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Bound states in a quantized Hall ferromagnet

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We report on a study of the quasielectron-quasihole and skyrmion-antiskyrmion bound states in the $\nu = 1$ quantum Hall regime. The short range attraction potential is assumed to be determined by a point magnetic impurity. The calculations are performed within the strong field approximation when the binding energy and the characteristic electron-electron interaction energy are smaller than the Landau level spacing. The Excitonic Representation technique is used in that case.

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1. Unique properties of the two-dimensional electron gas (2DEG) in strong magnetic fields attract much attention to its spectrum. In particular, the interaction of 2DEG with neutral short range impurities exhibits unusual features in comparison with its 3D prototype [1, 2]. In this paper we study the bound fermion states appearing in the *quantum Hall "ferromagnet"* (QHF) regime; i.e. the filling factor is $\nu = \mathcal{N}/N_\phi \simeq 2n+1$, where $\mathcal{N}$ and $N_\phi = L^2/2\pi l_B^2$ are the numbers of electrons and magnetic flux quanta (L^2 is the 2DEG area, l_B is the magnetic length). In the high magnetic field limit, which really represents the solution to the first order in the ratio $r_c = (e^2/\varepsilon l_B)/\hbar\omega_c$ considered to be small (ω_c is the cyclotron frequency, ε is the dielectric constant), we get the ground state with zeroth, first, second,... and $(n-1)$-th Landau levels (LLs) fully occupied and with nth level filled only by spin-up electrons aligned along $\mathbf{B}$.

In the clean limit, fermion excitations are classified by their spin-numbers $|\Delta S_z| = K + 1/2$ (K is an integer) [3], ranging from the simplest $|\Delta S_z| = 1/2$ case of quasielectrons or quasiholes to the $K \to \infty$ limit which corresponds to the so-called skyrmions. Certainly, the total energy of excitations incorporates the Zeeman energy $|\epsilon_Z \Delta S_z|$, and the spin number of lowest-lying fermions is thus determined by the actual value of the gap $\epsilon_Z = |g\mu_B B|$. As regards to the Coulomb exchange energy of the fermions, in the $\nu = 1$ case this part of the total energy decreases monotonically with the K number [3]. E.g., to the first order in $r_{\rm C}$ the exchange energies are $-\frac{1}{4}\sqrt{\pi/2}e^2/\varepsilon l_B$ and $\frac{3}{4}\sqrt{\pi/2}e^2/\varepsilon l_B$ (in the strict 2D limit) for electron-like and hole-like skyrmions, respectively [4 – 8]. For comparison, in the $K = 0$ case these ones are 0 (electrons) and $\sqrt{\pi/2}e^2/\varepsilon l_B$ (holes).

We calculate the bound-state energies in the presence of point impurities. In the single electron approximation (i.e. in the $\nu \ll 1$ case) this problem was studied in Ref.[2] for arbitrary strengh of the perpendcular magnetic field. In Refs.[9, 10] the authors investigated the genesis of the impurity potential in the 2D channel and gave a proof for the short range approach. The latter in terms of the "envelope" wave-function method means that the inpurity Hamiltonian can be modeled by the δ-function:

$$\widehat{H}_{\rm imp} = 2\pi\left(W\widehat{\mathbf{1}} + \frac{D}{2}\widehat{\sigma_Z}\right)\delta(\mathbf{r}) \qquad (1)$$

(here and in the following $\sigma_{x,y,z}$ are the Pauli matrices). The $D \neq 0$ case corresponds to a paramagnetic impurity with its own magnetic moment aligned parallel to the magnetic field. (We consider that $\mathbf{B} \parallel \widehat{z}$.) At the negative g-factor the positive value of D provides capture of the spin wave (spin exciton) by an impurity in the QHF case [9].

We will solve the problem in the "shallow" impurity approximation,

$$|W_\sigma| \ll \hbar^2/m_e^* \qquad (\sigma = \uparrow, \downarrow), \qquad (2)$$

where $W_{\uparrow,\downarrow} = W \pm D/2$. Actually one will see that this condition enables to employ the high magnetic field approach:

$$E_b \ll \hbar\omega_c\,, \qquad (3)$$

where E_b is the desired binding energy, and ω_c is the cyclotron frequency. The condition (3) allows us to employ the projection onto a single LL approach when calculating E_b in the leading approximation. At the same time we still consider that the point-impurity approximation is not disturbed, i.e. the impurity localization radius ρ_b is much smaller than the magnetic length: $\rho_b \ll l_B$.

