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SUPERSONIC WAVES OF MAGNETIZATION IN DARK INTERSTELLAR MOLECULAR CLOUDS

J.Yang[+*], *S.I.Bastrukov*[+*□]

[+]*Asia Pacific Center for Theoretical Physics*
Seoul 130-012, Korea

[*]*Center for High Energy Astrophysics and Isotope Studies and Department of Physics, Ewha Womans University*
Seoul 120-750, Korea

[□]*Joint Institute for Nuclear Research*
141980 Dubna, Russia

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We suggest that supersonic linewidths inferred from recent measurements of magnetic field toward core position in dark interstellar molecular clouds may be due to transverse waves of magnetization propagating in poorly ionized magnetically ordered gas-dust medium composed of tiny ferromagnetic dusty grains suspended in cold gaseous cloud of molecular hydrogen.

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The physical nature of supersonic OH linewidths detected in recent Zeeman measurements from dark molecular clouds [1 – 3] is the objective of intense current debate [4 – 7] (see also references therein), primarily around seminal suggestion of Arons and Max [8] to identify the supersonic velocity dispersion with the speed of transverse Alfvénic waves in gas-dust interstellar medium (ISM). While the assumptions underlying the hydromagnetic mechanism of wave motions ((i) the presence of regular magnetic field and (ii) the perfect conductivity of gas-dust ISM) leave a little doubt at the conditions typical of the giant molecular clouds and peripheral region of dark star-forming clouds, well-ionized by ultraviolet photons, this may be quite different in highly obscured inner region of dark molecular clouds where the ionizing UV is totally excluded [9]. In this letter we point out that an alternative mechanism of large-scale wave motions of ISM may also be worthy of consideration, particularly in connection with new measurements of magnetic field toward cores in magnetically supported dark interstellar clouds [1]. Specifically, we present arguments that the supersonic internal velocity dispersion inferred on the basis of recent data [1, 2] may be due to sub-Alfvénic transverse waves of magnetization travelling in poorly ionized gas-dust medium capable of sustaining, in the presence of regular magnetic field, the long-ranged magnetic ordering.

The physical motivation underlying our consideration is based on the well-known Jones and Spitzer arguments [10] regarding the existence of gas-dust interstellar medium with highly pronounced property of magnetic polarizability which can be thought of as a superparamagnetic dispersion of fine ferromagnetic grains suspended in gaseous cloud of molecular hydrogen. The regular galactic magnetic field threading such a medium introduces anisotropy in the orientation of permanently magnetized solid particles tending to align their magnetic moments. According to [10], the alignment of magnetic grains can be accompanied by filamentary agglomeration of dusty particles (presumably by means of dipole-dipole interaction of between magnetic moment of ferrograins) in the form of long-ranged magnetic chains extending along the direction of the regular magnetic field. Similar mechanism of linear, chain-like magnetic ordering is known in the physics of superparamagnetic ferrocolloidal suspensions placed in a uniform magnetic field, which is due to De Gennes and Pincus [11]. From standpoint of the condensed matter physics, the filamentary ordering of permanently magnetized dusty particles, in the presence of regular magnetic field, can be regarded as an effect of soft magnetic solidification of gas-dust matter imparting to gas-dust ISM the magnetoelastic properties generic to soft materials like uniformly magnetized ferronematic liquid crystals [12, 13] and magnetically saturated elastic insulators [14]. Therefore it is reasonable to expect that large-scale fluctuations of non-conducting magnetically polarized gas-dust ISM should manifest magnetomechanical behavior typical of above magnetoelastic materials.

Following this line of arguments we consider a model of magnetically supported cloud by identifying the two-component gas-dust intercloud medium with single-component superparamagnetic soft matter of equivalent density whose continuum mechanics is described in terms of the velocity of elastic displacements $\mathbf{u}(\mathbf{r},t)$, the bulk density $\rho(\mathbf{r},t)$, and the field of magnetization $\mathbf{m}(\mathbf{r},t)$ (magnetic moment per unit volume). These three quantities are considered on equal footing as independent dynamical variables of dissipative-free motions governed by coupled equations

$$\frac{d\rho}{dt}+\rho\nabla\cdot\mathbf{u}=0,\qquad \rho\frac{d\mathbf{u}}{dt}=\frac{1}{2}\nabla\times[\mathbf{m}\times\mathbf{B}],\qquad \frac{d\mathbf{m}}{dt}=[\boldsymbol{\omega}\times\mathbf{m}],\tag{1}$$

