Pis'ma v ZhETF, vol.61, iss.7, pp.552 - 555 © 1995 April 10

ULTRASONIC ATTENUATION IN d-WAVE SUPERCONDUCTORS

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Submitted 21 February 1995

The longitudinal ultrasonic attenuation at low temperatures for a d-wave superconducting order parameter is examined. The ratio of the superconducting to normal state attenuation, α_s/α_n, depends strongly on the direction of propagation of ultrasound with maxima corresponding to nodes of the order parameter on the Fermi surface. We propose that measurements of ultrasonic attenuation in a single crystal of the high-T_c superconductors can give additional information regarding the pairing symmetry.

An important aspect of the high-T_c superconductors that is currently receiving a great deal of attention is the angular dependence of the energy gap. Some experimental evidence [1, 2] has been brought forward for d-wave pairing instead of the more conventional BCS s-wave pairing. The purpose of this note is to point out that ultrasonic attenuation may offer a very effective and useful tool for the experimental determination of the variation of the energy gap around the Fermi surface. We demonstrate that, due to the layered structure of the high-T_c cuprates, the anisotropy of the d-wave order parameter should result in a very pronounced dependence of the ultrasonic attenuation on the direction of propagation (parallel to the CuO_2 layers) at low temperatures.

In order to concentrate on our main point, we assume that the scale of the quantum of sound energy, $\hbar\omega_q$, is negligibly small compared to the energy gap. Typical experimental values of ω_q are of the order of $10^9 - 10^{10}\,\mathrm{sec}^{-1} \approx 0.01 - 0.1\,\mathrm{meV}$, equivalent to a temperature of $1\,\mathrm{K}$ or less. (For simplicity, we ignore the immediate vicinity of a node.) This assumption leads to the picture of the ultrasonic attenuation as a kind of quasi-elastic scattering of the quasiparticles of energy E by the incident sound quantum. The initial states are counted in terms of increments of the normal state energy, $d\epsilon = (\partial\epsilon/\partial E)dE$. The integration for the total transition rate in the Fermi "golden rule" formula needs to include an additional factor $\partial\epsilon/\partial E$ for the final state density. But these two factors of $\partial\epsilon/\partial E$ are cancelled by the squared strength of the coupling of the sonic field with the quasi-particle energy [3], which is proportional to $\partial E/\partial\epsilon$. This follows from the fact that it is to ϵ, and only indirectly to E, that the lattice deformation is coupled. Consequently, the ratio of the longitudinal ultrasonic attenuation in the superconducting and normal state is given by the formula

$$\frac{\alpha_s}{\alpha_n} = -2\int_{\Delta}^{\infty} \frac{\partial f(E)}{\partial E} dE = 2f(\Delta) \equiv \frac{2}{\exp\beta\Delta + 1}, \qquad (1)$$

where $f(E)$ is the Fermi function and Δ is the superconducting energy gap. This derivation [3] of Eq. (1) does not involve the BCS coherence factors. They do,

[1)]Permanent address: Department of Physics, Indian Institute of Technology, Kanpur, 208016, U.P., INDIA however, enter for frequencies so high that the energy of the sound quantum is not negligibly small, as can be seen in the standard treatment [4]. When Δ differs, as presently under consideration, for different quasi-particles at the Fermi surface we take the average of the right hand side of Eq. (1), weighted by the contribution in the normal state.

It is possible to obtain the same result using Green's functions to calculate the density-density correlation function, the imaginary part of which is proportional to ultrasonic attenuation. The result for longitudinal ultrasonic attenuation in anisotropical superconductors is given by the following expression [5]:

$$\frac{\alpha_s(\theta)}{\alpha_n(\theta)} = \frac{< \delta(\mathbf{v_k} \cdot \mathbf{q}) \ |g(\mathbf{k},\mathbf{k})|^2 \ 2f(\Delta_\mathbf{k}) >_{FS}}{< \delta(\mathbf{v_k} \cdot \mathbf{q}) \ |g(\mathbf{k},\mathbf{k})|^2 >_{FS}}\,, \tag{2}$$

where $g(\mathbf{k},\mathbf{k})$ is the matrix element of electron-(acoustic) phonon interaction. The momentum $\mathbf{k}$ and velocity $\mathbf{v_k}$ are on the Fermi surface and $< \ldots >_{FS}$ is the integration over the Fermi surface. The angle θ describes the direction of q, the transfer momentum. In the case of a two-dimensional Fermi surface, the integrations in Eq. (2) are reduced because of the δ-functions and the matrix element $g(\mathbf{k},\mathbf{k})$ in Eq. (2) cancells. For concreteness, we consider the two-dimensional tight-binding electron spectrum

