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π-junction realization due to the tunneling through thin ferromagnetic layer

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It is demonstrated that in superconductor-ferromagnet-superconductor (S/F/S) systems in the case of low interface transparency the transition into π-phase is not related with the oscillations of the superconducting order parameter in F-layer. In consequence the π-phase may exist at very thin F-layer thickness. The crossover from π-to 0-phase results in the non-monotonous temperature dependence of the critical current.

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I. Introduction. The strong exchange field acting on the electrons in a ferromagnet provokes the damping oscillatory behavior of the superconducting order parameter. This effect is at the origin of the π-junction realization in superconductor-ferromagnet-superconductor (S/F/S) systems [1, 2]. Recently such π-junctions have been successfully fabricated and experimentally studied in [3 – 5]. In [6] it has been demonstrated that in a system composed of alternating superconducting and ferromagnetic atomic thickness layers the so-called π-phase may exist, wherein the superconducting order parameter changes its sign as we go from one superconducting layer to another. The π-phase shift in this case appears due to the tunneling through the atomic ferromagnetic layer. Tunneling through a barrier with paramagnetic impurities also may provoke the π-phase shift in Josephson junctions [7, 8]. The possibility of the π-state realization in complex SFIFS Josephson junctions has been recently predicted in [9, 10].

The particularity of the three later cases is the special scenario of the $0-\pi$ transition, where the oscillations of the superconducting order parameter in F-layer are absent.

Here we demonstrate that astonishingly this scenario works also in simple S/F/S Josephson junction with thin F- layer, when the transition into π-phase becomes possible in the case of very small transparency of the S/F boundary. Such situation becomes physically similar to the tunneling through atomic thickness F-layer [6] and the studies of corresponding Josephson junctions could provide experimental test of this mechanism of the π-phase shift.

II. General formalism. Let us consider the case of an S/F/S junction with a thin F-layer of thickness d and large superconducting electrodes. We suppose the dirty limit conditions hold and the thickness of F-layer being smaller than the characteristic length $\xi_f = \sqrt{D_f/h}$ of superconducting correlations decay (with oscillations) in F- layer, where h is the ferromagnetic exchange field acting on the electron spins in the F-layer and D_f the electron diffusion constant in F-layer. As it will be demonstrated, the crossover from 0- to π-phase occurs for small S/F interface transparency, i.e. when the induced superconductivity in F-layer is weak and may be described by the linearized Usadel equation for the anomalous function F_f (see for example [2]):

$$[|\omega| + i\text{sign}\,(\omega)\,h]\,F_f - \frac{D_f}{2}\frac{\partial^2 F_f}{\partial x^2} = 0, \qquad (1)$$

where $\omega = 2\pi T\,(n+1/2)$ are the Matsubara frequencies and F-layer corresponds to the region $-d/2 < x < d/2$. Moreover, the weak S/F interface transparency permits to neglect the proximity effect in S electrodes and consider the superconductivity there to be practically unperturbed by F-layer. The interface transparency enter through the general boundaries conditions at the S/F interfaces to Usadel equation [11]. Near T_c they can be written in the following form:

$$F_s\,(d/2) = F_f\,(d/2) + \xi_n \gamma_{B2}\left(\frac{\partial F_f}{\partial x}\right)_{d/2},$$

$$F_s\,(-d/2) = F_f\,(-d/2) - \xi_n \gamma_{B1}\left(\frac{\partial F_f}{\partial x}\right)_{-d/2}, \qquad (2)$$

$$\left(\frac{\partial F_s}{\partial x}\right)_{\pm d/2} = \frac{\sigma_n}{\sigma_s}\left(\frac{\partial F_f}{\partial x}\right)_{\pm d/2},$$

where the notation $F_s(F_f)$ is used for the anomalous Green function in a superconductor (ferromagnet) and σ_n (σ_s) is the conductivity of the F-layer

