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RELATIVE PHASE BETWEEN THE $J/\psi \to \rho\eta$ AND $\omega\eta$ AMPLITUDES

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It is shown that the study of the $\omega - \rho^0$ interference pattern in the $J/\psi \to (\rho^0 + \omega)\eta \to$ $\to \pi^+\pi^-\eta$ decay provides evidence for the large (nearly 90°) relative phase between the one-photon and the three-gluon decay amplitudes.

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In the last few years it has been noted that the single-photon and three-gluon amplitudes in the two-body $J/\psi \to 1^-0^-$ and $J/\psi \to 0^-0^-$ [1-3] decays appear to have relative phases nearly 90°.

This unexpected result is very important to the observability of CP violating decays as well as to the nature of the $J/\psi \to 1^-0^-$ and $J/\psi \to 0^-0^-$ decays [1-6]. In particular, it points to a non-adequacy of their description built upon the perturbative QCD, the hypothesis of the factorization of short and long distances, and specified wave functions of final hadrons. Some peculiarities of electromagnetic form factors in the J/ψ mass region were discussed in Ref. [7].

The analysis [1-3] involved theoretical assumptions relying on the strong interaction $SU_f(3)$-symmetry, the strong interaction $SU_f(3)$-symmetry breaking and the $SU_f(3)$ transformation properties of the one-photon annihilation amplitudes. Besides, effects of the $\rho - \omega$ mixing in the $J/\psi \to 1^-0^-$ decays were not taken into account in Ref. [1] while in Ref. [2] the $\rho - \omega$ mixing was taken into account incorrectly, see the discussion below. Because of this, the model independent determination of these phases are required.

Fortunately, it is possible to check the conclusion of Refs. [1, 2] at least in one case. We mean the phases between the amplitudes of the one-photon $J/\psi \to \rho^0\eta$ and three-gluon $J/\psi \to \omega\eta$ decays.

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Indeed, the $\omega-\rho$ interference pattern in the $J/\psi \to (\rho^0+\omega)\eta \to \rho^0\eta \to \pi^+\pi^-\eta$ decay is conditioned by the $\rho^0-\omega$ mixing and the ratio of the amplitudes of the ρ^0 and ω production. As for the $\rho^0-\omega$ mixing amplitude, it is reasonably well studied [8–14]. Its module and phase are known. The module of the ratio of the amplitudes of the ρ and ω production can be obtained from the data on the branching ratios of the J/ψ-decays. So, the interference pattern provides a way of measuring the relative phases of the ρ^0 and ω production amplitudes.

The $\pi^+\pi^-$-spectrum in the ω, ρ energy region is of the form

$$\frac{dN}{dm} = N_\rho(m)\frac{2}{\pi}m\Gamma(\rho\to\pi\pi\,,\,m)\left|\frac{1}{D_\rho(m)} + \frac{\Pi_{\omega\rho^0}(m)}{D_\rho(m)D_\omega(m)}\left[\frac{N_\omega(m)}{N_\rho(m)}\right]^{1/2}\exp\left\{i\left(\delta_\omega-\delta_\rho\right)\right\}+\right.$$

$$\left.+\frac{1}{D_\omega(m)}\frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}}\left[\frac{N_\omega(m)}{N_\rho(m)}\right]^{1/2}\exp\left\{i\left(\delta_\omega-\delta_\rho\right)\right\}\right|^2, \tag{1}$$

where m is the invariant mass of the $\pi^+\pi^-$-state, $N_\rho(m)$ and $N_\omega(m)$ are the squares of the modules of the ρ and ω production amplitudes, δ_ρ and δ_ω are their phases, $\Pi_{\omega\rho^0}(m)$ is the amplitude of the $\rho-\omega$ transition, $D_V(m) = m_V^2 - m^2 - im\Gamma_V(m)$, $V=\rho,\ \omega$.

