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# Spin polarization in quantum dots by radiation field with circular polarization

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For circular quantum dot (QD) with account of the Razhba spin-orbit interaction (SOI) an exact energy spectrum is obtained. For the small SOI constant the eigen functions of the QD are found. It is shown that application of radiation field with circular polarization lifts the Kramers degeneracy of the eigen states of the QD. Effective spin polarization of transmitted electrons through the QD by radiation field with circular polarization is demonstrated.

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The spin dependence of the electronic properties of artificial nanostructures is one of today's leading problems in the physics of electronic devices. The interest lays both on the improvement of actual devices, like the GaAs polarized electron source (GaAs-PES) [1], and on search for new devices like spin transistors [2]. The effects of the spin degree of freedom on the electron transport properties of semiconductor heterostructures in the presence of inhomogeneous magnetic fields have been intensively studied experimentally [3, 4]. Experiments focusing on fundamental issues used inhomogeneous magnetic fields created either by vortices in superconductors [5, 6] or by ferromagnetic layers [7 – 9]. Theoretically the spin dependent resonant tunneling through magnetic barriers was calculated [10, 11] and dependence of the spin polarization of the transmitted electrons

$$P=\sum_\sigma \sigma T_\sigma / \sum_\sigma T_\sigma \qquad (1)$$

on the magnetic configuration, the applied bias, the incident electron energy has been found [12].

The spin dependence of the electron transport across nonmagnetic semiconductor heterostructures at zero applied magnetic field originate from the spin-orbit interaction (SOI). Basically this phenomenon originates from well known phenomenon that the SOI has a polarization effect on particle scattering processes [13] and was considered for different microdevices [14 – 17].

In this letter we consider a possibility of resonant spin polarization of transmitted electrons by radiation field with circular polarization. It is well known in atomic spectroscopy that circular polarized radiation field can transmit electron from multiplet state with total half integer angular momentum to continuum with definite spin polarization [18]. In present article we consider similar phenomenon for electron ballistic transport in quantum dots and microelectronic devices which have bound states.

At first stage we consider a circular quantum dot with hard walls fabricated by metallic gates with applied negative electric potential. Because standard technique of fabrication of microelectronic devices with depletion of 2DEG is based on semiconductor $GaAs/Al_xGa_{1-x}As$ the SOI in the Razhba form [19]

$$V_{SL} = \hbar K\left[\sigma_x p_y - \sigma_y p_x\right] \qquad (2)$$

is important where $\sigma_x$ and $\sigma_y$ are Pauli spin-matrices. Parameter of the spin-orbit coupling $K$ depends on the confining potential profile along $z$-direction and e.g. estimation for InAs structure with effective mass $m^* = 0.023 m_0$ gives $\hbar^2 K \sim 6\cdot 10^{-3}\mathrm{eV}\cdot\mathrm{nm}$ [20] and $\hbar^2 K \sim 10^{-3}\mathrm{eV}\cdot\mathrm{nm}$ for GaAs structure.

Using a natural energy scale of the QD $E_0 = \hbar^2/2m^*R^2$ where $R$ is the radius of the QD and complex coordinates $z = x + iy$ we rewrite the SOI (2) as follows

$$V_{SL} = 2\beta \begin{pmatrix} 0 & -\partial/\partial z \\ \partial/\partial z^* & 0 \end{pmatrix} \qquad (3)$$

where space variables $x, y, z$ are referred to the QD radius $R$ and

$$\beta = 2m^* K R. \qquad (4)$$

The total Hamiltonian of the QD

$$H = -\nabla^2 + V(r) + V_{SL} \qquad (5)$$

1)e-mail: almsa@ifm.liu.se, almas@tnp.krasn.ru commutes with the $z$-projection of the total angular momentum

$$\hat{J}_z = L_z + \frac{1}{2}\sigma_z, \tag{6}$$

and with operator of time reversal

$$\hat{K} = -i\sigma_y C \tag{7}$$

where $C$ is the operator of complex conjugation. The first integral of movement (6) allows to present eigen states solution of the (5) as

$$\psi_m = \begin{pmatrix} u(r)e^{im\phi} \\ v(r)e^{i(m+1)\phi} \end{pmatrix}, \tag{8}$$

because of $\hat{J}_z\psi_m = (m+1/2)\psi_m$.

