Pis'ma v ZhETF, vol. 81, iss. 12, pp. 784 – 786 © 2005 June 25

No robust phases in aerogel: ^{3}He-A with orientational disorder in the Ginzburg–Landau model. (Comment on papers by I.A. Fomin on robust phases)

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Submitted 12 May 2005

In series of papers [2, 7] Fomin introduced and discussed the so-called robust phases in a system with frozen orientational disorder (with application to superfluid ^{3}He in aerogel). We show that his consideration is based on the erroneous overestimation of the fluctuation energy which comes from the interaction of the Goldstone modes with the frozen disorder. This interaction leads to the Imry-Ma effect, which destroys the orientational order, but is unable to destroy the local structure of ^{3}He-A. There is no ground for the robust phases.

PACS: 67.57.Pq

Following Fomin, let us discuss the superfluid ^{3}He in aerogel using the Ginzburg-Landau (GL) model supplemented by the interaction with the frozen orientational disorder field η_{ij}:

$$F = F_0 + F_{\rm grad} + F_\eta. \qquad (1)$$

Here F_0 and $F_{\rm grad}$ are condensation and gradient energies, and

$$F_\eta = \int \eta_{ij}(\mathbf{r}) A_{\mu i}(\mathbf{r}) A^*_{\mu j}(\mathbf{r}) d^3r, \qquad (2)$$

where $\langle \eta_{ij} \rangle = 0$, and we only consider the orientational anisotropy, i.e. the tensor η_{ij} is traceless: $\eta_{ii} = 0$.

We assume that the disorder is relatively small. Therefore we can start with homogeneous states which have spatially uniform order parameter $A_{\mu i} = A^{(0)}_{\mu i} =$ const. Since $\int \eta_{ij} d^3r = 0$, the energy of such state only comes from F_0:

$$F(A^{(0)}_{\mu i}) = F_0(A^{(0)}_{\mu i}). \qquad (3)$$

We consider here the proper range of the parameters of the GL functional F_0 (the β-parameters of 4-th order terms in F_0 [1]), for which ^{3}He-A has minimum energy. The energy of the uniform ^{3}He-A is smaller than the energy of any other uniform phase by the magnitude $\sim F_0 \sim N_F \tau^2 T_c^2$, where N_F is the density of states in normal Fermi liquid, and $\tau = 1 - T/T_c$. The quasi-isotropic robust phase determined by the condition $\eta_{ij}(\mathbf{r}) A^{(0)}_{\mu i} (A^{(0)}_{\mu j})^* = 0$ [2], has also higher energy.

Let us consider now the second-order (η^2) correction to the energy F_0. The uniform ^{3}He-A is not the minimum of the total GL functional (1), that is why its energy can be reduced by adding the non-uniform corrections (fluctuations), $A_{\mu i} = A^{(0)}_{\mu i} + a_{\mu i}$, with $a \propto \eta$ and thus $\langle a \rangle = 0$. The η^2 terms contain the linear and quadratic terms in $a_{\mu i}$. In $\mathbf{k}$ representation after diagonalization of the a^2 terms one obtains

$$F = F_0(A^{(0)}_{\mu i}) + F_{\rm fl}, \qquad (4)$$

where the fluctuation energy:

$$F_{\rm fl} = \frac{1}{2} \sum_{n,\mathbf{k}} a^2_{n,\mathbf{k}} \epsilon_n(\mathbf{k}) + \sum_{n,\mathbf{k}} \tilde{\eta}_{n,\mathbf{k}} a_{n,\mathbf{k}}. \qquad (5)$$

Here $\tilde{\eta}_{n,\mathbf{k}}$ comes from the product of η and $A^{(0)}_{\mu i}$ matrices, and $\epsilon_n(\mathbf{k})$ is the spectrum of the n-th mode. For Goldstone modes (GM),

$$\epsilon_G(\mathbf{k}) \sim N_F \xi_0^2 k^2, \qquad (6)$$

and for other modes with gaps:

$$\epsilon_{non-G}(\mathbf{k}) \sim N_F(\tau + k^2\xi_0^2) = N_F\tau(1 + k^2\xi^2), \qquad (7)$$

where $\xi = \xi_0/\sqrt{\tau}$ is the GL coherence length.

After minimization over a one obtains the contribution of fluctuations that reduce the ^{3}He-A energy:

$$F_{\rm fl} = -\frac{1}{2} \sum_{n,\mathbf{k}} \tilde{\eta}^2_{n,\mathbf{k}} \epsilon_n^{-1}(\mathbf{k}). \qquad (8)$$

