Pis'ma v ZhETF, vol.71, iss.8, pp.496 - 501

PHASE DIAGRAM AND SKYRMION ENERGY FOR BILAYER HETEROSTRUCTURES AT INTEGER FILLING FACTORS

S.V.Iordanski[1], *A.Kashuba*

Landau Institute for Theoretical Physics RAS, 117334 Moscow, Russia

Submitted 30 March 2000

Phase diagram of a bilayer heterostructure at integer filling factors was established using the hidden symmetry method. Three phases: ferromagnetic, canted antiferromagnetic (CAP) and spin-singlet, have been found. We confirm early results of Das.Sarma et al. Each phase violates the $SU(4)$ hidden symmetry and is stabilized by anisotropy interactions. A charged excitation in bilayer: skyrmion, has been found and its anisotropic energy gap has been calculated. The gap has a prominent minimum in the CAP.

PACS: 64.60.Cu, 73.40.Hm, 75.10.-b

Integer filling factors ν of Landau Level in hetero-structure 2D electron gas (2DEG) are of special interest. Here the ground state is non-degenerate and the Hartree – Fock approximation (HFA) can be applied with the accuracy limited only by a small ratio of the Coulomb interaction energy to the energy of cyclotron resonance. The ground state of a single layer at $\nu = 1$ is a ferromagnet with the elementary excitations being spin excitons or spin-waves. These are gapless [1] in the exchange approximation and do not interact with each other for vanishieg momenta [2]. Both are consequences of the exact symmetry under spin rotation. In Ref.[3] a special spin texture in 2DEG ferromagnet: skyrmion [4], was predicted as an elementary charged excitation. The energy of neutral skyrmion-antiskyrmion pair is just a half of the spin exciton energy.

In a bilayer 2DEG the HFA applies only in two cases. The first one corresponds to well separated layers which is a common setup in the experiment [5, 6]. Here one starts from the two single layer ferromagnets and makes a perturbation expansion in powers of interlayer interactions [7]. The second one is the symmetric case where the bilayer Hamiltonian is invariant under $SU(4)$ rotations in both layer and spin spaces. Here all symmetry breaking fields like Zeeman must be negligible. The first attempt in this direction dealt with the case $\nu = 1$ and spin polarized electrons [8, 9]. Recent works [10, 11] that specialize to the bilayer case $\nu = 2$ employ the HFA and predict three distinct phases: ferromagnetic, CAP and a spin-singlet phase. Our approach is similar to that of Refs.[11] but we show explicitly that the HFA is exact in the $SU(4)$ symmetric bilayer. Anisotropy terms reduce the symmetry to $SU(2) \otimes SU(2)$ and lift the eigen state degeneracy. But there are no Fermi-liquid type renormalizations of the anisotropy Hamiltonian due to the symmetric Hamiltonian. We prove the stability of all phases with respect to long-range spatial perturbations. Our work was motivated by recent measurement of the diagonal conductivity activation energy in bilayer [6], which we identify with the energy gap of topological excitation: skyrmion, in an $SU(4)$ Sigma Model. Calculated in this letter skyrmion's energy gap has a profound minimum in the CAP, in line with the findings of Ref.[6].

[1] e-mail: iordansk@itp.ac.ru

The Hamiltonain of a 2DEG in a confining potential $V(\boldsymbol{\rho})$ and in an external magnetic field H reads:

$$H = \int \psi^+_\alpha(\boldsymbol{\rho}) \left(\frac{1}{2m} [-i\boldsymbol{\nabla} + \boldsymbol{A}(\boldsymbol{\rho})]^2 + V(\boldsymbol{\rho}) - |g|\mu_B H \sigma^z_{\alpha\beta} \right) \psi_\beta(\boldsymbol{\rho})\, d^3\boldsymbol{\rho} + \frac{1}{2} \int\int \frac{e^2}{\kappa|\boldsymbol{\rho} - \boldsymbol{\rho}'|} \psi^+_\alpha(\boldsymbol{\rho}) \psi^+_\beta(\boldsymbol{\rho}') \psi_\beta(\boldsymbol{\rho}') \psi_\alpha(\boldsymbol{\rho})\, d^3\boldsymbol{\rho}\, d^3\boldsymbol{\rho}', \tag{1}$$

