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Nonperturbative a-expansion technique and the Adler D-function

A. N. Sissakian, I. L. Solovtsov, O. P. Solovtsova

Joint Institute for Nuclear Research, 141980 Dubna, Russia

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We show that the "experimental" D-functions corresponding to the e^+e^- annihilation into hadrons and the inclusive τ decay data are both in good agreement with results obtained in the framework of the nonperturbative a-expansion method.

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The specific feature of quantum field theory is that a sufficiently complete study of the structure of a quantum field model within the framework of perturbative approach is not enough, even in theories with a small coupling constant. Numerous publications are devoted to the problem of going beyond perturbation theory. However, many of them use model assumptions and phenomenological parameters which are not involved into the Lagrangian. Clearly, that it is desirable to use a theoretical method based on a minimal number of additional parameters and allowing a nonperturbative region to be considered. The theoretical method we will use is the nonperturbative expansion technique [1] based on the idea of variational perturbation theory (see [2] for a review) which, in the case of QCD, leads to a new small expansion parameter, a. Even going into the infrared region of small momenta where the running coupling becomes large and the standard perturbative expansion fails, the a-expansion parameter remains small and the approach holds valid [3].

In comparing theoretical predictions with experimental data, it is important to connect measured quantities with "simplest" theoretical objects to check direct consequences of the theory without using model assumptions in an essential manner. Some single-argument functions which are directly connected with experimentally measured quantities can play the role of these objects. A theoretical description of inclusive processes can be made in terms of functions of this sort. Let us mention, among them, the hadronic correlator $\Pi(s)$ and the corresponding Adler function [4], D, that appear in the process of e^+e^- annihilation into hadrons and the inclusive decay of the τ lepton.

The cross-section for e^+e^- annihilation into hadrons or its ratio to the leptonic cross-section, $R(s)$, have a resonance structure that is difficult to describe, at the present stage of a theory, without model considerations. Moreover, the basic method of calculations in quantum field theory, perturbation theory, becomes ill-defined due to the so-called threshold singularities. These problems can, in principle, be avoided if one considers a "smeared" quantity [5]

$$R_\Delta(s) = \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R(s')}{(s-s')^2+\Delta^2}\,. \tag{1}$$

However, a straightforward usage of conventional perturbation theory to calculate R_Δ is not possible. Indeed, if the QCD contribution to the function $R(s)$ in Eq. (1) is, as usual, parametrized by the perturbative running coupling that has unphysical singularities, it is difficult to define the integral on the right-hand side. Moreover, the standard method of the renormalization group gives a Q^2-evolution law of the running coupling in the Euclidean region, and there is the question of how to parametrize a quantity, for example, $R(s)$, defined for timelike momentum transfers [6]. To perform this procedure self-consistently, it is important to maintain correct analytic properties of the hadronic correlator which are violated in perturbation theory. Within the nonperturbative a-expansion it is possible to maintain such analytic properties and to self-consistently determine the effective coupling in the Minkowskian region [7][1)].

Another function, which characterizes the process of e^+e^- annihilation into hadrons and can be extracted from experimental data, is the Adler function

$$D(Q^2) = Q^2\int_0^\infty ds \frac{R(s)}{(s+Q^2)^2}. \tag{2}$$

The D-function defined in the Euclidean region for a positive momentum Q^2 is a smooth function, and thus it is not necessary to apply any "smearing" procedure in order to be able to compare theoretical results with

[1)] The analytic approach to QCD [8] also leads to a well-defined procedure of analytic continuation [9].

experimental data. Recently an "experimental" curve for this function has been obtained [10].

For massless quarks, one can write down the Minkowskian quantity $R(s)$ in the form

$$R(s) = 3\sum_f q_f^2 \left[1 + r_0 \lambda_s^{\rm eff}(s)\right], \qquad (3)$$

where the sum runs over quark flavors, q_f are quark charges and r_0 is the first perturbative coefficients that is renormalization-scheme independent. This expression includes the effective coupling defined in the Minkowskian region or, as we will say, in the s-channel, which is reflected in the subscript s. It should be stressed that, as it has been argued from general principles, the behavior of the effective couplings in the spacelike and the timelike domains cannot be symmetric [11].

