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# FINITE-TIME HEAVY QUARK INTERACTION IN QCD

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Heavy $q\bar{q}$ interaction is studied at finite time $T$. Using the nonperturbative background formalism the averages of Wilson loops to lowest order in $g^2$ are calculated and the generalized formula of perturbative interaction $V_p(R,T)$ is obtained. It is shown that for finite time, $T \lesssim R, V_p(R,T)$ essentially differs from Coulomb potential. Our analytical results are in good agreement with lattice calculations.

1. Static $q\bar{q}$ interaction $V(R)$ yields the important information on nonperturbative (NP) (confining) and perturbative forces between static quarks and their possible interference. Moreover the effects of $\alpha_s$ renormalization at small and large distances can be conveniently formulated in a gauge invariant way in terms of $V(R)$ [1,2]. However, in physical applications the time duration $T$ of $q\bar{q}$ interaction is never infinite and one should rather consider finite-time interaction $V(R,T)$ with typical $R$ and $T$ for a given process, e.g. heavy quark $c\bar{c}$ creation inside a nucleus or a hadron. In particular, in lattice QCD one defines $V(R,T)$ through the Wilson loop of size $R\times T$ and the time $T$ on the lattice is kept finite.

For our analytical calculation we use the definition of $V(R,T)$ analogous to the lattice one [3,4]

$$V(R,T) = -\ln \frac{W(R,T)}{W(R,T-a)}, \tag{1}$$

where $a$ is a small (lattice) unit of length and $W(R,T) \equiv < W(R,T) >$ is the average of the Wilson loop over vacuum fields.

Our purpose here is twofold. First, we shall derive the analytical expression for $V(R,T)$ using NP background formalism [2] and compare it with lattice data [4]. Secondly, we carefully study the finite-time corrections to the static Coulomb potential $V(R)$.

2. In the formalism of NP background [2] one represents the total gluonic field $A_\mu$ as a sum of NP background field $B_\mu$ and quantum fluctuations $a_\mu$ treated perturbatively:

$$A_\mu = B_\mu + a_\mu, \tag{2}$$

where $a_\mu$ transforms homogeneously under gauge transformations. To calculate Wilson loop average we keep the lowest order contribution in $a_\mu$

$$< W(B+a) > = < W(B) > -g^2 < W^{(2)}(a) > +0(g^4). \tag{3}$$

For $V(R,T)$ we have

$$V(R,T) = -\ln \frac{< W(B;R,T) > \left[1 - g^2 \frac{<W^{(2)}(R,T)>}{<W(B;R,T)>}\right]}{< W(B;R,T-a > \left[1 - g^2 \frac{<W^{(2)}(R,T-a)>}{<W(B;R,T-a)>}\right]} \equiv \tag{4}$$

$$\equiv V_{NP}(R,T) + V_P(R,T),$$

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where

$$V_{NP}(R,T) = -\ln \frac{< W(B;R,T) >}{< W(B;R,T-a) >}, \tag{5}$$

$$V_P(R,T) = g^2 \left\{ \frac{< W^{(2)}(R,T) >}{< W(B;R,T) >} - \frac{< W^{(2)}(R,T-a) >}{< W(B;R,T-a) >} \right\}. \tag{6}$$

The "perturbative term" $< W^{(2)}(R,T) >$ in fact depends on NP background, i.e. it contains gluon propagator in the background field, $G_{\mu\nu}(x,y;B)$, between points $x$ and $y$ on the contour of Wilson loop [2]

$$< W^{(2)}(R,T) >_B = < \int \phi_1 G_{\mu\nu}(x,y;B) \phi_2 dx_\mu dy_\nu >_B \tag{7}$$

where $\phi_1$ and $\phi_2$ represent the left and right parts of Wilson loop between points $x$ and $y$. As was shown in [5], in the limit of large $N_c$ the gluon line can be replaced by a double fundamental line and then in (7) one obtains an average product of two Wilson loops which are factorizing in this limit. As a result [2] for large $T$, $T > \sigma^{-1/2}$, perturbative and NP contributions in $< W^{(2)} >$ also factorize. The first factor of the product is simply
$< W(B;R,T) >$ and the second – perturbative term-reduces to the free one-gluon exchange at large $T$.

From (6) and (7) using results from Ref.[6] one can derive $V_P(R,T)$ (Feynman background gauge is taken for simplicity)

$$V_P(R,T) = +\frac{g^2 C_2}{8\pi^2} \int_C \int_C \frac{dx_\mu dy_\mu}{(x-y)^2} \quad , \quad T > \sigma^{-1/2}. \tag{8}$$

Here $C_2$ is the quadratic Casimir operator.