which have been derived in [14] by means of systematic application of the conservation laws of the continuum physics. In (1) $d/dt=\partial/\partial t+\mathbf{u}\cdot\nabla$ is the convective derivative and $\boldsymbol{\omega}(\mathbf{r},t)=(1/2)[\nabla\times\mathbf{u}(\mathbf{r},t)]$ stands for the vorticity. Notice that for both paramagnets and superparamagnets the linear constitutive equation holds [15]: $\mathbf{M}=\chi\mathbf{B}$, where χ is the magnetic susceptibility, essentially positive dimensionless constant. The difference is that for superparamagnets this parameter is 5-6 orders of magnitude greater than that for normal paramagnets, that is, for superparamagnets $\chi\sim 0.1-1$. The most important point to be stressed regarding the governing equations of magnetoelastodynamics (1) is that the bulk force originates from interaction between the magnetic field and the field of magnetization which is not a direct, but is mediated by rotational deformations of elastic medium resulting in precession motions of the magnetization under which the direction of $\mathbf{m}$ changes but the magnitude does not. To see that the magnetoelastic bulk force (inherently related with the body-torque density $\mathbf{m} \times \mathbf{B}$) provides stable oscillatory behavior of magnetically polarized gas-dust medium, we consider long-wavelength, non-radial oscillations of a spherical uniformly magnetized cloud. From electrodynamics of continuous media [16] it is known that in a homogeneous spherical mass of paramagnetic matter with constant magnetization $\mathbf{M}$ inside, the internal magnetic field is uniform and is expressed by the equations $\mathbf{B} + 2\mathbf{H} = 0$ and $\mathbf{B} - \mathbf{H} = 4\pi\mathbf{M}$, from which it follows

$$\mathbf{B} = \frac{8\pi}{3}\mathbf{M}. \tag{2}$$

With above reservations in mind, it would be not inconsistent to consider (2) as a constitutive equation of superparamagnetic continuum with $\chi = 3/8\pi \approx 0.1$. The advantage of this model is that it allows one to avoid uncertainty in the magnitude of χ.

Making use of the standard procedure of linearization: $\mathbf{u} \to \mathbf{u}_0 + \delta\mathbf{u}(\mathbf{r}, t)$, and $\mathbf{m} \to \mathbf{m}_0 + \delta\mathbf{m}(\mathbf{r}, t)$, where $\mathbf{u}_0 = 0$ and $\mathbf{m}_0 = \mathbf{M}$, we arrive at equations

$$\nabla \cdot \delta\mathbf{u}(\mathbf{r}, t) = 0, \qquad \nabla \cdot \delta\mathbf{m}(\mathbf{r}, t) = 0, \tag{3}$$

$$\rho\,\frac{\partial \delta\mathbf{u}(\mathbf{r}, t)}{\partial t} = \frac{4\pi}{3}\nabla \times [\delta\mathbf{m}(\mathbf{r}, t) \times \mathbf{M}], \tag{4}$$

$$\frac{\partial \delta\mathbf{m}(\mathbf{r}, t)}{\partial t} = \frac{1}{2}[[\nabla \times \delta\mathbf{u}(\mathbf{r}, t)] \times \mathbf{M}], \tag{5}$$

describing small-amplitude fluctuations of magnetically saturated incompressible elastic continuum which are not accompanied by appearance of density of magnetic poles (right of equations (3)). In (4) we have used (2). The period of magnetoelastic oscillations can be evaluated on the basis of the energy variational principle which is particularly efficient in study of long-wavelength non-radial vibrations of spherical masses of incompressible stellar material possessing elastic properties. In particular, this method has been utilized in [17, 18] to compute periods of Alfvén hydromagnetic vibrations of neutron stars and in [19, 20] the periods of their non-radial gravitational pulsations. The procedure is the following. Scalar multiplication of (4) with $\delta\mathbf{u}$ and integration over the cloud volume lead to equation of energy balance

$$\frac{\partial}{\partial t}\int \frac{\rho\delta\mathbf{u}^2}{2}\,dV = \frac{8\pi}{3}\int [\delta\mathbf{m} \times \mathbf{M}] \cdot \delta\boldsymbol{\omega}\,dV = -\frac{8\pi}{3}\int \delta\mathbf{m}^2\,dV. \tag{6}$$