where κ is the dielectric constant, $\alpha, \beta = \pm$ are spin indices and thereafter a sum over repeated indices is implied. We use units: $\hbar = 1$, $e = c$ and $H = B = 1$ and distances are expressed in terms of the magnetic length: $l_H = \sqrt{c\hbar/eH} = 1$. We split $\boldsymbol{\rho}$ into a coordinate ξ perpendicular to the layer and two in-plane coordinates $\boldsymbol{r} = (z, \bar{z})$ and assume the confining potential to be uniform over the plane: $V(\boldsymbol{\rho}) = V(\xi)$, with the two wells being separated by a distance d. Two eigen functions: the lowest energy symmetric and the lowest energy antisymmetric can be rotated into $\chi_{1,2}(\xi)$ eigen functions localized in one layer. We expand an electron operator in terms of these two eigen functions: $\psi_\alpha(\boldsymbol{\rho}) = \chi_\tau(\xi)\phi_p(\boldsymbol{r}) c_{\alpha\tau p}$, where $c^+_{\alpha\tau p}$ and $c_{\alpha\tau p}$ are electron creation and anhilation operators, $\phi_p(z\bar{z})$ is the lowest LL wave function number p, the index $\tau = 1, 2$ being the layer index. We assume the case of a sufficiently strong magnetic field with the cyclotron energy $1/m$ dominating over the Coulomb, Zeeman and the level splitting: $t = E_A - E_S$, energies.

The Coulomb interaction matrix can be projected onto $\chi_{1,2}(\xi)$:

$$V^{\mu\nu}(\boldsymbol{r} - \boldsymbol{r}') = \frac{e^2}{\kappa} \tau^\mu_{\tau_1\tau_4} \tau^\nu_{\tau_2\tau_3} \int\int \frac{\chi_{\tau_1}(\xi)\chi_{\tau_2}(\xi')\chi_{\tau_3}(\xi')\chi_{\tau_4}(\xi)}{\sqrt{(\xi - \xi')^2 + (\boldsymbol{r} - \boldsymbol{r}')^2}}\, d\xi d\xi'. \tag{2}$$

We use notations: τ^0 for the unit matrix, τ^x, τ^y and τ^z, for the Pauli matrices in the layer space and σ^x, σ^y and σ^z, for the Pauli matrices in the spin space. The Coulomb energy (2) is invariant under transformations: $\tau_1 \leftrightarrow \tau_4$, $\tau_2 \leftrightarrow \tau_3$ as well as $(\tau_1\tau_4) \leftrightarrow (\tau_2\tau_3)$. Hence, $V^{\mu\nu}$ is a 3×3 symmetric matrix with indices μ, ν running over a set $(0, z, x)$. If there is a symmetry of the Coulomb interaction under an exchange of layers: $(\xi, \xi') \leftrightarrow (-\xi, -\xi')$ and $1 \leftrightarrow 2$ then it restricts further values of the interaction matrix: $V^{0z} = 0$ and $V^{zx} = 0$. We note also, that $V^{0x} \sim t$, $V^{xx} \sim t^2$ and we neglect it thereafter, whereas $V^{zz} \sim d^2/|z|^3$ as $|z| \to \infty$.

We split the total Hamiltonian (1) into two parts. The first one is invariant under uniform rotations from the $SU(4)$ Lee group in the combined spin and layer space:

$$H^{sym} = \frac{1}{2m} c^+_{\alpha\tau p} c_{\alpha\tau p} + \frac{1}{2} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} V^{00}(\boldsymbol{q}) \exp\left(\frac{-\boldsymbol{q}^2}{4}\right) T^0(\boldsymbol{q}) T^0(-\boldsymbol{q}), \tag{3}$$

where (see e.g. [9])

$$T^\mu(\boldsymbol{q}) = \sum_p c^+_{\alpha\tau_1 p} \tau^\mu_{\tau_1\tau_2} c_{\alpha\tau_2 p - q_y} \exp\{-iq_x(p - \frac{q_y}{2})\}. \tag{4}$$