In the range of small times, $T \ll \sigma^{-1/2}$, Eq.(8) is not valid and $V_P(R,T)$ can be found from the hybrid string interaction as it was done in lattice calculations in [7]. Here we focus our attention only on times $T \gtrsim \sigma^{-1/2}$.

3. The perturbative interaction (8) is the one-gluon exchange between heavy quarks inside the Wilson loop which was studied analytically in [6]. It contains the linear divergence (which gives the constant additive term to $V(R,T)$) and also the logarithmic divergencies due to nonanalyticity of Wilson contour. Both have to be regularized and for this purpose we introduce the minimal cut–off distance $\varepsilon$ in the integral (8). Introducing also lattice units, $t = \frac{T}{a}$, $r = \frac{R}{a}$, $V \to aV$ , $e = \frac{4}{3}\alpha_s$, the expression (4) can be presented in the form:

$$V(r,t) = V_{NP}(r,t) + V_0 + V_P(r,t), \tag{9}$$

where the perturbative part consists of three terms

$$V_P(r,t) = V_s + V_t + V_{reg}. \tag{10}$$

From (8) for a rectangular contour the regularization term is

$$V_{reg} = \frac{e}{2\pi} \cdot 4 \ln \left( \frac{t-1-\delta}{t-\delta} \right) \quad , \quad \delta = \frac{\varepsilon}{a} \tag{11}$$

and $V_s$ and $V_t$ in (10) refer to the gluon exchange between space-like links and time-like links respectively. From (8) we find

$$V_t = -\frac{e}{\pi}\left\{\frac{2}{r}\mathrm{arctg}\frac{t-1}{r} + \frac{2t}{r}\left[\mathrm{arctg}\frac{t}{r} - \mathrm{arctg}\frac{t-1}{r}\right] + \ln\frac{r^2+(t-1)^2}{r^2+t^2}\right\}, \qquad (12)$$

$$V_s = -\frac{e}{\pi}\left\{\frac{2r}{t}arctg\frac{r}{t} - \frac{2r}{t-1}arctg\frac{r}{t-1} + \ln\frac{[(t-1)^2+r^2]t^2}{(t^2+r^2)(t-1)^2}\right\}. \qquad (13)$$

The asymptotic behaviour of Eqs.(11)-(13) at large $t$ can be easily found; for $t \gg r \gg 1$ one has

$$V_t \to \frac{e}{2\pi}\left\{-\frac{2\pi}{r} + \frac{4}{t} + \frac{2}{t^2} - \frac{4}{3}\frac{r^2}{t^3} + \ldots\right\} \qquad (14)$$

$$V_s \to \frac{e}{2\pi}\frac{4r^2}{t^3} + 0(\frac{1}{t^4})$$

$$V_{reg} \to -\frac{e}{2\pi}(\frac{4}{t} + \frac{2}{t^2} + 0(\frac{1}{t^3})).$$

It is interesting that for $t \gg r \gg 1$ the first two corrections (of the order of $1/t$ and $1/t^2$) cancel inside the perturbative interaction $V_p$, so from (10) it follows

$$V_p \to -\frac{e}{r} + 0(\frac{r^2}{t^3}), t \gg r \gg 1. \qquad (15)$$

This cancelation explains why the asymptotic value $V_p(\mathrm{asymp}) \equiv -e/r$ is approached rather fast: so for $t = r+2$ the difference between exact value $V_p(r,t)$ and $(-e/r)$ is less than 20% but for $t = r+4$ this difference is already less than 5% (see table) for any $r$ ($r = 2, 4, 6, 8$ were considered).

The perturbative part of interaction, $V_{pert}(r,t)$, as a function of time $t = T/a$ for different fixed distances $r = R/a$ ($e = 0.302$, $\delta = 0.25$).

| $r$ \ $t$ | 1.5 | 2.0 | 3.0 | 4.0 | 5.0 | 7.0 | 10 | 15 | $V_p(\mathrm{asym}) \to$ $\to -\frac{e}{r}(t \to \infty)$ |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.255 | -0.038 | -0.128 | -0.144 | -0.148 | -0.149 | -0.1507 | 0.151 | -0.151 |
| 4 | 1.083 | 0.296 | 0.018 | -0.039 | -0.058 | -0.070 | -0.0746 | -0.0751 | -0.0755 |
| 6 | 1.892 | 0.604 | 0.128 | 0.024 | -0.012 | -0.037 | -0.046 | -0.049 | -0.0563 |
| 8 | 2.699 | 0.908 | 0.220 | 0.079 | 0.024 | -0.015 | -0.030 | -0.035 | -0.03775 |

