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Nuclear Stochastic Resonance

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Fission decay of highly excited periodically driven compound nuclei is considered in the framework of Langevin approach. We used residual-time distribution (RTD) as a tool for studying the dynamic features in the presence of periodic perturbation. The structure of RTD essentially depends on the relation between Kramers decay rate and the frequency ω of periodic perturbation. In particular, the intensity of the first peak in RTD has a sharp maximum at certain nuclear temperature depending on ω. This maximum should be considered as first-hand manifestation of stochastic resonance in nuclear dynamics.

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The atomic nucleus since its discovery has been constantly used for verifying new physical ideas such as tunneling [1], superfluids [2], superconductivity [3], supersymmetry [4], dynamical chaos [5]. Thus it seems unnatural that one of the most recent and intriguing discoveries in nonlinear physics – stochastic resonance (SR) (see [6] for a recent review) have not still found response of the nuclear community. This is particularly strange because there is no doubt that the theory of the collective nuclear motion claiming to a consistent description of nuclear dynamics must be essentially nonlinear theory. The aim of the present work is to demonstrate the possibility of observation of SR in nuclear dynamics. As a specific example we consider the process of induced nuclear fission in the presence of weak periodic perturbation.

SR was introduced nearly 20 years ago to explain the periodicity of the Earth's ice ages [7, 8] and has found its numerous applications into such diverse fields like physics, chemistry and biology (see [6]).

The mechanism of SR can be explained in terms of the motion of a particle in a symmetric double-well potential subjected to noise and time periodic forcing. The noise causes incoherent transitions between the two wells with a well-known Kramers rate [9] r_k. If we apply a weak periodic forcing noise-induced hopping between the potential wells can become synchronized with periodic signal. This statistical synchronization takes place at the condition

$$r_k^{-1} \approx \pi/\omega \qquad (1)$$

where ω is the frequency of periodic forcing. Two prominent feature of SR arises from synchronization condition (1):

(i) signal-to-noise ratio does not decrease monotonously with increasing of noise amplitude (as it happens in linear system), but attains a maximum at a certain noise strength (optimal noise amplitude can be found from (1) as r_k is simply connected with it);

(ii) the residence-time distribution (RTD) demonstrates a series of peaks, centered at odd multiples of the half driving period $T_n = 2(n - \frac{1}{2})\frac{\pi}{\omega}$ with exponentially decreasing amplitude. Notice that if a single escape from a local potential well is the event of interest then RTD reveals the dynamics of considering system more transparently than the signal-to-noise ratio. These signatures of SR are not confined to the special models but occur in general bi- and monostable systems and for different types of noise [6].

Kramers [9] was the first to consider nuclear fission as a process of overcoming the potential barrier by the Brownian particle. A slow fission degree of freedom (with large collective mass) is considered as Brownian particle, and fast nucleon degrees of freedom – as a heat bath. Adequacy of such description is based on the assumption that the time of equilibrium achievement in nucleons degrees of freedom system is much less than the characteristic time scale of collective motion. The most general way of description of dissipative nuclear dynamics is Fokker-Planck equation [10]. However for demonstration of qualitative effects it is convenient to use Langevin equation [11] that is equivalent to Fokker-Planck equation but is more transparent. As it has been shown the description based on Langevin equation adequately represents nuclear dissipative phenomena such

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as heavy-ion reactions and fission decay [12–14] and possesses a number of advantages over Fokker-Planck description.

As we only intend to qualitatively demonstrate SR in nucleus let us consider the simplest type of Langevin equation – one-dimensional problem with inertial M and friction γ parameters independent of coordinates. Fission coordinate R is considered as a coordinate of Brownian particle. The rest degrees of freedom play a role of heat bath. Interaction of fission coordinate with this heat bath causes a friction γP and a random force $\xi(t)$.

