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UPPER BOUND ON THE SUPERSYMMETRY BREAKING SCALE IN SUPERSUMMETRIC $SU(5)$ MODEL

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The status of coupling constant unification in standard supersymmetric $SU(5)$ model and its extensions is discussed. Taking into account uncertainties related with the initial coupling constants and threshold corrections at the low and high scales we find that in standard supersymmetric $SU(5)$ model the scale of the supersymmetry breaking could be up to 10^8 GeV. In the extensions of standard $SU(5)$ model it is possible to increase the supersymmetry breaking scale up to 10^{11} GeV.

There has recently been renewed interest [1-12] in grand unification business related with the recent LEP data which allow to measure $sin^2(\theta_w)$ with unprecendented accuracy. Namely, the world averages with the LEP data mean that the standard nonsupersymmetric $SU(5)$ model [13] is ruled out finally and forever (the fact that the standard $SU(5)$ model is in conflict with experiment was well known [14, 15] before the LEP data) but maybe the most striking and impressive lesson from LEP is that the supersymmetric extension of the standard $SU(5)$ model [16-18] predicts the Weinberg angle θ_w in very good agreement with experiment. The remarkable success of the supersymmetric $SU(5)$ model is considered by many physicists as the first hint in favour of the existence of low energy broken supersymmetry in nature. A natural question arises: is it possible to invent nonsupersymmetric generalizations of the standard $SU(5)$ model nonconfronting the experimental data or to increase the supersymmetry breaking scale significantly. In the $SO(10)$ model the introduction of the intermediate scale $M_I \sim 10^{11}\,\mathrm{GeV}$ allows to obtain the Weinberg angle θ_w in agreement with experiment [19]. In refs.[20, 21] it has been proposed to cure the problems of the standard $SU(5)$ model by the introduction of the additional split multiplets $5 \oplus \bar{5}$ and $10 \oplus \overline{10}$ in the minimal $3(\bar{5} \oplus 10)$ of the $SU(5)$ model. In ref.[22] the extension of the standard $SU(5)$ model with light scalar coloured octets and electroweak triplets has been proposed.

In this paper we discuss the coupling constant unification in standard supersymmetric $SU(5)$ model and its extensions. Taking into account uncertainties associated with the initial gauge coupling constants and threshold corrections at the low and high scales we conclude that in standard supersymmetric $SU(5)$ model the scale of the supersymmetry breaking could be up to $10^8\,\mathrm{GeV}$. We find also that in the extensions of the standard $SU(5)$ supersymmetric model it is possible to increase the supersymmetry breaking scale up to 10^{11} GeV.

The standard supersymmetric $SU(5)$ model [16-18] contains three light supermatter generations and two light superhiggs doublets. A minimal choice of massive supermultiplets at the high scale is $(\bar{3}, 2, \frac{5}{2}) \oplus c.c.$ massive vector supermultiplet with the mass M_v, massive chiral supermultiplets $(8,1,0), (1,3,0), (1,1,0)$ with the masses m_8, m_3, m_1 (embeded in a 24 supermultiplet of $SU(5)$) and a

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$(3,1,-\frac{1}{3}) \oplus (-3,1,\frac{1}{3})$ complex Higgs supermultiplet with a mass M_3 embeded in $5 \oplus \bar{5}$ of $SU(5)$. In low energy spectrum we have squark and slepton multiplets $(\tilde{u},\tilde{d})_L, \tilde{u}^c_L, \tilde{d}^c_L, (\tilde{\nu},\tilde{e})_L, \tilde{e}^c_L$ plus the corresponding squarks and sleptons of the second and third supergenerations. Besides in the low energy spectrum we have $SU(3)$ octet of gluino with a mass $m_{\tilde{g}}$, triplet of $SU(2)$ gaugino with a mass $m_{\tilde{w}}$ and the photino with a mass $m_{\tilde{\gamma}}$. For the energies between M_z and M_{GUT} we have effective $SU(3) \otimes SU(2) \otimes U(1)$ gauge theory. In one loop approximation the corresponding solutions of the renormalization group equations are well known [18]. In our paper instead of the prediction of $\sin^2(\theta_w)$ following ref.[6] we consider the following one loop relations between the effective gauge coupling constants, the mass of the vector massive supermultiplet M_v and the mass of the superhiggs triplet M_3:

$$A \equiv 2(\frac{1}{\alpha_1(m_t)} - \frac{1}{\alpha_3(m_t)}) + 3(\frac{1}{\alpha_1(m_t)} - \frac{1}{\alpha_2(m_t)}) = \Delta_A, \quad (1)$$

$$B \equiv 2(\frac{1}{\alpha_1(m_t)} - \frac{1}{\alpha_3(m_t)} - 3(\frac{1}{\alpha_1(m_t)} - \frac{1}{\alpha_2(m_t)}) = \Delta_B, \quad (2)$$

where

$$\Delta_A = (\frac{1}{2\pi})(\delta_{1A} + \delta_{2A} + \delta_{3A}), \quad (3)$$

$$\Delta_B = (\frac{1}{2\pi})(\delta_{1B} + \delta_{2B} + \delta_{3B}), \quad (4)$$

$$\delta_{1A} = 44\ln(\frac{M_v}{m_t}) - 4\ln(\frac{M_v}{m_{\tilde{g}}}) - 4\ln(\frac{M_v}{m_{\tilde{w}}}), \quad (5)$$

$$\delta_{2A} = -12(\ln(\frac{M_v}{m_8}) + \ln(\frac{M_v}{m_3})), \quad (6)$$

$$\delta_{3A} = 6\ln(m_{(\tilde{u},\tilde{d})_L}) - 3\ln(m_{\tilde{u}^c_L}) - 3\ln(m_{\tilde{e}^c_L}), \quad (7)$$

$$\delta_{1B} = 0.4\ln(\frac{M_3}{m_h}) + 0.4\ln(\frac{M_3}{m_H}) + 1.6\ln(\frac{M_3}{m_{sh}}), \quad (8)$$

$$\delta_{2B} = 4\ln(\frac{m_{\tilde{g}}}{m_{\tilde{w}}}) + 6\ln(\frac{m_8}{m_3}), \quad (9)$$

$$5\delta_{3B} = -12\ln(m_{(\tilde{u},\tilde{d})_L}) + 9\ln(m_{\tilde{u}^c_L}) + 6\ln(m_{\tilde{d}^c_L}) - 6\ln(m_{(\tilde{\nu},\tilde{e})_L}) + 3\ln(m_{\tilde{e}^c_L}) \quad (10)$$

Here m_h, m_H and m_{sh} are the masses of the first light Higgs isodoublet, the second Higgs isodoublet and the isodoublet of superhiggses. The relations (1)-(10) are very convenient since they allow to determine separately two key parameters of the high energy spectrum of $SU(5)$ model, the mass of the vector supermultiplet M_v and the mass of the chiral supertriplet M_3. Both the vector supermultiplet and the chiral supertriplet are responsible for the proton decay in supersymmetric $SU(5)$ model [18]. In standard nonsupersymmetric $SU(5)$ model the proton lifetime due to the massive vector exchange is determined by the formula [23]

$$\Gamma(p \to e^+\pi^o)^{-1} = 4 \cdot 10^{29\pm0.7}(\frac{M_v}{2 \cdot 10^{14}\mathrm{GeV}})^4 yr \quad (11)$$

In supersymmetric $SU(5)$ model the GUT coupling constant is $\alpha_{GUT} \approx \frac{1}{25}$ compared to $\alpha_{GUT} \approx \frac{1}{41}$ in standard $SU(5)$ model, so we have to multiply the expression (11) by factor $(\frac{25}{41})^2$. From the current experimental limit [24] $\Gamma(p \to e^+\pi^o)^{-1} \geq 9 \cdot 10^{32} yr$

we conclude that $M_v \geq 1.2 \cdot 10^{15}\,\mathrm{GeV}$. The corresponding experimental bound on the mass of the superhiggs triplet M_3 depends on the masses of gaugino and squarks [25, 26]. In our calculations we use the following values for the initial coupling constants [24, 27]:

$$\alpha_3(M_z) = 0.120 \pm 0.07, \tag{12}$$

$$\sin^2_{\overline{MS}}(\theta_w)(M_z) = 0.2319 \pm 0.0005, \tag{13}$$

$$(\alpha_{em,\overline{MS}}(M_z))^{-1} = 127.79 \pm 0.13 \tag{14}$$

For the top quark mass $m_t = 174\,\mathrm{GeV}$ after the solution of the corresponding renormalization group equations in the region $M_z \leq E \leq m_t$ we find that