$$\epsilon_\mathbf{k} = -2t[\cos k_x + \cos k_y] - \mu\,, \tag{3}$$

where t is the hopping matrix element. μ is the shift of chemical potential as determined by the band-filling factor n (the Fermi surface is specified by $\epsilon_\mathbf{k} = 0$). The lattice constant is taken to be 1 for simplicity. The d-wave order parameter can be written in the form

$$\Delta_\mathbf{k} = \frac{\Delta_0(T)}{2}[\cos k_x - \cos k_y]\,, \tag{4}$$

where $\Delta_0(T)$ obeys the usual BCS temperature dependence. One can see

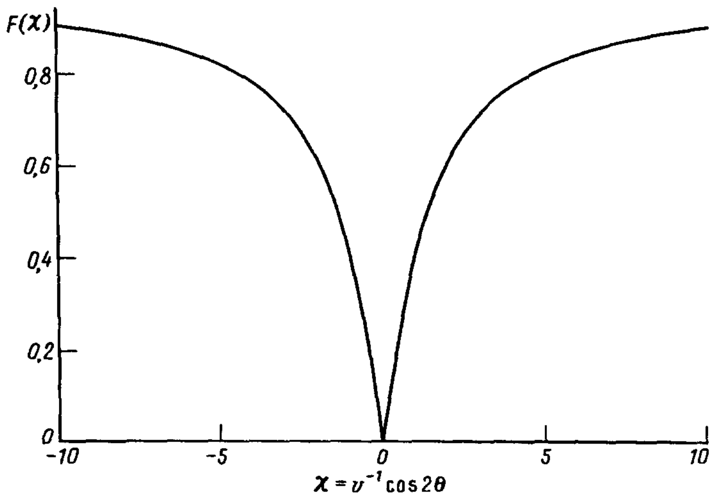

Fig.1. The scaling function for the angular dependence of the d-wave order parameter. This function characterizes the universal behavior which connects small doping and the angle dependence of the order parameter

that, in any given direction, the ultrasonic attenuation falls off exponentially with temperature except for directions close to those of the nodes of the d-wave order

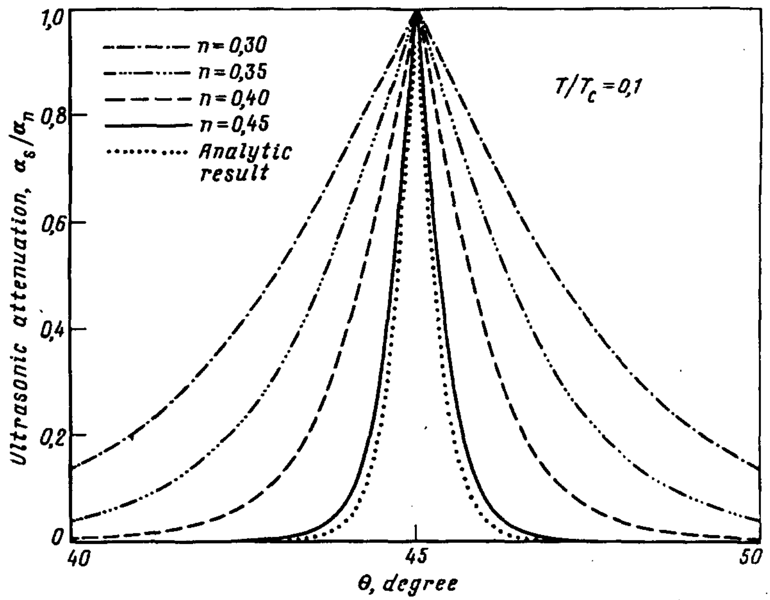

Fig.2. The dependence of longitudinal ultrasonic attenuation at $T/T_c = 0.1$ and different band-filling factors. The dotted line shows the analytic result at $n = 0.45$ (see Eq. (5)–(6))

parameter. Exactly at the nodes, $\alpha_s/\alpha_n = 1$, independent of T. This yields sharp spikes in the angular dependence for α_s/α_n, which become more pronounced with decreasing temperature. The analytic result for the case where the band-filling factor is close to one-half ($n \simeq 0.5 - 3\pi^{-2}\nu[1+\ln(2/\sqrt[3]{\nu})]$, with $\nu = |\mu|/4t \ll 1$) is a kind of universal scaling, so that all such small values of ν are described by the single scaling function