The particle motion in the presence of external periodic force $A\cos\omega t$ is described by Langevin equation for canonically conjugate variables $\{P, R\}$

$$dR/dt = P/M,$$

$$dP/dt = -\beta P - dV/dR + A\cos\omega t + \xi(t), \qquad (2)$$

$$\beta = \gamma/M,$$

$\xi(t)$ is stochastic force possessing statistical properties of white noise:

$$\langle\xi(t)\rangle = 0, \quad \langle\xi(t)\xi(t')\rangle = 2D\,\delta(t-t'), \quad D = \gamma T. \quad (3)$$

The nuclear temperature $T(\mathrm{MeV}) = \sqrt{E^*/a}$ where E^* is an excitation energy and the level density parameter $a = \widetilde{A}/10$ ($\widetilde{A}$ being a mass number). The deformation potential V is given as [12]

$$V(R) = \begin{cases} 37.46\,(R-1)^2(\mathrm{MeV}), & 0 < R < 1.27 \\ 8.0 - 18.73\,(R-1.8)^2\ (\mathrm{MeV}), & R > 1.27 \end{cases} \qquad (4)$$

(these are parameters of ^{205}At nucleus [12]).

Plausible sources of periodic perturbation are considered below.

The discretized form of the Langevin equation is given by [13, 14]

$$R_{n+1} = R_n + \tau\frac{P_n}{M},$$

$$P_{n+1} = P_n(1-\beta\tau) - \left[\left(\frac{dV(R)}{dR}\right)_n - A\cos\omega t_n\right]\tau + $$

$$+\sqrt{\frac{2\beta MT\tau}{N}}\,\eta(t_n). \qquad (5)$$

Here $t_n = n\tau$ and $\eta(t_n)$ is a normalized Gaussian-distributed random variable which satisfies

$$\langle\eta(t)\rangle = 0, \quad \langle\eta(t_n)\eta(t_{n'})\rangle = N\delta_{nn'}. \qquad (6)$$

Efficiency of numerical algorithm (5) was checked for the following cases:

(i) $V = 0$, $A = 0$, where numerical and analytical results for $\langle P^2\rangle$ and $\langle R^2\rangle$ can be compared [12];

(ii) $V \neq 0$, $A = 0$, where numerical and analytical values for Kramers decay rate r_k can be compared. According to [9]

$$r_k = \frac{\omega_{\min}}{2\pi}\left[\sqrt{\tilde\beta^2+1} - \tilde\beta\right]\exp(-\Delta V/T) \equiv$$

$$\equiv W\exp(-\Delta V/T), \quad \tilde\beta = \frac{\beta}{2\omega_{\max}}. \qquad (7)$$

Here ω_{min} and ω_{max} are the angular frequencies of the potential (4) at the potential minimum and at the top of barrier respectively, ΔV is the height of the potential barrier. Numerical values of Kramers decay rates ${r_k}^i$ for the time bin i are calculated by sampling the number of fission events $(N_f)_i$ in the i^{th} time bin width Δt normalized to the number of nuclei $N_{total} - \sum_{j<i}(N_f)_j$ which have not fissioned

$${r_k}^i = \frac{1}{N_{total} - \sum_{j<i}(N_f)_j}\frac{(N_f)_i}{\Delta t}. \qquad (8)$$

Comparison of (7) with asymptotic value of (8) was used for determination of the time interval τ, which provides saturation for numerical integration (5). From the results one could see that 20 steps per nuclear time $\hbar$/MeV provides a sufficient saturation.

Now let us proceed to the description of the expected effect – manifestation of SR in nuclear fission. For usually considered case of symmetric double well in the absence of periodic forcing, RTD $N(t)$ has the exponential form (see [6]) $N(t) \sim \exp(-r_k t)$. In the presence of the periodic forcing one observes a series of peaks centered at odd multiples of the half driving period $T_\omega = 2\pi/\omega$. The heights of these peaks decrease exponentially with their order number.