2. Any single-electron state may be presented in the form of the expansion

$$\chi = \sum_{ap} c_{ap}\phi_{ap}, \qquad (4)$$

where we choose the Landau-gauge functions ϕ_{ap} as the basis set. The subscript p distinguishes between different states belonging to a continuously degenerate Landau level, and the label a is a binary index $a = (n_a, \sigma_a)$, which represents both LL index and spin index. We have thus $\phi_{ap}(\mathbf{r}, \sigma) = \delta_{\sigma,\sigma_a}(l_B L)^{-1/2} e^{ipy} \varphi_{n_a}(pl_B + x/l_B)$, where

$$\varphi_n(x) = (2^n n! \sqrt{\pi})^{-1/2} e^{-x^2/2} H_n(x) \qquad (5)$$

$[H_n(x)$ is the Hermite polynomial]. In the following we employ the notation a_p, b_p, ... for the electron annihilation operator corresponding to sublevel a, b,... and use also the intra-LL "displacement" operators $\mathcal{A}^+_{\mathbf{q}}$, $\mathcal{B}^+_{\mathbf{q}}$, ..., where

$$\mathcal{A}^+_{\mathbf{q}} = N_\phi{}^{-1} \sum_p e^{-iq_x p l_B^2} a^+_{p+\frac{q_y}{2}} a_{p-\frac{q_y}{2}}, \qquad \mathcal{B}^+_{\mathbf{q}} = (a \to b), \ldots \qquad (6)$$

($\mathcal{A}^+_{\mathbf{q}} = \mathcal{A}_{-\mathbf{q}}$). Considering the quantity χ as an annihilation operator in the Schrödinger representation, we can substitute Eq. (4) into $\langle\chi|\widehat{H}_{\rm imp}|\chi\rangle$ to obtain the secondary quantized representation of the Hamiltonian $\mathcal{H}_{\rm imp}$, namely:

$$\widehat{\mathcal{H}}_{\rm imp} \approx l_B^{-2} \sum_{\mathbf{q}} e^{-q^2 l_B^2/4} \left(W_\uparrow \mathcal{A}^+_{\mathbf{q}} + W_\downarrow \mathcal{B}^+_{\mathbf{q}} \right). \qquad (7)$$

Here the sign of approximate equality means that we have omitted the terms corresponding to the LL mixing and have kept only those relevant to the projection onto the nth LL. Therefore, specifically we have in Eq. (7) that the labels a and b correspond to $a = (n, \uparrow)$ and $b = (n, \downarrow)$ sublevels. At $\nu = 2n + 1$ the "clean" ground state $|0\rangle$ is completely determined by the equations $\mathcal{A}_{\mathbf{q}}|0\rangle = \delta_{\mathbf{q},0}|0\rangle$ and $\mathcal{B}_{\mathbf{q}}|0\rangle = 0$, while electron and hole are defined as the

$$|f_e\rangle = \sum_p f_e(p) b_p^+ |0\rangle, \quad |f_h\rangle = \sum_p f_h(p) a_p |0\rangle \qquad (8)$$

states, respectively. Here the envelope functions are normalized as $\sum_p |f_{e,h}|^2 = 1$.

The quasiparticle states (8) satisfy the "clean" equations $\widehat{\mathcal{H}}_0 |f_{e,h}\rangle = (E_0 + E_{e,h})|f_{e,h}\rangle$, where $E_e = \epsilon_Z$ and $E_h = E_C$. Here E_C is the characteristic Coulomb energy: $E_C = \int e^{-q^2 l_B^2/2} V(\mathbf{q}) d\mathbf{q}$, where $2\pi V(\mathbf{q})$ is the 2D Fourier component of the averaged Coulomb potential (in the ideal 2D case $V = e^2/\kappa q$ and $E_C = \sqrt{\pi/2} e^2/\kappa l_B$). To obtain this result, it is convenient to employ the expression for the Coulomb interaction Hamiltonian in terms of the Excitonic Representation (see, e.g., Ref.[8]) and the commutation rules

$$\left[\mathcal{A}^+_{\mathbf{q}}, a_p\right] \equiv -\frac{1}{N_\phi} e^{-iq_x l_B^2 (p - q_y/2)} a_{p-q_y}, \qquad \left[\mathcal{B}^+_{\mathbf{q}}, b_p^+\right] \equiv \frac{1}{N_\phi} e^{-iq_x l_B^2 (p + q_y/2)} b^+_{p-q_y} \qquad (9)$$

and $\left[\mathcal{A}^+_{\mathbf{q}}, b_p^+\right] = \left[\mathcal{B}^+_{\mathbf{q}}, a_p\right] \equiv 0$.