Its eigen levels are hugely degenerate. Given any eigen state $|\Psi\rangle_0$ a set of related eigen states can be generated by applying rotations: $|\Psi\rangle = U|\Psi\rangle_0$, where $U \in SU(4)$. For $\nu = 1, 2, 3$ we assume that the ground state is uniform over p-orbitals:

$$\Psi = \prod_{i=1}^{\nu} \prod_p c^+_{\alpha_i\tau_i p} |empty\rangle. \tag{5}$$

Wave-function (5) is an eigen function of the H^{sym} (3). The second anisotropy part of the Hamiltonian (1) is treated like a perturbation:

$$H^{anis} = -c^{+}_{\alpha\tau_1 p}\left(t\tau^{x}_{\tau_1\tau_2} + \mu^{z}\tau^{z}_{\tau_1\tau_2}\right)c_{\alpha\tau_2 p} - |g|\mu_B H\, c^{+}_{\alpha\tau p}\sigma^{z}_{\alpha\beta}c_{\beta\tau p} + \\ + \frac{1}{2}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}V^{\mu\nu}(\boldsymbol{q})\exp\left(\frac{-\boldsymbol{q}^2}{4}\right)T^{\mu}(\boldsymbol{q})T^{\nu}(-\boldsymbol{q}). \tag{6}$$

Here t is the hopping constant. The electrostatic potential μ^z, which is the difference between chemical potentials in the two layers, breaks down the symmetry between the two wells of $V(\xi)$ potential. We assume that the energy of a capacitor formed by the two layers is much lower than the characteristic Coulomb energy: $e^2/\kappa l_H$. The Coulomb energy constants are:

$$E^{\mu\nu} = \int \frac{dzd\bar{z}}{2\pi l_H^2}V^{\mu\nu}(|z|)\exp\frac{-|z|^2}{2} \approx \int \frac{dzd\bar{z}}{2\pi}V^{\mu\nu}(|z|), \tag{7}$$

where the last approximation holds for $(\mu\nu) \neq (00)$ in the limit $d \ll l_H$.

First, we specialize to the $SU(4)$-symmetric part of the bilayer 2DEG Hamiltonian (3). Weakly non-homogeneous state is generated by a rotation matrix: $c_{\alpha\tau} = U^{\alpha\beta}_{\tau\tau'}(t,\boldsymbol{r})c_{\beta\tau'}$, which adds a gauge matrix field: $\Omega_\mu = -iU^{+}\partial_\mu U$, in the kinetic energy. An effective low-energy Goldstone Action has an expansion in powers of Ω. We calculated this action following step in step the Ref.[12] for $\nu = 1,2,3$ at once:

$$H_G = \frac{E_1}{2}\int \frac{d^2\boldsymbol{r}}{2\pi}\left[\mathrm{tr}\left((1-N)\Omega_\mu N\Omega_\mu\right) + i\,\mathrm{sgn}(B^z)\epsilon_{\mu\nu}\mathrm{tr}\left(\Omega_\mu\Omega_\nu N\right)\right], \tag{8}$$

where $E_1 = E^{00}/2$. The insertion matrices in (8) are non-negative diagonal ones and they represent the occupation number for the electron states:

$$N = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array}\right), \tag{9}$$

where diagonal blocks are 1×1 and 3×3 in the case $\nu = 1,3$ and 2×2 in the case $\nu = 2$. It can be proven that $H_G \geq 0$.

The matrices N and $1-N$ are projector operators that allow only physical rotations into the Hamiltonian (8) which do change the ground state. The vector field Ω_μ can be expanded in the basis of fifteen generators of $SU(4)$ Lee group. We subdivide them into two complementary sets: the first one includes generators that do commute with N, constitute an algebra itself and we called it a stabilizer sub-group S; whereas the remaining physical rotations constitute a coset: $U(4)/U(\nu)\otimes U(4-\nu)$, with dimension eight in the case $\nu = 2$ and six in the case $\nu = 1,3$.