In the discussion that follows, Eq. (1) is conveniently rewritten as

$$\frac{dN}{dm} = N_\rho(m)\frac{2}{\pi}m\Gamma(\rho\to\pi\pi\,,\,m)\left|\frac{1}{D_\rho(m)}\left(1-\varepsilon(m)\left[\frac{N_\omega(m)}{N_\rho(m)}\right]^{1/2}\exp\left\{i\left(\delta_\omega-\delta_\rho\right)\right\}\right)+\right.$$

$$\left.+\frac{1}{D_\omega(m)}\left(\varepsilon(m)+g_{\omega\pi\pi}/g_{\rho\pi\pi}\right)\left[\frac{N_\omega(m)}{N_\rho(m)}\right]^{1/2}\exp\left\{i\left(\delta_\omega-\delta_\rho\right)\right\}\right|^2, \tag{2}$$

where

$$\varepsilon(m) = -\frac{\Pi_{\omega\rho^0}(m)}{m_\omega^2-m_\rho^2+im\left(\Gamma_\rho(m)-\Gamma_\omega(m)\right)}. \tag{3}$$

As known [8–12], the imaginary part of the $\rho-\omega$ transition amplitude is due to the $\pi\pi$, 3π, $\gamma\pi$ and $\gamma\eta$ intermediate states

$$Im\left(\Pi_{\rho^0\omega}(m)\right) = m\left(\frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}}\Gamma(\rho\to\pi\pi\,,\,m)+\frac{g_{\rho\rho\pi}}{g_{\omega\rho\pi}}\Gamma(\omega\to\rho\pi\to3\pi\,,\,m)+\right.$$

$$\left.+\frac{g_{\rho\gamma\pi}}{g_{\omega\gamma\pi}}\Gamma(\omega\to\gamma\pi\,,\,m_\omega)+\frac{g_{\rho\gamma\eta}}{g_{\omega\gamma\eta}}\Gamma(\omega\to\gamma\eta\,,\,m)\right). \tag{4}$$

The quite conservative estimate of the contribution of the $\pi\pi$ and 3π intermediate states gives

$$m_\omega\frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}}\Gamma(\rho\to\pi\pi\,,\,m_\omega) = \pm m_\omega\cdot10^{-2}\cdot\Gamma(\rho\to\pi\pi\,,\,m_\omega) = \pm1.17\cdot10^{-3}\ \text{GeV}^{-2}\,,$$

$$m_\omega\frac{g_{\rho\rho\pi}}{g_{\omega\rho\pi}}\Gamma(\omega\to\rho\pi\to3\pi\,,\,m_\omega) =$$

$$= \pm m_\omega \cdot 10^{-2} \cdot \Gamma(\omega \to 3\pi\,,\, m_\omega) = \pm 5.84 \cdot 10^{-5}\,\mathrm{GeV}^{-2}. \qquad (5)$$

The constituent quark and vector meson dominance models both give the same result

$$m_\omega \frac{g_{\rho\gamma\pi}}{g_{\omega\gamma\pi}} \Gamma(\omega \to \gamma\pi\,,\, m_\omega) = m_\omega \cdot \frac{1}{3} \cdot \Gamma(\omega \to \gamma\pi\,,\, m_\omega) = 1.86 \cdot 10^{-4}\ \mathrm{GeV}^{-2}\,,$$
$$m_\omega \frac{g_{\rho\gamma\eta}}{g_{\omega\gamma\eta}} \Gamma(\omega \to \gamma\eta\,,\, m_\omega) = m_\omega \cdot 3 \cdot \Gamma(\omega \to \gamma\eta\,,\, m_\omega) = 1.28 \cdot 10^{-5}\ \mathrm{GeV}^{-2}\,. \qquad (6)$$

Notice that the predictions of the constituent quark and vector meson dominance models on the $\omega \to \gamma\pi(\eta)$ and $\omega \to \gamma\pi(\eta)$ decays agree adequately with the experiment.