(S-layer above T_c), and $\xi_s = \sqrt{D_s/2\pi T_c}$ being the superconducting coherence length of the S-layer, while $\xi_n = \sqrt{D_f/2\pi T_c}$. The parameter γ_B is related with S/F boundary resistance per unit area R_b via the following simple relation $\gamma_B = R_b\sigma_f/\xi_n$ [11] (note also the relation between γ_B and the transparency of the S/F interface $T = 1/(1+\gamma_B)$ [11]). Further on, for simplicity we suppose that both interfaces are identical, so $\gamma_{B1} = \gamma_{B2} = \gamma_B$ and we will use the notation $F = F_f$. Also for small interface transparency (if $\sigma_n\xi_s/\sigma_s\xi_n \ll \gamma_B$) we may use the rigid boundary conditions (see for example [12, 13]) with $F_s(-d/2) = \Delta e^{-i\varphi/2}/\sqrt{\omega^2+\Delta^2}$ and $F_s(d/2) = \Delta e^{i\varphi/2}/\sqrt{\omega^2+\Delta^2}$. This means that to find the anomalous Green function F_f it is in fact enough to use only two first conditions in (2).

The solution of (1), satisfying the boundary conditions is readily written as:

$$F(x) = \frac{\Delta\cos(\varphi/2)}{\sqrt{\omega^2+\Delta^2}\,(\mathrm{ch}\,(kd/2)+k\gamma_B\xi_n\mathrm{sh}\,(kd/2))}\mathrm{ch}\,(kx) + \frac{\Delta i\sin(\varphi/2)}{\sqrt{\omega^2+\Delta^2}\,(\mathrm{sh}\,(kd/2)+k\gamma_B\xi_n\mathrm{ch}\,(kd/2))}\mathrm{sh}\,(kx), \quad (3)$$

where the complex wave-vector

$$k = \sqrt{2\left(|\omega| + i\,\mathrm{sign}\,(\omega)\,h\right)/D_f}.$$

Note that in principle, at arbitrary temperature the boundaries conditions are different from that in (2), see for example [12]. However in the limit of low S/F interface transparency ($\gamma_B \gg 1$), when the amplitude of the F function in F-layer is small, we may use the linearized Usadel equation (1) at all temperatures. The only modification in the boundary conditions (2) is that F_s must be substituted by $F_s/|G_s|$ and γ_B by $\gamma_B/|G_s|$, where the normal Green function in superconducting electrode $G_s = \omega/\sqrt{\omega^2+\Delta^2}$. Taking this renormalization into account in the explicit form (3), we may put it in the formula for the supercurrent

$$J_s(\varphi) = ieN(0)D_f\pi T\sum_{-\infty}^{\infty}\left(F\frac{d}{dx}\widetilde{F} - \widetilde{F}\frac{d}{dx}F\right),$$

where $\widetilde{F}(x,h) = F^*(x,-h)$, and obtain the usual sinusoidal current-phase dependence with the critical current

$$I_c = eN(0)D_f\pi T\sum_{-\infty}^{\infty}\frac{\Delta^2}{\omega^2+\Delta^2}\frac{1}{G_s^2}\times \frac{2k}{\mathrm{sh}\,(kd)\left(1+(\gamma_B\xi_n/G_s)^2k^2\right)+2k\left(\gamma_B/|G_s|\right)\xi_n\mathrm{ch}\,(kd)}.$$

In the limit $kd \ll 1$ (i.e. $d < \xi_f$) the oscillations of anomalous function F are absent, but nevertheless if the boundary transparency is very low $1/\gamma_B \ll \xi_n d/\xi_f^2$, the critical current can change its sign. Indeed, in this limit the expression for the critical current reads

$$I_c = eN(0)D_f\pi T\sum_{-\infty}^{\infty}\frac{2\Delta^2}{\omega^2+\Delta^2}\frac{1}{\gamma_B^2\xi_n^2 d}\times\left(\frac{1}{k^2} - \frac{d^2}{6} - \frac{2}{\gamma_B\xi_n dk^4}\frac{|\omega|}{\sqrt{\omega^2+\Delta^2}}\right) = 2eN(0)D_f\pi T\sum_{\omega>0}\frac{\Delta^2}{\omega^2+\Delta^2}\frac{1}{\gamma_B^2\xi_n^2 d}\times\left(\xi_f^2\frac{(\omega/h)}{(\omega/h)^2+1} - \frac{d^2}{3} + \frac{\xi_f^4}{\gamma_B\xi_n d}\frac{1-(\omega/h)^2}{\left(1+(\omega/h)^2\right)^2}\frac{\omega}{\sqrt{\omega^2+\Delta^2}}\right). \quad (4)$$