The surface integral has been omitted, since in the outer region the superparamagnetic state should be, most likely, disordered by ambient ionizing ultraviolet radiation. The next step is to represent the velocity of elastic displacements in the following separable form

$$\delta\mathbf{u}(\mathbf{r}, t) = \boldsymbol{\xi}(\mathbf{r})\dot{\alpha}(t), \qquad \delta\boldsymbol{\omega}(\mathbf{r}, t) = \frac{1}{2}[\nabla \times \boldsymbol{\xi}(\mathbf{r})]\,\dot{\alpha}(t), \tag{7}$$

where $\boldsymbol{\xi}(\mathbf{r})$ is the field of instantaneous elastic displacements and $\alpha(t)$ is the harmonic in time amplitude. The expression for fluctuating vorticity in (7) is a consequence of separable representation of the fluctuating field of velocity. Inserting (7) into (5) and eliminating time derivative we obtain

$$\delta\mathbf{m}(\mathbf{r}, t) = \frac{1}{2}[[\nabla \times \boldsymbol{\xi}(\mathbf{r})] \times \mathbf{M}]\,\alpha(t). \tag{8}$$

After substitution (7) and (8) into (6), this latter is reduced to equation of normal vibrations

$$\frac{dH}{dt}=0, \qquad H=\frac{M\dot{\alpha}^2}{2}+\frac{K\alpha^2}{2} \quad \rightarrow \quad M\ddot{\alpha}+K\alpha=0, \tag{9}$$

with the inertia M and the stiffness K given by

$$M=\int \rho\,\boldsymbol{\xi}^2\,dV, \qquad K=\frac{2\pi}{3}\int [[\nabla\times\boldsymbol{\xi}]\times\mathbf{M}]^2\,dV. \tag{10}$$

Thus, to compute the frequency, $\omega^2=K/M$, it is necessary to specify the field of instantaneous displacements $\boldsymbol{\xi}$ which is, as it follows from above, of an essentially rotational character. With this in mind, we consider torsional long-wavelength vibrations around the polar axis z of a spherical cloud with the constant magnetization inside pointing to the same direction: $\mathbf{M}=[M_x=0, M_y=0, M_z=M]$. The kinematics of elastic torsional deformations of a spherical mass is described in details in [19, 20], and we take advantage of the explicit form for the velocity given in these latter papers

$$\delta\mathbf{u}(\mathbf{r},t)=\frac{1}{2}[\delta\boldsymbol{\omega}(\mathbf{r},t)\times\mathbf{r}], \quad \delta\boldsymbol{\omega}(\mathbf{r},t)=N_t\nabla r^L\,P_L(\mu)\dot{\alpha}(t), \quad N_t=\frac{1}{R^{L-1}}, \quad \mu=\cos\theta. \tag{11}$$

Hereafter $P_L(\mu)$ stands for the Legendre polynomial of the multipole degree L. The corresponding field of instantaneous torsional displacements has the form of the toroidal vector field

$$\boldsymbol{\xi}(\mathbf{r})=N_t\nabla\times[\mathbf{r}\,r^L\,P_L(\mu)]. \tag{12}$$

Inserting (12) in (10) we obtain

$$M=4\pi\rho R^5\frac{L(L+1)}{(2L+1)(2L+3)}, \qquad K=\frac{8\pi^2}{3}M^2R^3\frac{L(L-1)(L+1)^2}{(4L^2-1)}, \tag{13}$$

and the eigenfrequency is given by

$$\omega^2=\omega_M^2\,(L^2-1)\frac{(2L+3)}{(2L-1)}, \qquad \omega_M^2=\frac{2\pi}{3}\frac{M^2}{\rho R^2}=\frac{3}{32\pi}\frac{B^2}{\rho R^2}, \tag{14}$$

where ω_M is the natural unit of frequency of torsional magnetomechanical vibrations, so that corresponding period is evaluated according to $t_M=2\pi/\omega_M$.