As is seen from Eqs. (3) and (4), the contribution of the $\pi\pi$ intermediate state in $\mathrm{Im}\left(\Pi_{\omega\rho^0}(m)\right)$ and the $g_{\omega\pi\pi}$ direct coupling constant cancel considerably in the $g^{eff}_{\omega\pi\pi}$ effective coupling constant:

$$g^{eff}_{\omega\pi\pi}(m) = \varepsilon(m) \cdot g_{\rho\pi\pi} + g_{\omega\pi\pi} = -\frac{\Pi'_{\omega\rho^0}(m) \cdot g_{\rho\pi\pi} + i\Gamma(\rho \to \pi\pi\,,\, m) \cdot g_{\omega\pi\pi}}{m_\omega^2 - m_\rho^2 + im\left(\Gamma_\rho(m) - \Gamma_\omega(m)\right)} + g_{\omega\pi\pi} =$$

$$= -\frac{\Pi'_{\omega\rho^0}(m) + \left(m_\rho^2 - m_\omega^2 + im\Gamma_\omega(m)\right)\left(g_{\omega\pi\pi}/g_{\rho\pi\pi}\right)}{m_\omega^2 - m_\rho^2 + im\left(\Gamma_\rho(m) - \Gamma_\omega(m)\right)} g_{\rho\pi\pi} =$$

$$= -\frac{\Pi'_{\omega\rho^0}(m) \mp 1.87 \cdot 10^{-4}\,\mathrm{GeV}^{-2} \pm i6.6 \cdot 10^{-5}\,\mathrm{GeV}^{-2}}{m_\omega^2 - m_\rho^2 + im\left(\Gamma_\rho(m) - \Gamma_\omega(m)\right)} g_{\rho\pi\pi}\,, \qquad (7)$$

where $\Pi'_{\omega\rho^0}(m)$ is the amplitude of the $\rho^0 - \omega$ transition without the contribution of the $\pi\pi$ intermediate state in in the imaginary part, the numerical values are calculated at $m = m_\omega$.

The branching ratio of the $\omega \to \pi\pi$ decay

$$B\left(\omega \to \pi\pi\right) = \frac{\Gamma\left(\rho \to \pi\pi\,,\, m_\omega\right)}{\Gamma_\omega(m_\omega)} \left|\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi}\right|^2. \qquad (8)$$

It follows from Eqs. (6) and (7) that imaginary part of the numerator in Eq. (7) is dominated by the $\gamma\pi$ intermediate state to within 35%. This imaginary part gives $B\left(\omega \to \pi\pi\right) \simeq 5 \cdot 10^{-5}$ instead of the experimental value [14]

$$B(\omega \to \pi^+\pi^-) = 0.0221 \pm 0.003\,. \qquad (9)$$

So, one can get the module of the real part of the numerator in Eq. (7) which is clearly dominated by $Re\left(\Pi_{\omega\rho^0}(m)\right)$. Besides, the interference pattern of the ρ^0 and ω mesons in the $e^+e^- \to \pi^+\pi^-$ reaction and in the $\pi^+\pi^-$ photoproduction on nuclei shows [8 – 12] that the real part of the numerator in Eq. (7) is positive. So, from Eqs. (3), (7), (8) and (9) one gets

$$\mathrm{Re}\left(\Pi_{\rho^0\omega}(m_\omega)\right) = (3.80 \pm 0.27) \cdot 10^{-3}\,\mathrm{GeV}^2 \qquad (10)$$

and

$$\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi} = (3.41 \pm 0.24) \cdot 10^{-2} \exp\left\{i\,(102 \pm 1)^\circ\right\}. \qquad (11)$$

The data [15, 16] were fitted with the function

$$N(m) = L(m) + \left|(N_\rho)^{1/2} F_\rho^{BW}(m) + (N_\omega)^{1/2} F_\omega^{BW}(m) \exp\{i\phi\}\right|^2 , \qquad (12)$$

where $F_\rho^{BW}(m)$ and $F_\omega^{BW}(m)$ are the appropriate Breit – Wigner terms [15] and $L(m)$ is a polynomial background term.

