical potential μ assumed to be inside the excitation gap Δ_{eh} with the total density of electrons just like it is done in the case of usual Fermi liquid theory, considering the phase and analitical properties of the Green function [8, 9]. We consider a special topological invariant constructed from the Matsubara Green functions [10]:

$$\nu' = \frac{1}{2\pi i} \int\limits_C \mathrm{Tr}\left(G(p,\omega)\nabla G^{-1}(p,\omega)\right) d\mathbf{l}. \qquad (2)$$

Integrand is a logarithmic derivative of one particle Green function on the plane with two axis being the Matsubara frequency $\omega = i\omega$ and the continuous gauge index p. $G_{nn'}(\omega, p)$ is a matrix with inter Ll indices n and n', and the gradient is a vector in p, ω plane. If the contour C is drawn in a region p, ω free of singularities in $\log(\det G)$ then the integrand (2) is a closed form and therefore gives topological invariant quantity not depending on the form of closed contour C in this region. If there is no singularity inside the contour C then $\nu' = 0$, otherwise it gives some integer number because the complex matrix G is single-valued. Assuming Landau gauge, the variable p coincides with Y coordinate of the center of electron orbit, which is restricted to the area of the sample occupied by 2D electrons. Fermi liquid electron Green function (1) has no singularities or zeroes inside the sample and the only singularity of the integrand can be found on a boundary at small ω due to the existence of edge states. Therefore integration over a macroscopically large rectangular contour C with sides parallel to p and ω axes (see Figure with dashed



Contour C of integration for topologial Green function invariant. The dashed area coincides with the 2DEG sample

region representing the 2DEG) gives some integer number depending on the position of chemical potential and related to the edge plasmon modes. In this case one can neglect the contributions to (2) from the horizontal sides since G is essentially $1/\omega$ not depending on p deep inside the 2DEG due to the gauge invariance. The only nonzero contribution comes from the vertical side of C inside the 2DEG because the integration over the other vertical side outside the sample gives zero (no electrons in any state). Thus we get the integer:

$$\nu = \int\limits_{-\infty}^{\infty} \mathrm{Tr}\left(G(p,\omega)\frac{\partial}{\partial\omega}G^{-1}(p,\omega)\right)\frac{d\omega}{2\pi i}, \qquad (3)$$

for any p inside the sample. Using the definition of the density in terms of the Green function [8]:

$$N = \sum_p \int_{-\infty}^{\infty} \mathrm{Tr} G(p,\omega)\; e^{i\omega\delta}\;\frac{d\omega}{2\pi i}, \qquad (4)$$

where $\delta \to +0$, and the definition of the self energy:

$$G^{-1}_{nn'}(p,\omega) = (\omega - n\omega_H)\delta_{nn'} - \Sigma_{nn'}(s,p,\omega), \qquad (5)$$

we rewrite the density in a convenient form:

$$N = \sum_p \int_{-\infty}^{\infty} \mathrm{Tr}\left[G(p,\omega)\frac{\partial}{\partial\omega}G^{-1}(p,\omega) + \right.$$

$$\left. + G(p,\omega)\frac{\partial\Sigma(p,\omega)}{\partial\omega}\right] e^{i\omega\delta}\frac{d\omega}{2\pi i}. \qquad (6)$$

Due to the existence of the Luttinger and Ward functional [9] variation of which reads:

$$\delta X = \sum_p \int_{-\infty}^{\infty} \mathrm{Tr}\;\Sigma(p,s,\omega)\delta G(p,s,\omega)\;\frac{d\omega}{2\pi} \qquad (7)$$

we can eliminate the second term in the brackets as being zero exactly. Summing Eq.(3) over all p we get in accordance with Eq.(6) the total number of electrons

$$N = \nu\sum_p = \nu\frac{S}{2\pi l_H^2}, \qquad (8)$$

assuming the periodic conditions along y direction $p = n/2\pi L_y$, $L_x > l_H^2 p > 0$ and $l_H^2 = c\hbar/eH$. This gives a standard expression for the electron density $n_e = \nu/2\pi l_H^2$. Thus we have shown that our Fermi liquid assumptions for electron Green function are valid only for electron density corresponding to the integer fillings.