$$A = 184.45 \pm 0.68 \pm 0.92, \tag{15}$$

$$B = 13.31 \pm 0.24 \pm 0.92 \tag{16}$$

Here the first error is the "electroweak" error and the second error is the "strong coupling" error. An account of two loop corrections leads to the appearance of the additional factors

$$\delta_{4A,4B} = 2(\theta_1 - \theta_3) \pm 3(\theta_1 - \theta_3), \tag{17}$$

$$\theta_i = \frac{1}{4\pi}\sum_{j=1}^{3} \ln[\frac{\alpha_j(M_v)}{\alpha_j(m_t)}] \tag{18}$$

Here b_{ij} are the two loop β function coefficients. Let us start from the expression (1) and assume that the masses of the octet supermultiplet and the masses of the triplet supermultiplet coincide with the mass of the vector supermutiplet M_v. We shall neglect the variation of the low energy spectrum (we assume that all the squarks and the sleptons have the same mass). Numerically we find that

$$M_v = 2.0 \cdot 10^{16 \pm 0.05 \pm 0.07}\,\mathrm{GeV} \tag{19}$$

for the $M_{SUSY} \equiv (m_{\tilde{g}} m_{\tilde{w}})^{\frac{1}{2}} = 174\,\mathrm{GeV}$. From the lower bound on the value of the mass of the vector bosons responsible for the baryon number nonconservation we find an upper bound on the value of the supersymmetry breaking parameter $M_{SUSY} \leq 2 \cdot 10^8\,\mathrm{GeV}$. For $M_{SUSY} = 10^8; 10^7; 10^6; 10^5; 10^4; 10^3; 10^2\,\mathrm{GeV}$ we find that $M_v = (1.4 \cdot 10^{15}; 1.8 \cdot 10^{15}; 3.0 \cdot 10^{15}; 5.1 \cdot 10^{15}; 10^{16}; 1.6 \cdot 10^{16}; 2.4 \cdot 10^{16}) \cdot 10^{\pm 0.05 \pm 0.07}\,\mathrm{GeV}$. The uncertainty in the masses of the coloured octet m_8 and electroweak triplet m_3 leads to the uncertainty $(\frac{m_8 m_3}{M_v^2})^{\frac{1}{3}}$ for the supersymmetry breaking scale M_{SUSY}. The uncertainty due to the difference of squark and slepton masses is small for the realistic spectrum when the difference in masses is less than 3 and we shall neglect it.

Let us consider now the equation (2). For $M_{SUSY} = m_t$ and $m_{\tilde{g}} = \frac{\alpha_3(m_t)}{\alpha_2(m_t)} m_{\tilde{w}}$ [18] in the assumption that all squark and slepton masses coincide , the masses of the coloured octet m_8 and electroweak triplet m_3 are equal to the mass of the vector supermultiplet M_v and the masses of superhiggses and Higgs isodoublets are equal to M_t we find that

$$M_3 = 6.6 \cdot 10^{14 \pm 0.27 \pm 1.05}\,\mathrm{GeV} \tag{20}$$

It should be noted that from the nonobservation of the proton decay the bound on the mass of the Higgs triplet for $M_{SUSY} = m_t$ is [18] $M_3 \geq O(10^{16})\,\mathrm{GeV}$.

When we increase the value of M_{SUSY} two scenario are possible. According to the first scenario only the single Higgs isodoublet is light with a mass $O(M_z)$ and the masses of the second Higgs isodoublet and superhiggses are of the order of M_{SUSY}. In the second scenario the first Higgs isodoublet and superhiggses are relatively light with the masses $O(M_z)$ (or superhiggses are slightly heavier with the mass $O(1\,\text{TeV})$) and only the second Higgs isodoublet is relatively heavy with the mass $O(M_{SUSY})$. We have investigated two scenario. In our investigation we have used an upper bound $M_3 \leq 3M_v$ [5] which comes from the requirement of the applicapability of the perturbation theory. Taking into account uncertainties in the determination of the parameter B we have found that in the first scenario $M_{SUSY} \leq 10^5\,\text{GeV}$ and in the second scenario $M_{SUSY} \leq 10^8\,\text{GeV}$. If we assume that the difference between the masses of coloured octets and coloured triplets could be up to factor 3 then for $\frac{m_8}{m_3} = 3$ we find that in the first scenario the supersymmetry breaking scale could be up to $10^7\,\text{GeV}$. It should be noted that the proton lifetime due to the exchange of the Higgs supertriplet is proportional to m^2_{sq} (here we assume that $m_{sq} \sim M_{SUSY}$) and it does not contradict to the nonobservation of the proton decay for big M_{SUSY}.