There is no divergence at small k, and the integral is concentrated at large $k \gg 1/\xi$, if we assume that the frozen disorder is concentrated at $1/\xi_0 > k \gg 1/\xi$. It mainly gives the shift of the transition temperature T_c. Actually the traceless orientational disorder increases the transition temperature. Subtracting from Eq.(8) the integral with $\tau = 0$ in the denominator, one obtains the integral $\propto \int d^3k/k^2(1+k^2\xi^2)$ concentrated at $k \sim \xi^{-1}$:

$$\Delta F_{\text{fl}} \sim (A^{(0)})^2 \frac{\eta_0^2}{\tau \xi^3 N_F^2} \sim \alpha F_0, \tag{9}$$

where $\eta_0^2 = \int d^3r \, \langle \eta(\mathbf{r})\eta(0)\rangle$ and α is the Larkin-Ovchinnikov parameter [3]

$$\alpha = \frac{\eta_0^2}{\tau^{1/2}\xi_0^3 N_F^2} \ll 1. \tag{10}$$

We can already stop at this point, since the fluctuation energy is small compared to the condensation energy, and thus ^{3}He-A remains the only possible phase. However, Fomin points out that the interaction of the frozen disorder with GM changes the situation, because due to these modes the amplitude of fluctuations of the non-robust states diverges at small k: $\langle a^2 \rangle \propto \int d^3k/k^4 \sim \int dk/k^2 \sim L$, where L^{-1} is the infra-red cut-off parameter. This gives

$$\langle a^2 \rangle \sim \frac{\alpha L}{\xi}(A^{(0)})^2. \tag{11}$$

At $L \sim \xi$, fluctuations are small if $\alpha \ll 1$, and this is the condition for the applicability of the GL approach. But fluctuations become comparable to $A^{(0)}$ at

$$L \sim \frac{\xi}{\alpha} \gg \xi \ , \tag{12}$$

and this scale L provides the infrared cut-off.

This consideration is certainly true, but it is the well known Imry-Ma effect [4]: Since the Eq.(11) describes the fluctuations of the GM, it corresponds to the change in the orientation of the order parameter A without disturbing its structure. The scale L at which $\langle a^2 \rangle \sim (A^{(0)})^2$ thus indicates the scale at which the orientation of A changes by angle of order $\pi/2$. This is just the Imry-Ma length scale. The state looses the orientational long-range order due to interaction of the GM with the frozen orientational disorder. The similar destruction of the long-range translational order in the mixed state of superconductors by inhomogeneities was found even erarlier [5]. The Imry-Ma effect applied to ^{3}He-A in aerogel was discussed in [6].

Fomin claims that the GM also leads to the divergent contribution to fluctuation energy, which is absent in the robust phases. Let us see. The contribution of the GM with wavelength L to the fluctuation energy F_{fl} in Eq. (8) is proportional to $\int_0^{1/L} k^2dk/k^2 \sim 1/L$. The fluctuation energy in Eq.(9) comes from scale ξ and is proportional to $\int k^2dk/(k^2+1/\xi^2) \sim \int_0^{1/\xi} dk \sim 1/\xi$. Thus the contribution of GM with wavelength L is by factor $\xi/L = \alpha$ smaller, and gives the second-order in α correction to the GL energy. This is just the Imry-Ma energy gain due to the orientational disorder of the order parameter:

$$F_{\text{Imry-Ma}} \sim \alpha \Delta F_{\text{fl}} \sim \alpha^2 F_0 \ll F_0. \tag{13}$$

At the Imry-Ma wavelength L, the interaction with the frozen disorder is on the order of the gradient energy [4]. Thus the contribution of GM with wavelength $L \gg \xi$ to the energy is on the order of the gradient energy at this scale, and thus contains the small factor ξ^2/L^2 compared to the condensation energy F_0. This is demonstrated in Eq. (13), since $\xi/L = \alpha \ll 1$.

The equation (13) contradicts to the statement by Fomin [7], who erroneously concludes that the contribution of GM contains the large factor $1/\alpha$ compared to the contribution of the non-Goldstone modes: $F_{\text{fl-G}} \sim \alpha^{-1} F_{\text{fl-non-G}}$, and thus, due to GM, the fluctuation energy is comparable to the condensation energy: $F_{\text{fl-G}} \sim F_0$. This provides the justification for introduction of the robust phases where the disorder does not interact with GM, and thus there is no divergence in the amplitude of the order parameter. This justification is wrong and thus there is no basis for the robust phase. The same conclusion was made by Mineev and Zhitomirsky in their Comment [8].

In conclusion, the Goldstone modes, i.e. fluctuations in the direction of the degeneracy of the order parameter, do lead to the divergence of the amplitude of the order parameter. But their contribution to energy does not experience any divergence and is small compared to the condensation energy by the parameter $\alpha^2 \ll 1$. This is nothing but the Imry-Ma effect, which leads to disorder in the orientation of the order parameter at large length $L = \xi/\alpha \gg \xi$ without changing the local structure of the order parameter. Since the condensation energy F_0 is dominating, the local order parameter must be in the ^{3}He-A state everywhere (at least within the GL model (1)). The robust phase is not the extremum of F_0, and thus is not the solution of GL equations. Thus within the GL model with the frozen orientational disorder the Imry-Ma approach is valid and it does not leave any room for the robust phase.

I thank I. A. Fomin, N. B. Kopnin and V. P. Mineev for discussions.

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