Substituting (2) into equation $H\psi_m = \epsilon\psi_m$ one can obtain the following systems of radial equations

$$r^2u'' + ru' + (\epsilon r^2 - m^2)u + \\ + \beta r^2\left(\frac{d}{dr} + \frac{(m+1)}{r}\right)v = 0,$$

$$r^2v'' + rv' + (\epsilon r^2 - (m+1)^2)v - \\ - \beta r^2\left(\frac{d}{dr} - \frac{m}{r}\right)u = 0. \tag{9}$$

Taking

$$u = aJ_m(\mu r), \quad v = bJ_{m+1}(\mu r)$$

and using properties of the Bessel functions we have from (9)

$$\left[r^2\frac{d^2}{dr^2} + r\frac{d}{dr} + \left(\epsilon + \beta\mu\frac{b}{a}\right)r^2 - m^2\right]J_m(\mu r) = 0, \tag{10}$$

$$\left[r^2\frac{d^2}{dr^2} + r\frac{d}{dr} + \left(\epsilon + \beta\mu\frac{a}{b}\right)r^2 - (m+1)^2\right] \times \\ \times J_{m+1}(\mu r) = 0.$$

This equation can be fulfilled only if

$$\mu = (\epsilon + b\beta\mu/a)^{1/2}, \tag{11}$$
$$\mu = (\epsilon + a\beta\mu/b)^{1/2}. \tag{12}$$

Correspondingly we obtain $a = b$ with

$$\mu_{1\pm} = \beta/2 \pm \sqrt{\epsilon + (\beta/2)^2} \tag{13}$$

and $a = -b$ with

$$\mu_{2\pm} = -\beta/2 \pm \sqrt{\epsilon + (\beta/2)^2}. \tag{14}$$

As a result we obtain two couples of linearly independent solutions for (2). The first one is

$$\Phi^{\pm}_{1,m}(r,\phi) = \begin{pmatrix} J_m(\mu_{1\pm}r)e^{im\phi} \\ J_{m+1}(\mu_{1\pm}r)e^{i(m+1)\phi} \end{pmatrix}. \tag{15}$$

By similar way the next couple can be written.

We imply the Dirichlet boundary condition at $r = R$ for linear combination of solutions (15)

$$C\Phi^{+}_{1,m}(R,\phi) + D\Phi^{-}_{1,m}(R,\phi) = 0. \tag{16}$$

It gives us the following exact equation for energy spectrum of the QD with the SOI

$$J_m(\mu_{1+}R)J_{m+1}(\mu_{1-}R) - \\ - J_m(\mu_{1-}R)J_{m+1}(\mu_{1+}R) = 0. \tag{17}$$

A few lowest energy levels of the QD versus the SOI constant $\beta$ are shown in Fig.1a. It is easy to see that

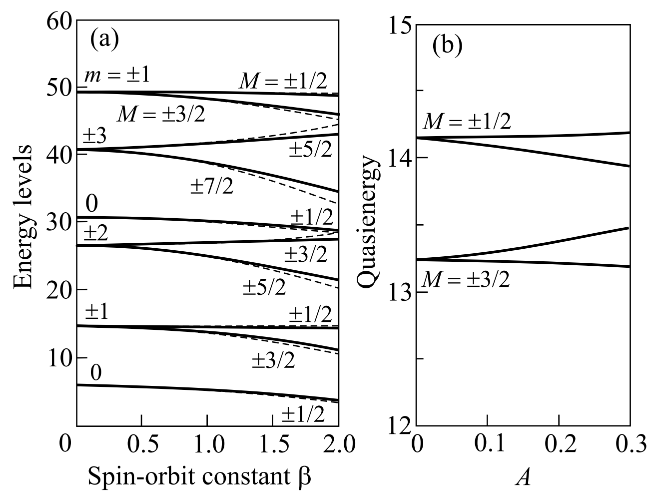


Fig.1. (a) Energy levels of the QD with the SOI versus the spin-orbit constant $\beta$. The exact spectrum (17) is shown by dashed lines while the approximated energy levels (18) are shown by solid lines. (b) The quasi-energy levels of the QD effected by the radiation field with circular polarization versus the amplitude $A$ of the radiation field for the spin-orbit coupling constant $\beta = 1$. In both cases the QD radius equals unit

next couple of equations leads to the same equation as (17).

Eq.(17) can be solved approximately for small constant of the SOI $\beta \le \sqrt{\epsilon}$. If to substitute (4) to this inequality we obtain for the GaAs dot that the approximation of small $\beta$ is valid for $R < 10^{-4}$ cm and the low eigen energies. Expanding (13) and the Bessel functions over small $\beta$ one can obtain after lengthy but elementary calculations the following expressions for approximated energy levels

$$\epsilon_{mn,1} \approx \frac{x_{nm}^2}{R^2} + $$
$$+ \frac{\beta^2}{4}\left[-1 + \frac{2x_{nm}J'_{m+1}(x_{nm})}{J_{m+1}(x_{nm})} - \frac{x_{nm}J''_m(x_{nm})}{J'_m(x_{nm})}\right],$$
$$\epsilon_{mn,2} \approx \frac{x_{nm}^2}{R^2} + \tag{18}$$
$$+ \frac{\beta^2}{4}\left[-1 + \frac{2x_{nm}J'_{m-1}(x_{nm})}{J_{m-1}(x_{nm})} - \frac{x_{nm}J''_m(x_{nm})}{J'_m(x_{nm})}\right]$$

and $x_{nm}$ is the $n$-th zero of the Bessel function $J_m(x)$. The approximated spectrum of energy levels (18) is shown in Fig.1a by dashed lines as dependent on the SOI constant $\beta$. One can see that for the lowest eigen energies the approximation is valid even for $\beta$ exceeded unit.