Usually, at experiment the Curie temperature Θ of ferromagnet is higher than the superconducting critical temperature T_c. For RKKY mechanism of ferromagnetic transition $\Theta \sim h^2/E_F$ and so the exchange field h occurs to be much larger than the superconducting critical temperature T_c. Taking into account the condition $h \gg T_c$ and performing summation over Matsubara frequencies of the first two terms in the brackets (4) we finally obtain

$$I_c = \frac{eN(0)D_f\Delta\xi_f^2}{2\gamma_B^2 d\xi_n^2}\times\left\{\frac{\Delta}{h}\left[\Psi\left(\frac{1}{2}+i\frac{h}{2\pi T}\right) - \Psi\left(\frac{1}{2}+i\frac{\Delta}{2\pi T}\right) + \mathrm{c.c}\right] - \frac{\pi}{3}\left(\frac{d^2}{\xi_f^2}\right)\mathrm{th}\left(\frac{\Delta}{2T}\right) + \frac{4\pi T\Delta\xi_f^2}{\gamma_B\xi_n d}\sum_{\omega>0}\frac{\omega}{(\omega^2+\Delta^2)^{3/2}}\right\}. \quad (5)$$

Let us start the analysis of I_c over d dependence in the limit of very large γ_B (more precisely when $\gamma_B \gg (h/T_c)$). In such a case we may neglect the term proportional to $1/\gamma_B$ in the bracket of (5) and then we obtain that at $T \to 0$ the transition into the $\pi-$ phase occurs (I_c changes its sign) at

$$d_c \approx \xi_f\sqrt{\frac{2\Delta(0)}{h}\ln\left(\frac{h}{\Delta(0)}\right)},$$

and indeed the condition $d < \xi_f$ is satisfied. Note that in the case of very low boundary transparencies the relevant formula obtained in [14] near the critical temperature in the limit $T_c/h \to 0$ also reveals the crossover between 0- and π-phase. On the other hand, no transition into $\pi-$phase has been obtained in the analysis of S/F/S system [13], which is apparently related with the gradient expansion of the F-function in ferromagnet when only the first term has been retained.

It is interesting to note that the critical F-layer thickness d_c, when the transition from 0- to π-phase occurs, depends on the temperature. The corresponding temperature dependences are presented in Fig.1 for different

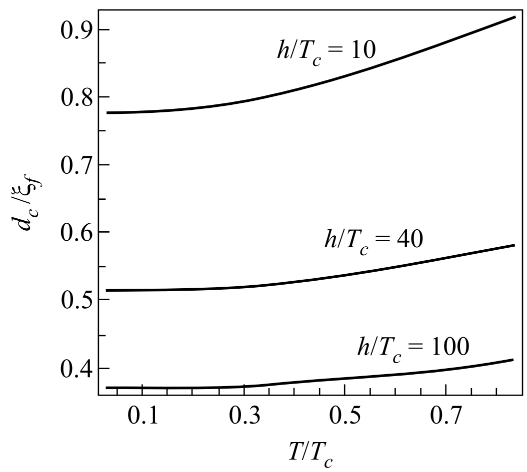

Fig.1. Temperature dependences of the critical thickness d_c of F-layer, corresponding to the crossover from 0- to π-phase in the limit of very small boundary transparency for different values of the exchange field

value of T_c/h ratios. We see that $d_c(T)$ decreases when the temperature decreases. So for some range of F-layer thickness the transition from 0- to π-phase is possible when temperature lowers. This resemble the situation with atomic thickness S/F multilayers [15] and we may therefore expect the non-monotonous $I_c(T)$ dependences revealing for $d_c(T=0) < d < d_c(T_c)$ the crossover between 0- and π-phases.