Let us consider propagation of plane-wave magnetoelastic perturbations in the cloud bulk. Substitution into equations (3)–(5) the plane-wave form of fluctuating variables

$$\delta\mathbf{u}=\mathbf{u}'\exp(i\omega t-i\mathbf{k}\cdot\mathbf{r}), \quad \delta\mathbf{m}=\mathbf{m}'\exp(i\omega t-i\mathbf{k}\cdot\mathbf{r}), \tag{15}$$

with $\mathbf{u}'$ and $\mathbf{m}'$ being some small constant vectors, after some algebra leads to

$$\omega\rho\delta\mathbf{u}+\frac{4\pi}{3}(\mathbf{k}\cdot\mathbf{M})\delta\mathbf{m}=0, \qquad \omega\delta\mathbf{m}+\frac{1}{2}(\mathbf{k}\cdot\mathbf{M})\,\delta\mathbf{u}=0. \tag{16}$$

The method of obtaining of these equations is very much similar to that utilized in [12] to derive dispersion relationship of magnetotorsion waves in uniformly magnetized liquid crystals (see also [13]). By eliminating $(\mathbf{k}\cdot\mathbf{M})$ from equations (16), one finds that magnetoelastic oscillatory motions satisfy the principle of energy equipartition

$$\frac{\rho\,\delta\mathbf{u}^2}{2}=\frac{4\pi}{3}\delta\mathbf{m}^2, \tag{17}$$

which states that in the magnetoelastic wave the kinetic energy of fluctuating elastic displacements equals to the mean potential energy of fluctuating magnetization. From (16) it follows

$$\omega^2 = \frac{(\mathbf{k}\cdot\mathbf{M})^2}{4\chi\rho} = v_M^2\, k^2 \cos^2\theta, \qquad v_M^2 = \frac{2\pi}{3}\frac{M^2}{\rho} = \frac{MB}{4\rho} = \frac{3}{32\pi}\frac{B^2}{\rho}, \tag{18}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{M}$. The wave is transmitted most efficiently when $\mathbf{k} \parallel \mathbf{M}$. The dispersion relation (18) describes transverse wave of magnetization in which the vectors of magnetization and velocity undergo coupled oscillations in the plane perpendicular to the axis of magnetic anisotropy directed along $\mathbf{M}$. Both directions $\mathbf{M}$ and $-\mathbf{M}$ are energetically equivalent for this wave. On the other hand, oscillatory motions in magnetoelastic wave bears strong resemblence to that for the oscillations of incompressible flow in perfectly conducting fluid transmitting Alfvén wave in the presence of uniform magnetic field. Thus, the magnetoelastodynamics provides consistent mathematical treatment and physical insight into the nature of waves capable of propagating in the magnetically saturated non-conducting ISM which can be regarded as a counterpart of Alfvén's waves in magnetoactive plasma. However, the very existence of hydromagnetic waves is attributed to the perfect conductivity of cosmic dusty plasma, whereas the considered magnetoelastic waves owe their existence to the magnetic polarizability of non-conducting interstellar medium.

To show that presented model can provide proper account of the recent 43 m Green Bank telescope data, reported by Crutcher [1], first we notice that from analytic estimate for v_M, equation (18), it follows that, at equal B and ρ, the considered wave motions are sub-Alfvénic: $v_M \approx 0.6\, v_A$, where $v_A = B/(4\pi\rho)^{1/2}$ is the speed of Alfvén's wave. This prediction is in line with data summarized in [2]. Taking the bulk density $\rho = n\,\mu_{H_2} \approx 3.9\times 10^{-21}\,\mathrm{g/cm^3}$ (where $n = 10^3\,\mathrm{cm^{-3}}$ and μ_{H_2} is the mass of the hydrogen molecule [1]) and the magnetic field $B = 10\,\mu$G [1], one finds that the speed of wave of magnetization $v_M \approx 0.28$ km/sec, that is, exceeds the isothermal sound speed $c_s = (k_B T/\mu_{H_2})^{1/2} \approx 0.19$ km/sec, at the average intercloud temperature $T \approx 10$ K [1]. So, under conditions typical of inner region of dark star-forming molecular clouds, the waves of magnetization are, most likely, supersonic. The characteristic time (period) of long-wavelength oscillations of magnetic cloud is estimated (in seconds) to be $t_M = 2\pi/\omega_M = 2\pi\, R/v_M = 22.4\cdot 10^{-5}\, R$, where R is the cloud radius. The fact that above predictions are not inconsistent with available data (Crutcher et al., 1996, Crutcher, 1999) suggests that supersonic motions observed toward core of dark molecular clouds, poorly ionized by ultraviolet photons, may be due to considered here wave motions.

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