First we obtain the correction to the ground state energy in the case of single impurity. Substituting expression (7) for $\widehat{\mathcal{H}}_{\rm imp}$ into the equation $\left(\widehat{\mathcal{H}}_0 + \widehat{\mathcal{H}}_{\rm imp}\right)|0\rangle = (E_0 + \Delta E_0)|0\rangle$ (where $\widehat{\mathcal{H}}_0$ is the "clean" QHF Hamiltonian *including the Zeeman and Coulomb interaction part*) we obtain the correction to the "clean" ground state: $\Delta E_0 = W_\uparrow / l_B^2$.

Finally, to calculate the bound electron state we solve the equation $\widehat{\mathcal{H}}_{\rm imp}|f_e\rangle = E'_e|f_e\rangle$, which with help of Eqs. (7)-(9) is reduced to the integral equation

$$\frac{W_\downarrow}{l_B\sqrt{\pi}} \int_{-\infty}^{+\infty} ds f_e(s) e^{-(s^2+p^2) l_B^2/2} = E'_e f(p). \qquad (10)$$

The latter has the solution $f_e(p) = (\pi N_\phi)^{-1/4} e^{-p^2 l_B^2/2}$ at $E'_e = W_\downarrow / l_B^2$ [$(\pi N_\phi)^{-1/4}$ is the normalization factor]. Hence the electron binding energy is

$$E_b^{(e)} = -W_\downarrow / l_B^2. \qquad (11)$$

Naturally, this state is realized if $W_\downarrow < 0$. Let us note that in the leading approximation the obtained $E_b^{(e)}$ value is equal to the binding energy in the single electron problem [2]. In a similar way we obtain that the equation $\widehat{\mathcal{H}}_{\rm imp}|f_h\rangle = E'_h|f_h\rangle$ has the solution $f_h(p) = (\pi N_\phi)^{-1/4} e^{-p^2 l_B^2/2}$ at $E'_h = -W_\uparrow / l_B^2$. So, the total energy of the $|f_h\rangle$ bound state is $E_0 + E_C$; i.e. it is just the same as in the "clean" hole state. The bound energy of the hole is

$$E_b^{(h)} = W_\uparrow / l_B^2. \qquad (12)$$

This state exists under the condition $W_\uparrow > 0$.

The physical meaning of the envelope functions $f_{e,h}$ obtained above becomes evident if we change the Landau gauge to the symmetric gauge. In the latter case we have to change the vector potential $\mathbf{A} = (0, Bx, 0)$ for $\mathbf{A} = (-By/2, Bx/2, 0)$. Then the single electron states of the n's LL are described by the basis spatial function

$$\phi_{nm} = l_B^{-1} \left[\frac{n!}{2^{m+1}(m+n)!\pi}\right]^{1/2} \times (ir/l_B)^m L_n^m (r^2/l_B^2 2) e^{-im\varphi - r^2/4l_B^2} \quad (n + m \geq 0), \qquad (13)$$

where $\mathbf{r} = (r\cos\varphi, r\sin\varphi)$, L_n^m is the Laguerre polynomial and m runs over N_ϕ integer numbers: $m = -n, 1-n, 2-n, \ldots, N_\phi - n - 1$. All these states have the same cyclotron energy, and the Fermi creation and annihilation operators acquire index m (instead of p in the Landau gauge) now. For example, one can find the expression for old Landau gauge operator a_p in terms of new operators a_m:

$$a_p = N_\phi^{-1/2} \sum_{m=0}^{\mathcal{N}-1} i^{m-n} \varphi_m(pl_B) a_{m-n} \tag{14}$$

where φ_m is the oscillatory function (5). Now, if we substitute this expression (and analogous one for b_p^+) into Eqs. (8) with the above found functions, we obtain that only the $m=0$ harmonic of Eq. (13) contributes to the $|f_{e,h}\rangle$ bound states. Indeed, the summation over p in the expansion (8) turns out to be proportional to the integral $\int\limits_{-\infty}^{\infty} e^{-p^2} H_m(p)$. The latter vanishes at any m except for $m=0$. Certainly, this feature reflects the well known fact that the point impurity Hamiltonian is diagonal exactly in the symmetric basis (13). Besides, only the zero (axially symmetric) harmonic contributes to the bound state energy calculated within the single LL approximation.