Within the a-expansion method the s-channel running coupling can be written as

$$\lambda_s^{(i)}(s) = \frac{1}{2\pi i}\,\frac{1}{2\beta_0}\left[\phi^{(i)}(a_+) - \phi^{(i)}(a_-)\right], \qquad (4)$$

where $a_\pm$ obey the equation [2]

$$F(a_\pm) = F(a_0) + \frac{2\beta_0}{C}\left(\ln\frac{s}{Q_0^2} \pm i\pi\right). \qquad (5)$$

At the level $O(a^3)$, the function $\phi(a)$ has the form

$$\begin{aligned}\phi^{(3)}(a) &= -4\ln a - \frac{72}{11}\frac{1}{1-a} + \\ &+\frac{318}{121}\ln(1-a) + \frac{256}{363}\ln\left(1+\frac{9}{2}a\right).\end{aligned} \qquad (6)$$

Similarly, a more complicated expression for the $O(a^5)$ level, we will use, can be derived.

The convenient way to incorporate quark mass effects is to use an approximate expression [5]

$$\begin{aligned}\tilde{R}(s) &= 3\sum_f q_f^2\,\Theta(s-4m_f^2)\,\mathcal{R}_f(s),\\ \mathcal{R}_f(s) &= T(v_f)\,\left[1 + g(v_f) r_f(s)\right],\end{aligned} \qquad (7)$$

where

$$T(v) = v\frac{3-v^2}{2},$$

$$g(v) = \frac{4\pi}{3}\left[\frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right], \qquad (8)$$

$$v_f = \sqrt{1 - \frac{4m_f^2}{s}}.$$

The quantity $r_f(s)$ is defined by the s-channel effective coupling $\lambda_s^{\rm eff}(s)$. The smeared quantity (1) and the D-function (2) can be calculated by using (7) in the corresponding integrands. For MS-like renormalization schemes, one has to consider some matching procedure. To perform this matching procedure, we can require the s-channel running coupling and its derivative to be continuous functions in the vicinity of the threshold [7, 12].

A description of quark-antiquark systems near the threshold requires to take into account of the resummation factor. In a nonrelativistic approximation, this is the well known Sommerfeld – Sakharov factor [13]. For a systematic relativistic analysis of quark-antiquark systems, it is essential from the very beginning to have a relativistic generalization of this factor. A new form for this relativistic factor in the case of QCD has been proposed in [14] by using the quasipotential approach to quantum field theory formulated in the relativistic configuration representation [15]. The local Coulomb potential defined in this representation is specified by its QCD-like behavior in the momentum space [16].

The relativistic S-factor has the form [14]

$$S(\chi) = \frac{X(\chi)}{1 - \exp\left[-X(\chi)\right]}, \qquad X(\chi) = \frac{4\pi\,\alpha_S}{3\sinh\chi}, \qquad (9)$$

where χ is the rapidity which is related to s by $2m\cosh\chi = \sqrt{s}$.

To take into account the threshold resummation factor, we modify the expression (7) as follows

$$\mathcal{R}(s) = T(v)\left[S(\chi) - \frac{1}{2}X(\chi) + g(v)\,r(s)\right]. \qquad (10)$$

As the mass $m \to 0$, this expression leads to Eq. (3). We use Eq. (10) in our analysis.

The non-strange vector contribution for the inclusive τ-lepton decay can be described in analogy with the e^+e^- annihilation into hadrons process. Using the theoretical expression for R_τ-ratio [17]

$$R_\tau^V = R^{(0)}\int_0^{M_\tau^2}\frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right)\mathcal{R}(s), \qquad (11)$$

where $R^{(0)}$ corresponds to the parton level, and measured value $R_\tau^V = 1.775 \pm 0.017$ [18], as an input, we extracted the value of parameter a_0 in Eq. (5) at the τ mass scale, $Q_0 = M_\tau$.