These peaks can be simply explained [15]. The best time for the particle to escape the potential well is when the potential barrier assumes a minimum. Thus $t = 1/2\,T_\omega$ is a preferred residence time interval. Following "good opportunity" to escape occurs in a full period, when potential barrier achieves its minimum again. The second peak in the RTD is therefore located at $3/2\,T_\omega$. The location of the other peaks is evident. The peak heights decay exponentially because the probabilities of the particle to jump over a potential barrier are statistically independent. As shown for symmetric double-well potential [16], the strength P_1 of the first peak at $1/2\,T_\omega$

(the area under peak) is a measure of the synchronization between the periodic forcing and the switching between the wells. So, if the mean residence time (MRT) of the particle in one potential well is much larger than the period of the driving, the particle is not likely to jump over the first time the relevant potential barrier assumes its minimum. The RTD exhibits in such a case a larger number of peaks where P_1 is small. If the MRT is much shorter than the period of the driving RTD has already decayed practically to zero before the time $1/2\,T_\omega$ is reached and the weight P_1 is again small. Optimal synchronization, i.e. maximum P_1, is reached when the MRT matches half driving period, i.e., condition (1). This resonance condition can be achieved by varying the noise intensity D (or ω).

For RTD constructing (and following P_1 calculation) we use the numerical solutions of Langevin equation (5). We studied the evolution of P_1 within the temperature interval 1 MeV $\leq T \leq$ 6 MeV. Let us fix the frequency of periodic perturbation $\omega = 0.0267\,\mathrm{MeV}/\hbar$ ($T_\omega/2 \approx 117.7\,\hbar/\mathrm{MeV}$) that is the resonant frequency at $T = 3$ MeV (following eq.(1)). The results of numerical procedure for RTD under fixed parameters of periodic perturbation ($A = 3$, $\omega = 0.0267$ – determined from resonance condition(1)) are presented in Fig.1. Nuclear friction β in all numerical calculation is chosen 1 MeV$/\hbar$.

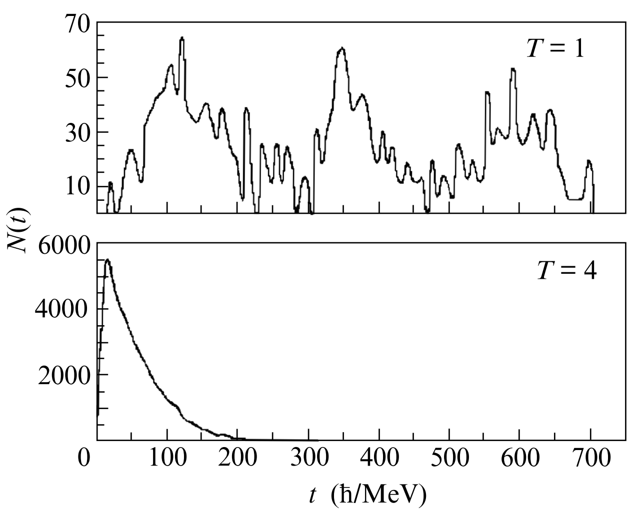

Fig.1. RTD for $T = 1$ and $T = 4$

In accordance with expected behavior at $T = 1$MeV (for low r_k) one can distinctly see three peaks located near $t = T_\omega/2(\sim 117.7)$, $3/2\,T_\omega(\sim 353)$, $5/2\,T_\omega(\sim 588)$, and at $T = 4$ MeV almost all RTD is concentrated near $t = 0$ (with width less than $T_\omega/2$). Connected with this variations of P_1 (that represents the measure of synchronization between the periodic forcing and Kramers transitions and in such a way the measure of SR) are depicted on Fig.2 for two frequencies of periodic perturbation (corresponding to temperatures 2 and 3 MeV from (1)). Maxima of intensities P_1 are close to predicted values of temperature.

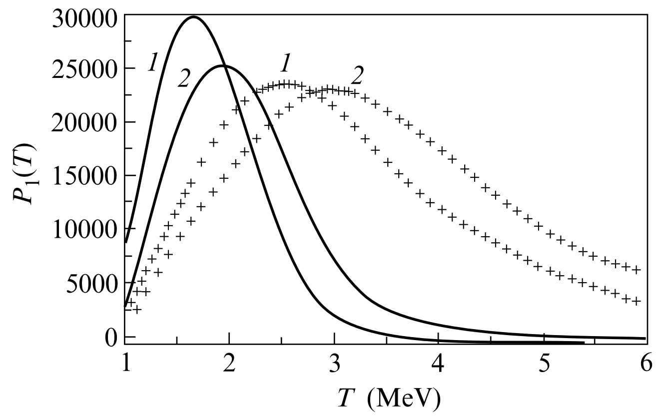

Fig.2 $P_1(T)$ at $\omega = 0.0267$ (crosses) and $\omega = 0.007$ (solid); *1* corresponds to eq.(12), *2* – to numerical calculation

Calculated above P_1 can be estimated theoretically for one well situation using a model similar to two-states model [6].