3. To study a bound skyrmionic excitation we present it in accordance with Refs.[7, 8] as a smooth rotation in the 3D spin space

$$\boldsymbol{\psi}(\mathbf{r}) = \widehat{U}(\mathbf{r})\boldsymbol{\chi}(\mathbf{r}), \quad \mathbf{r} = (x, y)\,. \tag{15}$$

Here $\boldsymbol{\psi}$ is a spinor given in the stationary coordinate system and $\boldsymbol{\chi}$ is a new spinor in the local coordinate system following this rotation. The rotation matrix $\widehat{U}(\mathbf{r})$ ($\widehat{U}^\dagger\widehat{U} = 1$) is parameterized by three Eulerian angles [11]:

$$\widehat{U} = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i(\varphi+\eta)/2} & \sin\frac{\theta}{2} e^{i(\eta-\varphi)/2} \\ -\sin\frac{\theta}{2} e^{i(\varphi-\eta)/2} & \cos\frac{\theta}{2} e^{i(\varphi+\eta)/2} \end{pmatrix}. \tag{16}$$

These angles $\theta(\mathbf{r})$, $\varphi(\mathbf{r})$ and $\eta(\mathbf{r})$ present continuum field functions. The skyrmion state is thus determined by the continuum matrix $\widehat{U}(\mathbf{r})$ and by the local quantum state $\boldsymbol{\chi}$ determined in terms of small gradient $(l_B\nabla\widehat{U})$ corrections to the local QHF, where all electron spins are parallel $\left[\boldsymbol{\chi} \propto \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)\right]$.

The solution may be found from the reformulated variational principle. Namely, we divide the 2DEG area into a large number G_i of domains which are much smaller than the total 2DEG area but still remain much larger than the magnetic flux quantum area $2\pi l_B^2$. The energy of excitations of this type may be found through the minimization procedure in the following way:

$$E = \min_U \left[\sum_i \min_\psi \left(\frac{\langle\psi|H_i|\psi\rangle_{G_i}}{\langle\psi|\psi\rangle_{G_i}}\right)\right]. \tag{17}$$

Here averaging is performed over the domain G_i. All the G_i areas add up to the total 2DEG area. H_i is the Hamiltonian corresponding to the G_i domain. The state $|\psi\rangle$ presents here a many-electron quantum state built by the single electron spinors (15). So, the state $|\psi\rangle$ is parametrized by $\widehat{U}$ and by the derivatives of $\widehat{U}$ (generally, up to any order) considered as external parameters for every G_i. The procedure of the inner minimization in Eq. (17) is equivalent to the solution of the Schrödinger equation within the area $\Delta x\Delta y = G_i$.

In our case the local Hamiltonians are just the same as in the clean case except for the Hamiltonian $H_{i=0}$ corresponding to the one domain G_0 which involves the point impurity. The procedure of the minimization (17) only differs from the clean case by adding the energy

$$\Delta E_{\rm imp}^{(U)} = \frac{\langle\chi|\widehat{U}^+\widehat{H}_{\rm imp}\widehat{U}|\chi\rangle_{G_i}}{\langle\chi|\chi\rangle_{G_i}} \tag{18}$$

to the sum within the square brackets in Eq. (17). Here we have used Eq. (15) and then should substitute

$$\widehat{U}^+\widehat{H}_{\rm imp}\widehat{U} = \begin{pmatrix} W + \dfrac{D}{2}\cos\theta & -\dfrac{D}{2}\sin\theta e^{-i\varphi} \\ -\dfrac{D}{2}\sin\theta e^{i\varphi} & W - \dfrac{D}{2}\cos\theta \end{pmatrix}\delta(\mathbf{r}) \tag{19}$$