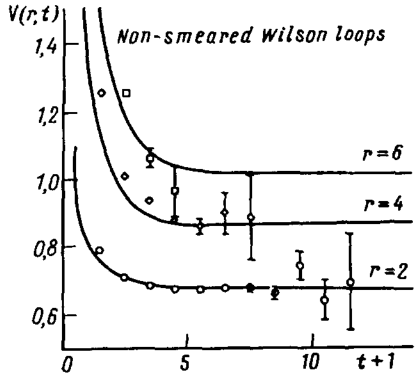
Nevertheless the most striking consequence of our calculations is following: for values $t \leq r$ (any $r$) the exact value of $V_p$ (10) differs several times from $V_p(\mathrm{asymp})$ and even has another (positive) sign for $t < r$ ( see table).

As to $NP$ interaction, it was studied previously [1] via cluster expansion and for large $r, t$ it was obtained there

$$V_{\mathrm{NP}}(r,t) \to (\sigma a^2)r + C_0 \ , \quad r > t_g. \qquad (16)$$

Here $t_g$ is the characteristic gluonic correlation length which can be found from the quadratic field correlator. Lattice calculations of this correlator in the Ref. [8] has shown that $t_g$ is smaller than $\sigma^{-1/2}$ therefore in (9) we can use the asymptotic behaviour (16) for $V_{\mathrm{NP}}(r,t)$.

4. For comparison with lattice calculations [4] we consider $V(r,t)$ defined by formula (9) at fixed $r$ and vary $t$ in the interval $1.5 \le t \le 15$. The results are presented in Figure by solid curve with the marked values of $r = 2, 4, 6..$ The lattice data from [4] are also shown by circles for $r = 2$, diamonds ($r = 4$) and squares ($r = 6$). From Fig. we can see good agreement between our analytical calculations and the available lattice ones.




The comparison of finite-time interaction $V(r,t)$, given by the analytical expression (9), with lattice data from Ref. [4]. Solid lines represent analytical calculations for different fixed $r$ ($e = 0.302$; $\sigma a^2 = 0.0589$; $aV_0 = 0.70; \delta = 0.25$); lattice data are shown by circles for $r = 2$, diamonds for $r = 4$ and squares for $r = 6$

From formulas (15),(16) asymptotically one has

$$V(r,t) \to -\frac{e}{r} + (\sigma a^2)r + V_0 \qquad (t \gg r \gg 1). \tag{17}$$

As follows from our calculations this behaviour is achieved already at $t \gtrsim r + 4$. Here we would like to stress several points:

1) For rather large $r$, e.g. $r = 6$ and $r = 8$, the perturbative term is much smaller than NP one, so we cannot feel the essential difference between $V_{pert}$ (exact) and $(-e/r)$ in the total interaction, $V$, even for small times $(t < r)$ when they have the opposite sign.

Hence to study pure perturbative effects at finite $T$ on the lattice and, in particular, the problem of freezing of $\alpha_s(R)$ at large distances [2, 9], it is necessary to separate perturbative an $NP$ terms with good accuracy.

2) For large $r$ ($r = 6$ or 8) the perturbative asymptotics is achieved only for the values of $t \gtrsim r + 4 (t \gtrsim 10)$ meanwhile on the lattice [4] the fit of parameters $(V_0, e)$ is usually done for smaller times $t = 4$ or 5 (where the use of asymptotical interaction is not valid).

3) In (17) our constant term $V_0$ is universal for any large $t$ and $r$ ($V_0 = 0.70$). So for large $r$ ($r$ fixed) when Coulomb term in (17) can be neglected, $V(r,t)$ is approaching the plateau value: $V(r$ fixed, any $t) \cong [(\sigma a^2)r + V_0]$. This simple asymptotics agrees well with the plateau values in lattice calculations [4] both for smeared and nonsmeared Wilson loops.

4) Our conclusions about the complex structure of perturbative interaction at finite $T$ is in agreement with results of recent analysis in [10] where lattice Coulomb contributions $V_s(R,T)$ and $V_t(R,T)$ (analougs to formulas (12) and (13)) have been measured and the important role of "unphysical" term $V_s(r,t)$ was stressed. Besides this term there is another "unphysical" term $V_{reg}$ (11), absent in [10], which is extremely important in achieving the asymptotic regime (15).

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