Let us evaluate RTD for one-well case. Rate equation for the number of fissile nuclei should be written as

$$\frac{dn}{dt} = -nr_k e^{-\epsilon\cos\omega t} \qquad (9)$$

where $\epsilon = A/T$. At low temperature (9) properly describes the process being modeled below (though it is inappropriate at T comparable with ΔV when r_k is not much smaller than the relaxation time within the well). The solution of (9) is

$$\ln\frac{n(t)}{n_0} = -r_k\int\limits_0^t \exp(-\epsilon\cos\omega t)\,dt =$$

$$= -r_0 t + \sum_{n=1}^{\infty}(-1)^n I_n(\epsilon)\frac{2r_k}{n\omega}\sin n\omega t \qquad (10)$$

where $r_0 = r_k I_0(\epsilon) > r_k$; $I_n(\epsilon)$ are the modified Bessel functions. RTD in this model is given by $N(t) = -dn/dt$, so

$$N(\pi/\omega) = r_k\exp(-r_0\pi/\omega + \epsilon). \qquad (11)$$

Using (9)–(11) we obtain a new condition for resonant temperature $T_{RES}(\omega)$, that provides maximal value for

$N(\pi/\omega)/\omega$, dependence of which on ω and T properly represents that of $P_1(\omega, T)$ calculated above:

$$r_k^{-1} = \frac{\pi}{\omega} \frac{\lambda I_0(\epsilon) - I_1(\epsilon)}{\lambda - 1} \tag{12}$$

instead of (1); $\lambda = \Delta V/A$. Numerical solution of (12) for $T_{RES}(\omega)$ is presented in Fig.3 (together with a solution of

$$r_k^{-1} = 2\pi/\omega \tag{13}$$

that much better than 1 approximates the curve 12)).

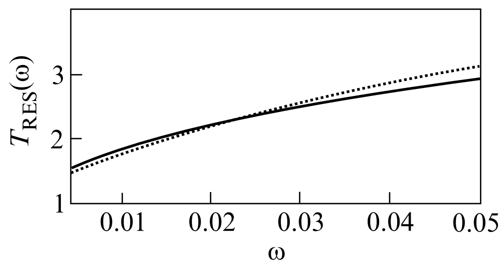

Fig.3. Resonant conditions (12) and (13) for $T_{RES}(\omega)$

$P_1(T)$ (obtained from (11)) is depicted in Fig.2 to be compared with numerical results; the scale of $P_1(T)$ is chosen in such a way that the height of (12) in its maximum for $\omega = 0.0267$ coincides with that of numerical data. Higher resonant T in the latter case is connected with non-equilibrium distribution within a long interval near $t = 0$ (that can be easily seen in Fig.1).

The first maximum in RTD is shifted from π/ω, so it may seem more reasonable to evaluate the height in true maximum. Such calculation shows that this height dependence on ω resembles presented in Fig.3, excepting the region of high T where the curve $N(t)$ does not possess any maxima. Nevertheless, $N(\pi/\omega)$ is easily defined observable and studying its dependence on T allows one to determine necessary characteristics of the nucleus.

In conclusion, let us briefly consider the possible sources of periodic perturbation. The first possibility is the fissile nucleus as a component of double nuclear system formed, for example, in heavy-ion collisions [17]. In this case, the deformation potential will experience periodic perturbation similar to tide-waves on the Earth caused by the Moon rotation. In the case of asymmetric fission alternating electric field may be the source of periodic perturbation. The problem of choice of periodic perturbation would be discussed separately.

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