The "light" D-function with three active quarks is shown in Fig. 1, where we draw the experimental curve, as dashed line, which was extracted in [19] from the ALEPH data, and our theoretical result (solid line) obtained by using the following effective masses of light

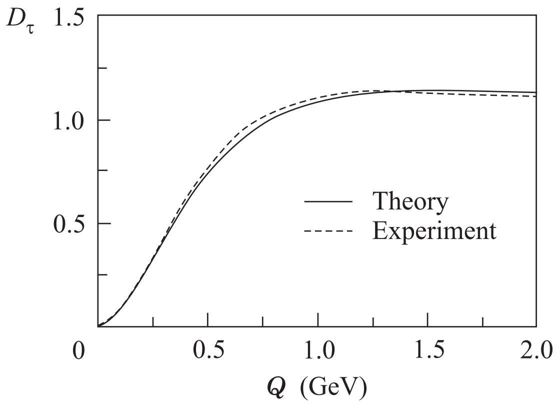

Fig.1. The plot of the "light" D-function. The experimental curve corresponding to ALEPH data is taken from [19]

quarks $m_u = m_d = 260\,\mathrm{MeV}$ and $m_s = 400\,\mathrm{MeV}$[2]. These values are close to the constituent quark masses and incorporate some nonperturbative effects. The shape of the infrared tail of the D-function is sensitive to the value of these masses.

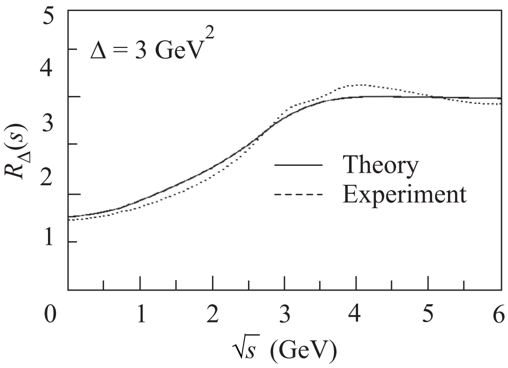

Fig.2. The smeared quantity $R_\Delta(s)$ for $\Delta = 3$ GeV^2. The solid curve is our result. The smeared experimental curve is taken from [22]

In Fig. 2, we have presented the smeared function $R_\Delta(s)$ for $\Delta = 3$ GeV^2. We use the same masses for the light quarks as before and the following masses for heavy quarks $m_c = 1.3\,\mathrm{GeV}$ and $m_b = 4.7\,\mathrm{GeV}$. The smeared $R_\Delta(s)$ function for $\Delta \simeq 1$–3 GeV^2 is less sensitive to the value of light quark masses as compared with the infrared tail of the D-function. The result for the D-function of the e^+e^- annihilation process which includes both the light and heavy quarks is plotted in Fig. 3. The experimental curve is taken from [10].

[2]Practically the same values were used to describe the experimental data in [20, 21].

The experimental D-function turned out to be a smooth function without traces of the resonance structure of $R(s)$. One can expect that this object more precisely reflects the quark-hadron duality and is convenient for comparing theoretical predictions with experimental data. Note here that any finite order of the operator product expansion fails to describe the infrared tail of the D-function. Within the framework of nonperturbative a-expansion technique, we have obtained a good agreement between our results and the experimental data down to the lowest energy scale both for Minkowskian and Euclidean quantities.

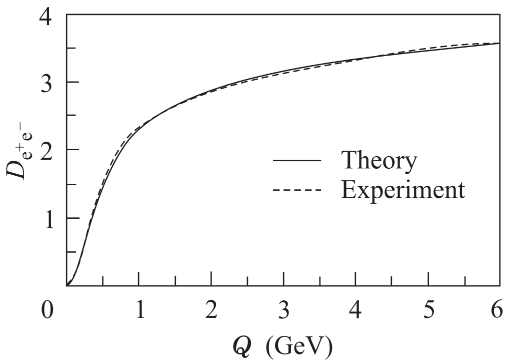

Fig.3. The D-function for the process of e^+e^- annihilation into hadrons. The solid curve is our result for five active quarks. The experimental curve is taken from [10]

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