The results are

$$\phi = (46 \pm 15)^\circ , \quad N_\omega(m_\omega)/N_\rho = 8.86 \pm 1.83 \ \ [15] ,$$
$$\phi = -0.08 \pm 0.17 = (-4.58 \pm 9.74)^\circ , \quad N_\omega(m_\omega)/N_\rho = 7.37 \pm 1.72 \ \ [16] . \qquad (13)$$

From Eqs. (2), (8), and (12) follows

$$N_\rho = N_\rho(m_\rho) \left|1 - \varepsilon(m_\rho) \left[N_\omega(m_\rho)/N_\rho(m_\rho)\right]^{1/2} \exp\{i\,(\delta_\omega - \delta_\rho)\}\right|^2 , \qquad (14)$$

$$N_\omega = B(\omega \to \pi\pi) N_\omega(m_\omega) , \qquad (15)$$

$$\phi = \delta_\omega - \delta_\rho + \arg\left[\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi}\right] -$$
$$-\arg\left\{1 - \varepsilon(m_\rho) \left[N_\omega(m_\rho)/N_\rho(m_\rho)\right]^{1/2} \exp\{i\,(\delta_\omega - \delta_\rho)\}\right\} \simeq$$

$$\simeq \delta_\omega - \delta_\rho + \arg\left[\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi}\right] - \arg\left\{1 - |\varepsilon(m_\omega)| \left[N_\omega(m_\omega)/N_\rho\right]^{1/2} \exp\{i\phi\}\right\} . \qquad (16)$$

From Eqs. (11), (13) and (16) we get that

$$\delta_\rho - \delta_\omega = \delta_\gamma = (60 \pm 15)^\circ \ \ [15] , \qquad (17)$$
$$\delta_\rho - \delta_\omega = \delta_\gamma = (106 \pm 10)^\circ \ \ [16] . \qquad (18)$$

A large (nearly 90°) δ_γ was obtained in Ref. [1, 2]. So, both the MARK III Collaboration [15] and the DM2 Collaboration [16], see Eqs. (17) and (18), provide support for this view.

The DM2 Collaboration used statistics only half as high as the MARK III Collaboration, but, in contrast to the MARK III Collaboration, which fitted N_ω as a free parameter, the DM2 Collaboration calculated it from the branching ratio of $J/\psi \to \omega\eta$ using Eq. (15).

In of Ref. [2] the effect of the $\rho - \omega$ transition in the $J/\psi \to \omega\pi^0$ decay is neglected for it is assumed that this effect is less significant than in the $J/\psi \to \rho\eta$ one. Actually the effect of the $\rho - \omega$ transition on the $J/\psi \to \omega\pi^0$ decay intensity is

$$\bar{N}_\omega \simeq \bar{N}_\omega(m_\omega) \left|1 + \varepsilon(m_\omega) \left[\bar{N}_\rho(m_\omega)/\bar{N}_\omega(m_\omega)\right]^{1/2} \exp\{i\,(\bar{\delta}_\rho - \bar{\delta}_\omega)\}\right|^2 , \qquad (19)$$

where $\bar{N}_\omega(m_\omega)$ and $\bar{N}_\rho(m_\omega)$ are the squares of the modules of the $J/\psi \to \omega\pi^0$ and $J/\psi \to \rho\pi^0$ amplitudes, $\bar{\delta}_\omega$ and $\bar{\delta}_\rho$ are their phases. In Refs. [1, 2] it is suggested that $\delta_\rho - \delta_\omega = \bar{\delta}_\omega - \bar{\delta}_\rho = \delta_\gamma$. Experimentally [14] $\bar{N}_\rho(m_\omega)/\bar{N}_\omega(m_\omega) \simeq 10$. So, the corrections due to the $\rho - \omega$ transition in Eqs. (14) and (19) have the approximately equal modules. For $\delta_\gamma = 90^\circ$ they are approximately equal to 20% but opposite in sign. It is significant that $\bar{N}_\omega$ and N_ρ are measured with an accuracy of 14% and 12% respectively. In Ref. [2] it is mistaken that the ρ contribution had not been isolated from the $J/\psi \to (\rho+\omega)\eta \to \pi^+\pi^-\eta$ decay. An isolation is done there by some means and a magnitude of $N_\rho(m_\rho)$ is obtained which differs from the true one by three standard deviations.

In summary, we should emphasize that it is urgent to study this fundamental problem once again with BES in Beijing . Needless to say the τ-CHARM factory [14] would solve this problem in the exhaustive way.

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