It is easy to obtain that the SOI gives rise to splitting of degenerated energy levels of the QD with $M = m \pm 1/2$, expect the level with $m = 0$, with value of splitting as

$$\Delta_{mn} = \epsilon_{mn,2} - \epsilon_{mn,1} =$$
$$= \frac{\beta^2}{2}x_{nm}\left[\frac{J'_{m-1}(x_{nm})}{J_{m-1}(x_{nm})} - \frac{J'_{m+1}(x_{nm})}{J_{m+1}(x_{nm})}\right]. \tag{19}$$

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Again using smallness of the SOI constant $\beta$ one can obtain from (16)

$$C = 1, \quad D = (-1)^m\left[1 + \beta R\frac{J'_{m+1}(x_{nm})}{J_{m+1}(x_{nm})}\right] \tag{20}$$

and from (15) the eigenstates

$$\Psi_{M=m+1/2} = \begin{pmatrix} J_m(x_{nm}r/R)e^{im\phi} \\ \frac{\beta}{2}e^{i(m+1)\phi}(rJ'_{m+1}(x_{nm}r/R) - RJ'_{m+1}(x_{nm})\frac{J_{m+1}(x_{nm}r/R)}{J_{m+1}(x_{nm})} \end{pmatrix},$$
$$\Psi_{M=m-1/2} = \begin{pmatrix} \frac{\beta}{2}e^{i(m-1)\phi}(rJ'_{m-1}(x_{nm}r/R) - RJ'_{m-1}(x_{nm})\frac{J_{m-1}(x_{nm}r/R)}{J_{m-1}(x_{nm})}) \\ J_m(x_{nm}r/R)e^{im\phi} \end{pmatrix}. \tag{21}$$

The next couple of degenerate states with $M = -(m \pm \pm 1/2)$ can be easily obtained by applying of the Kramers operator (7) to states (21).

Next consider application of the radiation field with circular polarization

$$\mathbf{A}(t) = A(\sin\omega t, \quad \cos\omega t, \quad 0). \tag{22}$$

Note that below we are using the dimensionless radiation field amplitude $A \to edA/c\hbar$ [21] where $d$ is the width of leads attachment of which will be considered below. Similar to the two-level system an effect of this radiation field can be considered exactly by transformation to the rotating coordinate system by the unitary operator $\exp(i\omega t\widehat{J}_z)$ to give rise to the following effective Hamiltonian

$$\widetilde{H} = H - \omega\widehat{J}_z + 2iA\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial z^*}\right), \tag{23}$$

where $H$ is given by (5). Since

$$\frac{\partial}{\partial z}J_m(\mu r)e^{im\phi} = \frac{\mu}{2}J_{m-1}(\mu r)e^{i(m-1)\phi},$$
$$\frac{\partial}{\partial z^*}J_m(\mu r)e^{im\phi} = -\frac{\mu}{2}J_{m+1}(\mu r)e^{i(m+1)\phi} \tag{24}$$

it is obviously follows that the perturbation $V$ can mix only states $M$ and $M'$ differing by $\Delta M = \pm 1$. One can see from (23) that the radiation field with circular polarization effects the QD like an external magnetic field, i.e. lifts the Kramers degeneracy. This phenomenon firstly was considered by Ritus for an atom effected by the radiation field with circular polarization [22]. Because of $[J_z, V] \neq 0$ we can not present exact eigen states of the Hamiltonian (23). However it is clear that the splitting of degenerated quasienergy states $\pm M$ can be found in the second order of the perturbation theory to dive rise to $\Delta E \sim A^2$. In fact numerical calculation of eigenvalues of the effective Hamiltonian (23) clearly demonstrates the squared behavior of the quasienergy levels versus the amplitude of the radiation field as it is shown in Fig.1b. Moreover the eigen states $\tilde{\psi}$ of the Hamiltonian (23) are spin polarized ones. In particular we calculated numerically the spin polarization $< S_z >=< \tilde{\psi}|\widehat{S}_z|\tilde{\psi} >$ for a few lowest states of the QD and found that $< S_z >\approx \pm 0.9$ for the first doublet and very slightly depends on $A$.