into Eq. (18). (We have set $\eta = \varphi$ without any loss of generality.) Under the conditions (2), (3), it is sufficient to use as $|\chi\rangle$ the unperturbed QHF state determined in the local coordinate system of domain G_0. That is, in our case $|\chi\rangle = \widehat{\chi}|0\rangle$, where $\widehat{\chi}$ is the annihilation operator. If employing again the expansion of Eq. (4), then with the help of equation

$$\widehat{\mathcal{H}}_{\rm imp} = \int\limits_{\text{over } G_0} d\mathbf{r}\,\widehat{\boldsymbol{\chi}}^+\widehat{U}^+\widehat{H}_{\rm imp}\widehat{U}\widehat{\boldsymbol{\chi}}, \tag{20}$$

we can obtain the impurity Hamiltonian in terms of the secondary quantized representation. We will study only the $\nu = 1$ case. Substituting Eqs. (19) and (4) into

Eq. (20) and considering that $a=(0,\uparrow)$ and $b=(0,\downarrow)$, we find within the single LL approximation

$$\widehat{\mathcal{H}}_{\rm imp}=l_B^{-2}\sum_{\mathbf{q}}e^{-q^2l_B^2/4}\times$$

$$\times\left[(W+\frac{D}{2}\cos\theta_0)\mathcal{A}^+_{\mathbf{q}}+(W-\frac{D}{2}\cos\theta_0)\mathcal{B}^+_{\mathbf{q}}-\right.$$

$$\left.-\frac{D}{2}\sin\theta_0 e^{-i\varphi_0}N_\phi^{-1/2}\mathcal{Q}_{\mathbf{q}}-\frac{D}{2}\sin\theta_0 e^{i\varphi_0}N_\phi^{-1/2}\mathcal{Q}^+_{\mathbf{q}}\right]. \tag{21}$$

Definitions of the operators $\mathcal{A}^+_{\mathbf{q}}$ and $\mathcal{B}^+_{\mathbf{q}}$ are identical to those given by Eq. (6) with the number of magnetic flux quanta $N_\phi=\Delta x\Delta y/2\pi l_B^2$ being non-zero only for the domain G_0. The notation of the spin-exciton creation $\mathcal{Q}^+_{\mathbf{q}}=N_\phi{}^{-1/2}\sum_p e^{-iq_xpl_B^2}b^+_{p+\frac{q_y}{2}}a_{p-\frac{q_y}{2}}$ and annihilation $\mathcal{Q}_{\mathbf{q}}=\left(\mathcal{Q}^+_{\mathbf{q}}\right)^+$ operators have also been used in Eq. (21). θ_0 and φ_0 are the Hermitian angles (determined by the given $\widehat{U}$ matrix) corresponding to the domain G_0. Now in accordance with Eq. (18) we get

$$\Delta E^{(U)}_{\rm imp}=\langle 0|\widehat{\mathcal{H}}_{\rm imp}|0\rangle=\left(W+\frac{D}{2}\cos\theta_0\right)/l_B^2. \tag{22}$$

(Only the $\sim\mathcal{A}^+_0$ term contributes to this result.)

The outer minimization in Eq. (17) is thereby presented as $\min\limits_U\left(E_U+\Delta E^{(U)}_{\rm imp}\right)$, where E_U is the "clean" energy obtained for a given function $\widehat{U}(\mathbf{r})$ after the summation over all G_i in Eq. (17). Meanwhile the minimization of E_U and $\Delta E^{(U)}_{\rm imp}$ may be fulfilled independently. Indeed, the clean skyrmion state is degenerate having the energy which does not depend on the "skyrmion center" position. The latter is the point of the total 2DEG area where $\theta=\pi$, i.e. local electron spins are aligned in the direction opposite to the direction at the infinity (where $\theta=0$). In contrast to this, the energy $\Delta E^{(U)}_{\rm imp}$ just depends on the relative positions of the impurity site (the $\mathbf{r}$=0 point in our coordinate system) and the skyrmion center site. Thus the impurity lifts the degeneracy. Hence by varying the position of the skyrmion center site, we find that $\min\limits_U\left[\Delta E^{(U)}_{\rm imp}\right]=(W-|D|/2)/l_B^2$. If $D<0$, this value is reached at $\theta_0=0$, i.e. the skyrmion center is located at the infinity. Evidently in this case the skyrmion does not form a bound state. At $D>0$, the minimum energy is realized for $\theta_0=\pi$, i.e. where the impurity site and the skyrmion center coincide (of course, to within the length smaller than the characteristic skyrmion radius R^* but perhaps larger than l_B). This means that the bound skyrmion state takes place. The binding energy should be obtained by subtraction of the minimum energy from the $\theta_0=0$ energy $(W+D/2)l_B^2$, i.e.