It is possible to establish the properties of low energy collective excitations in the large r_s limit. One of these excitations is the magnetoplasmon mode or Kohn exciton associated with the following operators in the Landau gauge

$$\Pi^{\pm}(\mathbf{q}) = \frac{1}{\sqrt{2}}\int e^{\pm i\mathbf{qr}}(j_y \mp i j_x) d^2\mathbf{r}. \qquad (9)$$

Here $\mathbf{j}(\mathbf{r})$ is the current density operator in the second quantized representation and Landau gauge

$$j_x = \frac{1}{2}\left\{\psi^+\left(-i\frac{\partial}{\partial x}\right)\psi + i\frac{\partial\psi^+}{\partial x}\psi\right\},$$

and

$$j_y = \frac{1}{2}\left\{\psi^+\left(-i\frac{\partial}{\partial y} + x\right)\psi - \left[\left(-i\frac{\partial}{\partial y} - x\right)\psi^+\right]\psi\right\}.$$

Operators $\Pi^{\pm}(\mathbf{q})$ raises/lowers Ll index by one unit whereas operators $\Pi^{\pm}(\mathbf{0})$ commute with the Coulomb part of Hamiltonian

$$H_c = \frac{1}{2}\int V(\mathbf{q})\rho(\mathbf{q})\rho(-\mathbf{q})\frac{d^2\mathbf{q}}{(2\pi)^2}, \qquad (10)$$

where $\rho(\mathbf{q})$ is Fourrier component of the density operator $\psi^+(\mathbf{r})\psi(\mathbf{r})$. Commutator of $\Pi^+(0)$ with the kinetic energy can be easily calculated and we get: $H\Pi^+(0)|0\rangle = (\omega_H + E_0)\Pi^+(0)|0\rangle$, where $|0\rangle$ is the ground state wave function, and E_0 is the ground state energy. Thus $\Pi^+|0\rangle$ is also an eigenstate of the Hamiltonian – a statement known as celebrated Kohn theorem [11]. It gives the lowest energy of magnetoplasmon. Similarly we find: $\Pi^-(0)|0\rangle = (E_0 - \omega_H)\Pi^-(0)|0\rangle$. This is compatible with the assumption that $|0\rangle$ is the ground state only if $\Pi^-(0)|0\rangle = 0$. This property originates from the ideal Fermi gas and survives switching on interaction.

Kohn exciton being neutral can be classified by momentum vector $\mathbf{q}$ and the operator $\Pi^+(\mathbf{q})$ is essentially the first term in expansion of true exciton creation operator in powers of $\mathbf{q}$. Finite momentum gives rise to a dispersion of Kohn exciton $\omega_{ex}(\mathbf{q}) = \omega_H + \delta\omega_{ex}(\mathbf{q})$. It is possible to find the main part of this dispersion at small $\mathbf{q}$ exactly.

The derivation can be done in terms of density operator $\rho(\mathbf{q}, t)$ and current density operator $\mathbf{j}(\mathbf{q}, t)$ in Heisenberg representation. There are two equation of motion – the continuity equation for density:

$$\frac{\partial\rho}{\partial t} + i\ \mathbf{q}\cdot\mathbf{j}(\mathbf{q}) = 0 \qquad (11)$$

and the equation of motion for current:

$$\frac{\partial\mathbf{j}}{\partial t} = \frac{e}{mc}\mathbf{H}\times\mathbf{j}(\mathbf{q}) + n_e\mathbf{q}\ V(q)\rho(\mathbf{q}). \qquad (12)$$

It is straightforward to find a 'hydrodynamic' expression for the frequency of magnetoplasmon at small $\mathbf{q}$ ($ql_H \ll 1$):

$$\omega_{ex}(\mathbf{q}) = \sqrt{\omega_H^2 + \frac{n_e}{m}V(q)\ \mathbf{q}^2}. \qquad (13)$$

One can check that the corresponding two particle Green function has a pole at frequency (13). At very small $\mathbf{q}$ we get $\delta\omega_{ex}(\mathbf{q}) = e^2\nu|\mathbf{q}|/2$. The hydrodynamic Eqs.(11), (12) are valid only at $q \ll \sqrt{n_e}$ that gives the Kohn exciton limiting energy bound: $\omega_{ex}(\mathbf{q}) < \Delta_{ex} \approx \approx \sqrt{e^2 n_e\sqrt{n_e}/m}$.