Now let us attach leads with a width $d$ to the QD and consider a transmission of electrons unpolarized by the spin through the QD. If coupling of leads with the QD is weak, we have resonant transmission of electrons. Because of strong spin polarization of the eigen states of the QD effected by the radiation field we can expect the resonant transmission with corresponding spin state while electrons with opposite spin state are reflecting. The above said establishes the basic principle of the spin polarization via the resonant transmission through the QD effected by the radiation field with the circular polarization.

Here we consider a case with tangential attachment as it is shown in the inset of Fig.2a. This case was

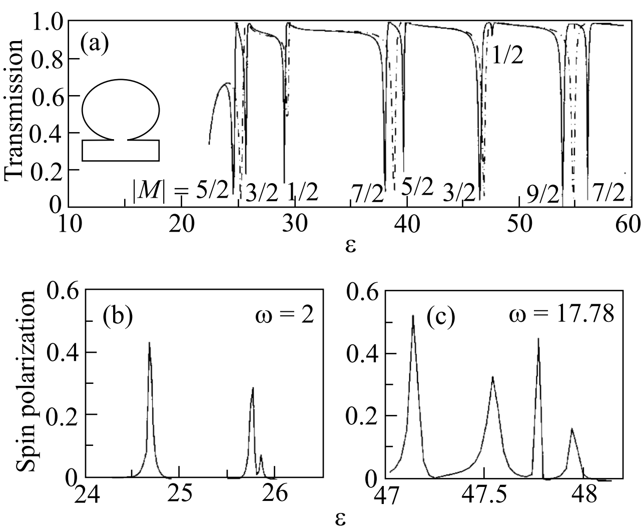


Fig.2. (a) The transmission probability through the QD without (dashed line) and with the spin-orbit interaction ($\beta = 0.75$, solid line). The radiation field is turned off. (b) The spin polarization versus the energy of incident electrons for the case of the radiation field is not resonant ($\beta = 0.75$). (c) The spin polarization versus the energy of incident electrons for the case of the frequency resonant to transition between the state $|M| = 3/2$ and $|M| = 1/2$ ($\beta = 0.75$)

considered in [23] and gives rise to resonant dips of the transmission probability. The computer calculations show that such kind of geometry is more effective for the spin polarization in comparison with standard case of symmetrical attachment of leads to the QD. The only difference is that the electrons with the spin state coincided with that of the eigen state of the QD are resonantly reflecting while electrons with the opposite spin are transmitted giving rise to the spin polarization of outgoing electron beam. Since tangential attachment of leads violates symmetry of the QD relative up to down there should spin polarizartion of transmitted electrons even without the radiation field [14−17]. However this effect is very negligible in comparison with effect of the radiation field.

A processes of the electron transmission through the QD with application of radiation field is complicated because of appearance of new satellite channels in electron transmission specified by quasi energies [24] $E_n = E_F + n\hbar\omega, n = \pm1, \pm2, ...$ where

$$E_F = \hbar^2/2m^*d^2[k^2 + (\pi p/d)^2],\ p = 1, 2, 3, ... \quad (25)$$

Detailed computational procedure of the electron transmission with application of the radiation field is described in [21]. Here we present only results of computation shown in Fig.2. Since formula (1) is obtained for the spin state of incident electron up and down relative the $z$-axis it follows that $P =< S_z >$. Therefore it necessary to apply operator $\exp(i\widehat{\mathbf{S}}\theta)$ to the incident spin state in order to obtain the spin polarization along the $x$ and $y$ axes. As a result one can obtain the total spin polarization described by a value $P_{tot} = (S_x^2 + S_y^2 + S_z^2)^{1/2}$ which is shown in Fig.2b, c.

Fig.2b clearly demonstrates that for arbitrary frequency of the radiation field but *nonresonant* to transition between the eigen energies $E_M$ of the QD shown in Fig.1a we have the resonant spin polarization for $E_F \approx E_M$. Moreover one can see that the energy dependence of the spin polarization is splitted in accordance with the Fig.1b with a value of the splitting of order $A^2$. Because of smallness of the radiation field amplitude the first resonant peak of the spin polarization in Fig.2b is not resolved.

However if $\hbar\omega \approx E_{M'} - E_M$ a picture of the resonant spin polarization of the transmitted electrons changes crucially as it is shown in Fig.2c. As a result we have enhanced spin polarization for the case $E_F \approx E_M$, for the frequency of the radiation field is tuned to transition between the states $M = 1/2, m = 0, \epsilon_{1/2} = 29.33$ and $M = 3/2, m = 1, \epsilon_{3/2} = 47.12$ for the spin-orbit constant $\beta = 0.75$. Moreover since the frequency of the radiation field is resonant to transition between the QD eigenstates we observe strong splitting of peaks of the spin polarization because of the Raby splitting. The last is linear to the radiation field amplitude.

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