For the case of moderately large γ_B, i.e. when $1 \ll \gamma_B \ll h/T_c$, the terms with Ψ functions in (5) can be neglected and at $T = T_c$ the critical thickness d_c is

$$d_c\,(T = T_c\,) = \xi_f\,(3\xi_f/\gamma_B\xi_n)^{1/3}\,,$$

while at $T \to 0$ the critical thickness is somewhat smaller $d_c\,(T=0) = \xi_f\,(6\xi_f/\pi\gamma_B\xi_n)^{1/3}$. The examples of different non-monotonous $I_c(T)$ dependences for low barrier transparency limit $\gamma_B \gg (h/T_c)$ are presented in Fig.2. We may see that the transition into π-phase state occurs roughly starting the thickness $\xi_f/2$ and rather unusual $I_c(T)$ dependencies may be expected.

The interface S/F transparency plays an important role for π-phase realization and finite transparency soften the conditions of the transition into π-state. The important particularity of the discussed mechanism is

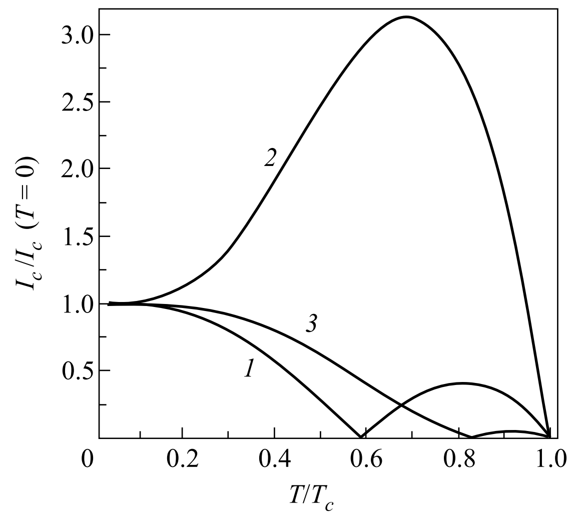

Fig.2. Non-monotonous temperature dependencies of the normalized critical current for low boundary transparency limit: curve *1*: $h/T_c = 10$ and $d/\xi_f = 0.84$; curve *2*: $h/T_c = 40$ and $d/\xi_f = 0.5$; curve *3*: $h/T_c = 100$ and $d/\xi_f = 0.41$

the existence of the temperature crossover in the limit $h \gg T_c$, which is relevant to the experimental situation. In the paper [3] the observed temperature crosssover has been explained by a model with a small exchange field $h \sim T_c \sim 8$ K. It is difficult to believe that the exchange field in Cu_xNi_{1-x} alloy, used in [3] was so small, when its Curie temperature was $\Theta \sim 20-30$ K. Such values of Curie temperatures imply the exchange field higher 100 K.

Note that the discussed mechanism of π-phase appearance in the limit of large γ_B is quite robust toward the F-layer thickness fluctuations, as the π-phase must exist in the quite large interval $d_c < d \lesssim \xi_f$.

The condition of applicability of Usadel equation is $l \ll d$ but qualitatively our analysis should be valid up to $d \sim l$ and it may provide an alternative explanation of the results of experiments [3].

The considered situation is analogous, in some sense to the mechanism of π-phase realization due to the tunneling through ferromagnetic layer in atomic S/F multilayer structure. There are very little layered systems with alternating superconducting and ferromagnetic layers and up to now there are no experimental evidences of π-phase existence in such systems. On the other hand, the Josephson S/F/S junctions with low interface transparency could be promising candidates for the observation of the discussed effect.

In conclusion we propose a new mechanism of π-phase state formation in S/F/S junctions with a large interface barrier.

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