$$F(\chi) = \frac{\chi}{1+\sqrt{1+\chi^2}}\,, \tag{5}$$

which is plotted in Fig. 1 versus the scaling variable $\chi = \nu^{-1}|\cos 2\theta|$. In this case of the square lattice, θ is the angle between q and the y-axis. Thus, Eq. (2) becomes

$$\frac{\alpha_s(\theta)}{\alpha_n(\theta)} \simeq 2\Big[1+\exp\Big(\frac{\Delta_0(T)}{T}F(\chi)\Big)\Big]^{-1}\,, \tag{6}$$

It follows from Eqs. (5) and (6) that, in the low-temperature region ($T \ll T_c$), very sharp peaks in $\alpha_s(\theta)/\alpha_n(\theta)$ appear at $\theta = \pm 45°, \pm 135°$. The sharpness of the peaks is the result of the anisotropy of both the gap and the Fermi surface. The dotted curve in Fig. 2 illustrates the application of Eqs. (5) and (6) at the low temperature $T/T_c = 0.1$ for $n = 0.45$ (or $\nu = 0.065$). The solid curve is calculated numerically for the same filling without recourse to the small ν approximation. The closeness of the solid and dotted curves is an indication of the accuracy for $\nu = 0.065$, of this approximation and of the scaling function that follows from it. The numerical computations for other fillings are exhibited by the other three curves in Fig. 2. These curves clearly demonstrate the salient feature mentioned above, namely, that at low temperature, and even at fillings significantly far from half-filling, a sharp peak can be expected in α_s/α_n at the sonic propagation

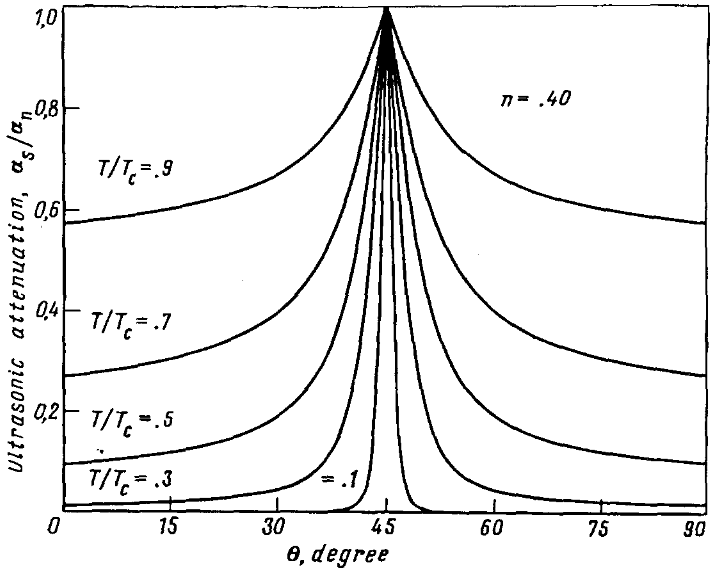

Fig.3. The angle dependence of the ratio of longitudinal ultrasonic attenuation in the superconducting state to the normal state value at n = 0.4 and different temperatures. Note the exponential decrease of ultrasonic attenuation with temperature far from nodes, as well as the closeness to one of the ratio α_s/α_n near the nodes

direction that corresponds to the nodes in the energy gap. Figure 3 shows the development of this peak, for $n = 0.4$, as the temperature is lowered.

To conclude, we note that effect of anisotropy of the order parameter upon ultrasonic attenuation was observed in various metals [6] and in superfluid ^{3}He [7]. Some recent progress has been made in ultrasonic measurements of single crystals of UPt_3 [8], where a three-dimensional d-wave order parameter is assumed. The earliest ultrasonic measurements on high-T_c crystals found a quite small velocity change at the superconducting transition temperature, which indicates that the influence of superconductivity on the acoustic processes is not yet fully understood[9]. Nevertheless, the sharp spike-like nature of the angular dependence of the longitudinal ultrasonic attenuation in the case of the possible d-wave order parameter should be observable and should provide a clear signature for the latter.

The authors are grateful to G.M.Eliashberg for discussions. This work is supported by NASA Grant NAG3-1395.

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