The Hamiltonian (8) is invariant under the time reversal symmetry, which can be chosen as a complex conjugate operator: $U \to U^*$. It follows that $\Omega_\mu \to -\Omega_\mu^T$. Thus, the time reversal changes the sign of the trace and the sign of the magnetic field B^z in the second term of (8).

The first term in H_G (8) is the gradient energy whereas the second term is proportional to the topological index of a non-homogeneous state:

$$\mathcal{Q} = \mathrm{sgn}(B^z)\int \mathrm{curl}\,\Omega^z\,\frac{d^2\boldsymbol{r}}{2\pi} = Z, \tag{10}$$

where Z is the set of integer numbers. The case $\mathcal{Q} = \pm 1$ corresponds to the simplest spin skyrmion in the first layer being rotated by a $SU(4)$ matrix to become a general bilayer skyrmion. The energy constant in H_G (8) coincide identically with that of the one-layer case [12], which means that the bilayer skyrmion energy is the same as found for one layer.

We introduce a local bilayer order parameter: $Q(\boldsymbol{r}) = U(\boldsymbol{r})NU^{+}(\boldsymbol{r})$, very much like magnetization in the theory of magnetism. Rotations from the denominator sub-group S leave Q intact. The total bilayer Hamiltonian in terms of this order parameter reads:

$$H_G = \frac{E_1}{4}\int \mathrm{tr}\,(\boldsymbol{\nabla} Q \boldsymbol{\nabla} Q)\,\frac{d^2\boldsymbol{r}}{2\pi} + \mathrm{sgn}(B^z)\frac{E_1}{2}\int \epsilon_{\mu\nu}\mathrm{tr}\,(Q\partial_\mu Q \partial_\nu Q)\,\frac{d^2\boldsymbol{r}}{2\pi}. \qquad (11)$$

In this representation the index selection rule (10) is a consequence of the homotopy group identity. Finally, we include the direct Coulomb energy of charge inside a skyrmion core:

$$H_{ee} = \frac{1}{2}\int\int d^2\boldsymbol{r}d^2\boldsymbol{r}'\,\frac{\mathrm{curl}\,\Omega^z(\boldsymbol{r})}{2\pi}\,\frac{e^2}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\frac{\mathrm{curl}\,\Omega^z(\boldsymbol{r}')}{2\pi}. \qquad (12)$$

We can cast the anisotropic part of the bilayer Hamiltonian (6) in terms of the order parameter matrix Q as well:

$$\begin{aligned} H^{anis}/\mathcal{N} = &-\left(t + (\nu-1)E^{0x}\right)\mathrm{tr}\,(Q\tau^x) - \left(\mu^z + (\nu-1)E^{0z}\right)\mathrm{tr}\,(Q\tau^z) - \\ &-|g|\mu_B H\,\mathrm{tr}\,(Q\sigma^z) + \frac{1}{2}E^{zz}\left[\mathrm{tr}\,(Q\tau^z)\,\mathrm{tr}\,(Q\tau^z) - \mathrm{tr}\,(Q\tau^z Q\tau^z)\right], \end{aligned} \qquad (13)$$

where $\mathcal{N}$ is the number of degeneracy of the LL, τ^μ acts on four-spinor as $\tau^\mu \otimes \sigma^0$. The Eqs.(11)–(13) defines the effective long-range Hamiltonian of a bilayer at integer ν.

The order parameter can be parameterized by six or eight angles in the case $\nu = 1, 3$ or $\nu = 2$. Actually, not every of those rotations corresponds to a physically distinct eigen state. The total bilayer energy is given by real diagonal matrix elements. One generates all real eigen states from a reference state by rotations from the $SO(4)$ sub-group of the $SU(4)$ group. This group has 6 parameters with two of them being from the stabilizer. Thus, only four global rotations do change the total bilayer energy. We start with the case $\nu = 2$ and we use a set of trial many electron wave functions parameterized by the three angles of relevant in our case rotations: $\theta_\pm$ and ϑ:

$$\prod_p U(\vartheta, -\vartheta)R(\theta_+, \theta_-)\; c^+_{+1p}c^+_{-2p}\,|empty\rangle, \qquad (14)$$