The other collective mode is Goldstone spin wave created by an operator:

$$S^{\pm}(\mathbf{q}) = \int e^{\mp i\mathbf{qr}}\psi^+_\alpha(r)\sigma^{\pm}_{\alpha\beta}\psi_\beta(r)d^2\mathbf{r}, \qquad \sigma^{\pm} = (\sigma_x \pm i\sigma_y)/2, \qquad (14)$$

where σ_i are Pauli matrices. These operators commute with the total Hamiltonian at $\mathbf{q} = 0$ in the exchange approximation (neglecting spin-orbit and Zeeman interaction terms). This fact is a consequence of the global symmetry of the Hamiltonian with respect to rotations in a spin space. Spin wave excitation are also neutral and therefore are classified by momentum $\mathbf{q}$. The dispersion curve is quadratic at small $\mathbf{q}$ and for small r_s was calculated in Refs.[2, 3] to be $\varepsilon_{sw}(q) \approx E_c(ql_H)^2$. However in the opposite limit of large r_s spin wave dispersion is $\varepsilon_{sw} = \omega_H(ql_H)^2/2$ at small $\mathbf{q}$, as we show below.

In 2D ferromagnet special topological textures of spin order parameter field known as Skyrmions with nontrivial mapping of the entire 2D plain onto the unit sphere of spin directions are allowed [4]. In order to find the energy of such topological excitations it is necessary to start from microscopical quantum Hamiltonian for electron spinors, because phenomenological non-linear sigma model and it's parameters must be derived. Our approach is the same as in Ref.[12] where the case of small r_s has been considered. But that publication contains few faults and we repeat briefly the main points here. To establish a procedure for construction of the electron wave function describing skyrmion and to calculate it's energy it is usefull to introduce a unitary matrix $U(r)$ which rotates initially uniform spinor field $\chi_\uparrow(r)$ at every point of 2D plane. The Coulomb energy is invariant under any nonuniform rotation $\psi(\mathbf{r}) = U(\mathbf{r})\chi(\mathbf{r})$ because local density $\rho(\mathbf{r}) = \psi^+(\mathbf{r})\psi(\mathbf{r})$ is obviously invariant. Therefore the total transformed Hamiltonian takes the form

$$H = \frac{1}{2m}\int \chi^+(\mathbf{r})(-i\boldsymbol{\nabla} + \mathbf{A}(\mathbf{r}) + \boldsymbol{\Omega}(\mathbf{r}))^2\chi(\mathbf{r})\ d^2\ \mathbf{r} +$$

$$+\frac{1}{2}\int V(\mathbf{r}-\mathbf{r}')\chi^+_\alpha(\mathbf{r})\chi^+_\beta(\mathbf{r}')\chi_\beta(\mathbf{r})\chi_\alpha(\mathbf{r})\ d^2\mathbf{r}d^2\mathbf{r}', \qquad (15)$$

where the matrix field $\boldsymbol{\Omega}(\mathbf{r}) = -iU^+\boldsymbol{\nabla}U = \boldsymbol{\Omega}^l\sigma^l$ can be expanded in terms of the Pauli matrices σ^l. We consider only the case of large skyrmion core compared to

the magnetic length l_H, i.e. only small gradients of $U(\mathbf{r})$ and small vector functions $\mathbf{\Omega}^l$. The main assumption is that it is possible to use a perturbation theory in powers of $\mathbf{\Omega}(\mathbf{r})$ starting from the ferromagnetic state as a leading approximation. Matrix U depends on three Euler angles and it is topologically non-trivial only if there exists some nonzero winding number for two of its Euler angles. Using the frame with spin direction at large distances as z axis we can parametrize U as three consecutive rotations

$$U(\alpha,\beta,\gamma) = U_z(\alpha)U_y(\beta)U_z(\gamma),$$

where angles α and γ describe rotation around z direction having the same winding number in 2D plain, whereas the angle $0 \le \beta \le \pi$ describes a rotation around some perpendicular direction taken to be y. The condition of identical winding number for α and γ is related to the requirement for matrix U to be nonsingular over the whole 2D plain which is essential to allow for a perturbation theory in Ω. In this case

$$\mathbf{\Omega}^z = \frac{1}{2}(1+\cos\beta)\boldsymbol{\nabla}\alpha,$$

$$\mathbf{\Omega}^x = \frac{1}{2}(\sin\beta\cos\alpha\boldsymbol{\nabla}\alpha - \sin\alpha\boldsymbol{\nabla}\beta),$$

$$\mathbf{\Omega}^y = \frac{1}{2}(\sin\beta\sin\alpha\boldsymbol{\nabla}\alpha + \cos\alpha\boldsymbol{\nabla}\beta),$$

We see that vectors $\mathbf{\Omega}^l$ are well defined and smooth provided point singularities of α coinciding with the points where $\beta(r)=\pi$. At $\mathbf{r}\to\infty$, $\beta(r)\to 0$, therefore $U(r)$ is uniquely defined over entire 2D plain. The nonsingular part of angle $\alpha-\gamma$ is irrelevant and we set it to be zero. The topological integer invariant is given by

$$Q = \frac{1}{2\pi}\int \boldsymbol{\nabla}\times\mathbf{\Omega}^z \; d^2\mathbf{r}. \tag{16}$$