It is instructive to consider the supersymmetric $SU(5)$ model with relatively light coloured octet and triplets. For instance, consider the superpotential

$$W = \lambda\sigma(x)[\text{Tr}(\Phi^2(x)) - c^2], \tag{21}$$

where $\sigma(x)$ is the $SU(5)$ singlet chiral superfield and $\Phi(x)$ is chiral 24-plet in the adjoint representation. For the superpotential (21) the coloured octet and electroweak triplet chiral superfields remain massless after $SU(5)$ gauge symmetry breaking and they acquire the masses $O(M_{SUSY})$ after the supersymmetry breaking. So in this scenario we have additional relatively light fields. Lower bound on the mass of the vector bosons leads to the upper bound on the supersummetry breaking scale $M_{SUSY} \leq 10^{11}\,\text{GeV}$ In order to satisfy the second equation for the mass of the Higgs triplets let us introduce in the model two additional superhiggs 5-plets. If we assume that after $SU(5)$ gauge symmetry breaking the corresponding Higgs triplets acquire mass $O(M_v)$, the light Higgs isodoublet has a mass $O(M_z)$, the second Higgs isodoublet and superhiggses have masses $O(M_{SUSY})$ then we can satisfy the equation (2) for $M_{SUSY} \sim 10^{11}\,\text{GeV}$. According two other scenario playing with relatively light octets and triplets we can increase the grand unification scale up to $O(10^{18})\,\text{GeV}$ that is welcomed from the point of view of the string unification scenario. In this case the supersymmetry breaking scale is $M_{SUSY} \sim 10^8\,\text{GeV}$. For such value of the supersymmetry breaking scale one can satisfy the equation (2) even without the introduction of the additional superhiggs 5-plets. It is possible also to have grand unification scale $M_v = 10^{17}\,\text{GeV}$ and $M_{SUSY} \leq 1\,\text{TeV}$ if octet and triplet are lighter than the vector supermultiplet by factor 100.

In conclusion let us formulate our main results. We have found that in standard supersymmetric $SU(5)$ model with coloured octet and triplet masses $O(M_v)$ the nonobservation of the proton decay leads to the upper bound $M_{SUSY} \leq 2 \cdot 10^8\,\text{GeV}$ on the supersymmetry breaking scale. This bound does not contradict to the equation for the superhiggs triplet mass in the second scenario. In the first scenario it is possible to have the supersymmetry breaking scale M_{SUSY} up to $O(10^7)\,\text{GeV}$ provided that the difference between the coloured octet and electroweak triplet masses is $\frac{m_{\tilde{g}}}{m_{\tilde{w}}} = O(3)$. For the case when the octets and triplets have the masses $O(M_{SUSY})$ it is possible to increase the supersymmetry breaking scale up to $O(10^{11})$GeV, however in this case in order to satisfy the equation for the superhiggs triplet mass we have to introduce additional relatively light pair of the superhiggs doublets. It is possible also to increase the value of the grand unification scale up to $O(10^{18})$GeV for the case when octets and triplets have the masses $O(M_{SUSY})$ and $M_{SUSY} = O(10^8)$GeV without the introduction of the additional light Higgs superdoublets. It should be noted that estimate on the supersymmetry breaking scale in supersymmetric $SU(5)$ model $M_{SUSY} = 10^{3.0\pm0.8\pm0.4}$GeV has been obtained in ref.[28] in the assumption that $M_v = M_3 = m_3 = m_8$. Our analysis demonstrates that the value of M_v depends rather weekly on the high and low energy uncertainties in the determination of the spectrum and low energy effective coupling constants. In the extraction of the bound on the value of M_{SUSY} our crusial assumption was the inequality [5] $M_3 \leq 3M_v$. The obtained bound on the M_{SUSY} depends rather strongly on the details of the high energy spectrum (on the splitting between octet and triplet masses) and on the initial value of the strong coupling constant. It should be noted that for $M_{SUSY} \geq O(1)$ Tev we have the fine tuning problem for the electroweak symmetry breaking scale.

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