where $\pm$-spin components of electron are first rotated by angles $\theta_\pm$, in the layer space and, then, spins in the layer 1,2 are rotated by angles $\pm\vartheta$. Plugging $Q = URNR^{+}U^{+}$ into (13) we find:

$$\begin{aligned} E^{anis}/\mathcal{N} = &-E^{zz}\cos\theta_+\cos\theta_- - (t + E^{0x})\cos\vartheta(\sin\theta_+ + \sin\theta_-) - \\ &-(\mu^z + E^{0z})(\cos\theta_+ - \cos\theta_-) - |g|\mu_B H\sin\vartheta(\cos\theta_+ + \cos\theta_-). \end{aligned} \qquad (15)$$

The minimum of this energy corresponds to three phases: a) ferromagnetic at $\vartheta = \pi/2$, $\theta_+ = \theta_- = 0$; b) spin singlet at $\vartheta = 0$, $\theta_+ = \pi - \theta_- = \theta$; and c) CAP otherwise, as it is shown on Fig.1. A line of continuous phase transitions between the ferromagnetic phase and CAP is given by:

$$\left[(E^{zz} + |g|\mu_B H)^2 - \left(\mu^z + E^{0z}\right)^2\right]|g|\mu_B H = \left(t + E^{0x}\right)^2(E^{zz} + |g|\mu_B H). \qquad (16)$$

In the spin singlet phase the mixing phase θ, is determined by the equation:

$$\left(E^{zz}\sin\theta + t + E^{0x}\right)\cos\theta = \left(\mu^z + E^{0z}\right)\sin\theta. \tag{17}$$

A line of continuous phase transition between the spin singlet phase and CAP is given parametrically by the equation:

$$\left((t+E^{0x})\sin\theta - E^{zz} + \left(\mu^z + E^{0z}\right)\cos\theta\right)\left(t+E^{0x}\right) = \left(|g|\mu_B H\right)^2\sin\theta, \tag{18}$$

whith θ being determined from (17). In the case $\nu = 1, 3$ there is only one phase which is ferromagnetic in both the spin and the layer spaces.

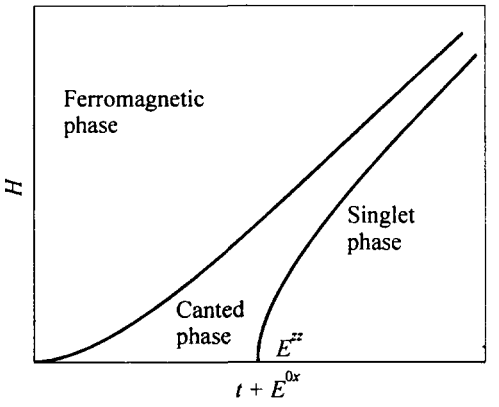

Fig.1. Phase Diagram in the $\nu = 2$ gate-symmetric case

Next, we find a skyrmion energy. Skyrmion's order parameter is given by the Belavin – Polyakov (BP) solution for $|\mathcal{Q}| = 1$ [4]:

$$Q_{\rm BP}(z\bar{z}) = \frac{R^2}{R^2+|z|^2}\begin{pmatrix} |z|^2 & zR \\ \bar{z}R & R^2 \end{pmatrix}, \tag{19}$$

with only one free parameter: the radius of a skyrmion core R. We omit inessential rotation angle between spin and orbital frames arbitrary in the absence of spin-orbit interaction. This Q has to be rotated by a homogeneous matrix RU (see above) in order to minimize the anisotropy energy away from the core. In addition, we allow all homogeneous rotations W from the stabilizer S that transform the BP skyrmion solution (19): $Q(\boldsymbol{r}) = RUWQ_{\rm BP}(z\bar{z})W^+U^+R^+$.