In this construction $U(\mathbf{r})$ is defined as an external classical matrix field in electron Hamiltonian. Complete quantum description of matrix field $U(\mathbf{r},t)$ or equivalently skyrmion wave function is a difficult problem. In quantum field theory different topological sectors are considered separately without transitions between [13]. We use this approach assuming a large skyrmion core making these transitions improbable. In Hamiltonian (15) Ω^z and Ω^l with $l\neq z$ describe different effects and can be treated separately up to the second order of perturbation theory. That allows us to cast the kinetic part of Hamiltonian in terms of $\mathbf{\Omega}^l$:

$$H = \frac{1}{2m}\int \chi^+(r)\Big[-i\boldsymbol{\nabla}+\mathbf{A}(r)+\mathbf{\Omega}^z\sigma_z\Big]^2\chi(r)\; d^2\mathbf{r} +$$

$$+\frac{1}{m}\sum_{l\neq z}\int \chi^+(\mathbf{r})\mathbf{\Omega}^l\sigma_l\Big(-i\boldsymbol{\nabla}+\mathbf{A}\Big)\chi(\mathbf{r})\; d^2\mathbf{r} +$$

$$+\frac{1}{2m}\sum_{l\neq z}\int \chi^+(\mathbf{r})\chi(\mathbf{r})\Big(\mathbf{\Omega}^l\Big)^2\; d^2\mathbf{r}. \tag{17}$$

We see that $\mathbf{\Omega}^z$ defines additional effective vector potential and corresponding effective magnetic field having opposite sign for two spin states. Up to second order in Ω^z we can consider only reference ferromagnetic spin up configuration. Any term in skyrmion energy can be expanded in gauge invariant terms therefore it depends only on $\nabla\times\mathbf{\Omega}^z$ and it's derivatives. The second term in kinetic energy can be rewritten in terms of excitonic like creation operators

$$T = \frac{1}{2m}\sum_{l\neq z}\int\Big\{\Omega^l_+\chi^+(\mathbf{r})\sigma_l\pi^-\chi(\mathbf{r}) +$$

$$+\Omega^l_-(\mathbf{r})\chi^+\sigma_l\pi^+\Big\}\chi(\mathbf{r})\; d^2\mathbf{r}, \tag{18}$$

where $\Omega_\mp = \Omega_y \pm i\Omega_x$ and

$$\begin{aligned}\pi^- &= \partial/\partial x - i\partial/\partial y + x,\\ \pi^+ &= -\partial/\partial x - i\partial/\partial y + x.\end{aligned} \tag{19}$$

Kinetic term (18) can be expressed in terms of a complicated spin-flip magnetoplasmon exciton – excitation that combines both charge and spin and is created by an operator

$$\Lambda^\pm(0) = \int \psi^+_\alpha(\mathbf{r})\sigma^\pm_{\alpha\beta}\pi_\pm\psi_\beta(\mathbf{r})\; d^2\mathbf{r}, \tag{20}$$

Operators $\Lambda^\pm(0)$ do not commute with the Coulomb part of Hamiltonian. Therefore their dispersion is determined by diagrams with large internal momenta and cannot be found analytically even at small $\mathbf{q}$. Even in the limit $r_s\to\infty$ any excitation frequency must depend on the kinetic part of the Hamiltonian because pure potential interaction leads to extreem degeneracy of electron states with zero velocity. Therefore the energy of this spin-flip excitons must include the kinetic part of the Hamiltonian. The proper scale is given by the energy of Kohn exciton at large $q\sim 1/l_H$: $\Delta_{ex}\sim\sqrt{e^2 n_e\sqrt{n_e}/m}$.

In the first order of perturbation theory there are three terms in QHF energy. The first is

$$\delta E_1 = \frac{1}{2m}\sum_{l\neq z}\int\langle\rho(\mathbf{r})\rangle\left(\mathbf{\Omega}^l\right)^2 d^2\mathbf{r}, \tag{21}$$

where $\langle\rho(r)\rangle$ is the ground state average density, and two terms due to the change of effective magnetic field: the local change of cyclotron energy

$$\delta E_H = \frac{1}{m}\sum_{n,p}\Big(n+\frac{1}{2}\Big)\int \boldsymbol{\nabla}\times$$

$$\times\,\boldsymbol{\Omega}^z\langle 0|\chi^+_{np}(r)\chi_{np}(r)|0\rangle\varphi^*_{np}(r)\varphi_{np}(r)\;d^2\mathbf{r}, \qquad (22)$$

where χ_{np}, χ^+_{np} are creation operators for the n's Ll state, and the correction to the local exchange energy

$$\delta E_{ex} = -\frac{1}{2}\int \frac{\partial E_{ex}}{\partial H}\nabla\times\boldsymbol{\Omega}^z(r)\;d^2\mathbf{r}, \qquad (23)$$

where $E_{ex}(H)$ is the exchange energy density in a uniform ferromagnetic ground state.