$$E_b^{\rm sk}=D/l_B^2. \tag{23}$$

So, in the adopted approximation only the magnetic impurity with $D>0$ captures the skyrmion (cf. Ref.[9] where the similar result is obtained in the case of the bound spin exciton). It is worth to note that the result (23) does not depend on the skyrmion charge [12]: electron-like and hole-like skyrmions have the same bound energy (23).

Analysis reveals that the charge dependence arises only in the second order approximation in terms of $l_B\nabla$, being determined by the relative correction $(l_B/R^*)^2$. This can be easily found by means of the renormalization procedure for the magnetic length $l_B\to\tilde{l}_B$. Indeed, the effective local magnetic length is determined by the effective magnetic field, including the additional part proportional to the second-order spatial derivatives of the field $\widehat{U}$ [8]:

$$\frac{1}{\tilde{l}_B^2}=\frac{1}{l_B^2}+\boldsymbol{\nabla}\times\boldsymbol{\Omega}^z\,. \tag{24}$$

Here

$$\boldsymbol{\Omega}^z=\frac{1}{2}\left(1+\cos\theta\right)\boldsymbol{\nabla}\varphi \tag{25}$$

(see Refs.[7, 8]) and l_B is the magnetic length at the infinity (far from the skyrmion center). Let us assume that the impurity is non-magnetic, i.e. $D=0$. The binding energy (if the bound state would be realized) should be determined exactly by the desired correction. We substitute into Eqs. (24), (25) for the angles their expressions in terms of functions of $\mathbf{r}$ [13]:

$$\cos\theta=\frac{R^{*2}-r^2}{R^{*2}+r^2}\,,\quad \phi=-q\arctan(y/x)$$

(where $q=\pm$ is the skyrmion charge). Eventually, by comparing the skyrmion energy at $\mathbf{r}=0$ with the energy at the infinity, we find the impurity correction: $\Delta E_{\rm imp}=W\boldsymbol{\nabla}\,\boldsymbol{\Omega}^z|_{\mathbf{r}=0}=2qW/R^{*2}$. The binding energy is thus

$$E_b^{\rm sk}=-2qW/R^{*2}\quad\text{if}\quad D=0\,.$$

Therefore the bound electron/hole-like skyrmion arises when W is positive/negative.

In reality, the concentration of point impurities in the 2D channel can be considerable. At least it seems to have the values at which the mean distance between impurities is well shorter than the effective skyrmion radius (the latter is determined by small but still non-zero Zeeman gap ϵ_Z^*). In this case the impurity contribution to the total energy is equal to $\int d\mathbf{r}\lambda l_B^{-2}\left[W+\frac{D}{2}\cos\theta(\mathbf{r})\right]$, where $\lambda(\mathbf{r})$ is the concentration of impurities. It involves also the correction to the ground state energy $l_B^{-2}\left(W+\frac{D}{2}\right)\int d\mathbf{r}\lambda$ which should be subtracted. The impurity correction to the proper skyrmion energy is thereby

$$\Delta E^{\rm sk}_{\rm imp}=\frac{D}{2l_B^2}\int\left[\cos\theta(\mathbf{r})-1\right]\lambda d\mathbf{r}\,.$$

If we compare this value with the skyrmion Zeeman energy

$$\frac{\epsilon_Z^*}{2}\int(1-\cos\theta)d\mathbf{r}/2\pi l_B^2\,,$$

it becomes evident that the magnetic impurities dispersed in the 2D channel lead to the correction to the Zeeman gap:

$$\epsilon_Z^*\to\epsilon_Z^*-2\pi\lambda D$$

(assuming a homogeneous concentration λ). Due to the supposed smallness of ϵ_Z^*, this can be substantial. Under certain conditions it could change the sign of effective g-factor in the 2DEG.

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