We retain only logarithmically divergent spatial integrals and we calculate the minimum of the skyrmion anisotropy energy over the seven free parameters of matrix W: $\mathcal{E}^{skyr}_{min}$, numerically. Then, we add the direct Coulomb energy (12) and minimize the total skyrmion energy with respect to R:

$$\Delta = \frac{\mathcal{Q}+|\mathcal{Q}|}{2}E_1 + 3\left(\mathcal{E}^{skyr}_{min}\left(\frac{3\pi^2e^2}{128\kappa l_H}\right)^2\log\frac{e^2}{\kappa l_H\mathcal{E}^{skyr}_{min}}\right)^{1/3}. \tag{20}$$

In the case of antiskyrmion $\mathcal{Q} = -|\mathcal{Q}|$, the gap consists only from a relatively small anisotropic energy. The total anisotropic skyrmion gap is shown on Fig.2 for the gate symmetric case $\mu^z + E^{0z} = 0$. Note, that the two cusp-like lines coincide with the two

phase transition lines on the Fig.1. A skyrmion in the ferromagnetic phase is a spin-skyrmion with spin rotations being localized in one layer, whereas a skyrmion in the spin-singlet phase is a layer-skyrmion with the electron density being rotated into the other layer.

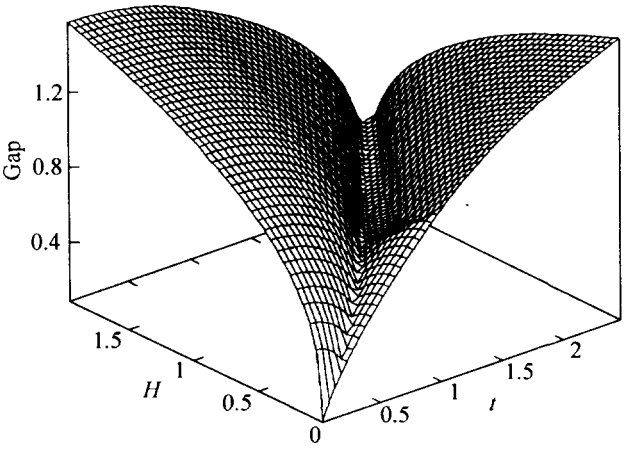

Fig.2. Total anisotropic anti-skyrmion gap energy in the $\nu = 2$ gate symmetric case

In the case $\nu = 1, 3$ we find the minimum of skyrmion's energy to be:

$$\Delta = \frac{\mathcal{Q} + |\mathcal{Q}|}{2} E_1 + \min\left(2\sqrt{t^2 + \mu_z^2}\,,\ 2|g|\mu_B H\right). \tag{21}$$

Valuable discussions with V.T.Dolgopolov and A.Maltsev are gratefully acknowledged. This work was supported by INTAS (grant #97-31980).

1. Yu.A.Bychkov, S.V.Iordanski, and G.M.Eliashberg, JETP Lett. **33**, 152 (1982).
2. I.V.Lerner and Yu.E.Lozovik, Sov. Phys. JETP **82**, 1188 (1982).
3. S.Sondhi, A.Kahlrede, S.Kivelson, and E.Rezai, Phys. Rev. **B47**, 16418 (1993).
4. A.A.Belavin and A.M.Polyakov, JETP Lett. **22**, 245 (1975).
5. V.Pellegrini et al., Phys. Rev. Lett. **78**, 310 (1997).
6. V.S.Khrapai et al., Phys. Rev. Lett. **84**, 729 (2000).
7. S.Das Sarma, S.Sachdev, and L.Zheng, Phys. Rev. Lett. **79**, 917 (1997).
8. M.Rasolt, B.Halperin, and D.Vanderbilt, Phys. Rev. Lett. **57**, 126 (1986).
9. Yu.A.Bychkov and S.V.Iordanski, Fizika Tverdogo Tela **2968**, 2442 (1987).
10. Zheng et al., Phys. Rev. Lett. **78**, 2453 (1997); S.DasSarma et al., Phys. Rev. **B58**, 4672 (1998); E.Demler and S.Das Sarma, Phys. Rev. Lett. **82**, 3895 (1999).
11. A.H.MacDonald, R.Rajaraman, and T.Jungwirth, Phys. Rev. **B60**, 8817 (1999).
12. S.V.Iordanski, S.G.Plyasunov, and V.I.Falko, Sov. Phys. JETP **115**, 716 (1999).