The calculation of skyrmion energy in the limit of small r_s has been done in [12] up to second order except for missing correction (23) to the exchange energy. Adding this to the results of [12] in the case of $\nu = 1$ for the 2D Coulomb interaction gives skyrmion energy for small r_s:

$$E_{sk} = \sqrt{\frac{\pi}{8}}\,\frac{e^2}{l_H}(|Q| - 2Q) \qquad (24)$$

that coincides with the earlier results of Refs.[14, 15].

The main new feature for the case of large r_s appears in the second order perturbation term in T:

$$\delta E_2 = \langle 0|T\frac{1}{E_0 - H}T|0\rangle, \qquad (25)$$

T is connected to spin-flip magnetoplasmon operators (20) with small $\mathbf{q}$ (18) because Fourrier transform of $\Omega^l(\mathbf{r})$ contains only small $\mathbf{q}$. Operator T acts on the ground ferromagnetic state and therefore only the term with σ^+ that reverses the spin is essential. We get for second order term the estimate

$$\delta E_2 \approx -\frac{\omega_H^2}{\sqrt{\Delta_{ex}}} \ll \omega_H. \qquad (26)$$

Though this contribution is negative and thus decreases the skyrmion energy we can neglect it in the limit of strong Coulomb interaction. We want to emphasize here that in the opposite limit of small r_s this term gives an essential contribution to total skyrmion energy.

Spin structure of skyrmion and antiskyrmion is identical except for the sign of winding number. Therefore their additional Zeeman and direct Coulomb energies are the same. In the activation energy for the creation of skyrmion antiskyrmion pair with opposite topological charges Q and $-Q$ all terms that are proportional to $\boldsymbol{\nabla}\times\boldsymbol{\Omega}^z$ cancel:

$$\Delta_{sk} = E_{sk}(Q) + E_{sk}(-Q) =$$

$$= \frac{\nu}{4\pi m}\int(\Omega_x^2 + \Omega_y^2)\;d^2\mathbf{r} = \nu\omega_H|Q|. \qquad (27)$$

The last equality in (27) holds for a special Belavin–Polyakov ansatz for skyrmion's matrix $U(\mathbf{r})$ that minimizes the energy of QHF. Result (27) coincides with large r_s extrapolation of skyrmion energy for the case of small r_s at $\nu = 1$ obtained in Ref.[16].

If we consider nonuniform rotation $U(r)$ with vanishing winding number we obtain a gradient energy in nonuniform ferromagnet:

$$\delta E_1 = \frac{\rho}{8m}\int\left(\frac{\partial\mathbf{n}}{\partial x_k}\right)^2 d^2\mathbf{r} = \frac{1}{16\pi}\nu\omega_H\int\left(\frac{\partial\mathbf{n}}{\partial x_k}\right)^2\;d^2\mathbf{r}, \qquad (28)$$

where $\mathbf{n} = (\sin\beta\cos\alpha,\ \sin\beta\sin\alpha,\ \cos\beta)$ is the unit vector in the direction of local spin. This gradient energy gives a spin wave dispersion: $\omega_{sp} = \omega_H\mathbf{q}^2/2$.

The local electron density is determined by the local magnetic field:

$$\rho(r) = \frac{\nu}{2\pi l_H^2}\Big(1 + \boldsymbol{\nabla}\times\boldsymbol{\Omega}^z\cdot\frac{\mathbf{H}}{H}\Big) \qquad (29)$$

according to our classification assumption that the density coincides with that for ideal Fermi gas. Therefore the electric charge of skyrmion is

$$q = \frac{e\nu}{2\pi}\int\nabla\times\boldsymbol{\Omega}^z\;d^2\mathbf{r} = e\nu Q \qquad (30)$$

and the activation energy per electron or hole charge is given by $\Delta_{activ} = \omega_H/2$. This quantity is proportional to magnetic field and is small compared to Coulomb exchange energy E_c. That is qualitatively in accordance with experimental data for activation energy. We do not discuss here the mobility of skyrmions that may alter qualitatively the mechanism behind experimentally observed activation energy.

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