ПИСЬМА
В ЖУРНАЛ ЭКСПЕРИМЕНТАЛЬНОЙ
И ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОСНОВАН В 1965 ГОДУ *ТОМ 70, ВЫПУСК 1*
ВЫХОДИТ 24 РАЗА В ГОД *10 ИЮЛЯ, 1999*

Pis'ma v ZhETF, vol.70, iss.1, pp.3 - 6 © 1999 July 10

ON INDUCED *CPT*-ODD CHERN-SIMONS TERMS IN 3+1 EFFECTIVE ACTION

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Submitted 31 May 1999

This paper was originally designated as Comment to the paper by R.Jackiw and V.A.Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999) [1]. We provide an example of the fermionic system, the superfluid ^{3}He-A, in which the CPT-odd Chern-Simons terms in the effective action are unambiguously induced by chiral fermions. In this system the Lorentz and gauge invariances both are violated at high energy, but the behavior of the system beyond the cut-off is known. This allows us to construct the CPT-odd action, which combines the conventional 3+1 CS term and the mixed axial-gravitational CS term discussed by G.E.Volovik and A.Vilenkin, hep-ph/9905460 [2]. The influence of CS term on the dynamics of the effective gauge field has been experimentally observed in rotating ^{3}He-A.

PACS: 11.15.-q, 11.30.Er, 67.57.-z, 98.80.Cq

Recently the problem of the radiatively induced CPT-odd Chern-Simons (CS) term in 3+1 quantum field theory has been addressed in a number of papers [1, 3 – 7]. The CS term $L_{\rm CS} = \frac{1}{2}k_\mu e^{\mu\alpha\beta\gamma}F_{\alpha\beta}A_\gamma$ in the 3+1 electromagnetic action, where k^μ is a constant 4-vector, is induced by the CPT- and Lorentz-violating axial-vector term $b^\mu\gamma_\mu\gamma_5$ in the Dirac Lagrangian for massive fermions. In the limit of small and large b compared with mass m of Dirac fermions, it was found that

$$k^\mu = \frac{3}{16\pi^2}b^\mu \quad , \quad b \ll m \; , \tag{1}$$

$$k^\mu = -\frac{1}{16\pi^2}b^\mu \quad , \quad b \gg m \; . \tag{2}$$

However it has been concluded that the existence of CS term depends on the choice of regularization procedure – "a renormalization ambiguity". This means that the result for k_μ depends on physics beyond the cut-off.

The above CPT-odd term can result not only from the violation of the CPT symmetry in the vacuum. The nonzero density of the chiral fermions violates the CPT invariance and thus can also lead to the CS term, with b^0 being determined by the chemical potential μ and temperature T of the fermionic system [8 – 10].

Here we provide an example of the fermionic system, in which such CS term is unambiguously induced by fermions. In this system the Lorentz and gauge invariances both are violated at high energy, but the behavior of the system beyond the cut-off is known. This allows the calculation of CS term in different physical situations. The influence of this CS term on the dynamics of the effective gauge field has been experimentally observed.

This is the superfluid ^{3}He-A, where in the low-energy corner there are two species of fermionic quasiparticles: left-handed and right-handed Weyl fermions [11]. Quasiparticles interact with the order parameter, the unit $\hat{\mathbf{l}}$ – vector of the orbital momentum of Cooper pairs, in the same manner as chiral relativistic fermions interact with the vector potential of the $U(1)$ gauge field: $\mathbf{A} \equiv p_F\hat{\mathbf{l}}$, where p_F is the Fermi momentum. The "electric charges" – the charges of the left and right quasiparticles with respect to this effective gauge field – are $e_R = -e_L = -1$. The normal component of superfluid ^{3}He-A consists of thermal fermions, whose density is determined by T and by the velocity $\mathbf{v}_n - \mathbf{v}_s$ of the flow of the normal component with respect to the superfluid vacuum. The velocity of the counterflow in the direction of $\hat{\mathbf{l}}$ is equivalent to the chemical potentials for left and right fermions in relativistic systems:

$$\mu_R = -\mu_L = p_F\hat{\mathbf{l}} \cdot (\mathbf{v}_n - \mathbf{v}_s) \ . \tag{3}$$

As in the relativistic theories, the state of the system of chiral quasiparticles with nonzero counterflow velocity (an analogue of chemical potential) violates Lorentz invariance and CPT symmetry and induces the CPT-odd CS term. This term can be written in general form, applicable both for the relativistic systems where it was found in Ref.[8,9] and for ^{3}He-A [11]:

$$\frac{1}{4\pi^2}\left(\sum_L \mu_L e_L^2 - \sum_R \mu_R e_R^2\right) \mathbf{A} \cdot (\nabla \times \mathbf{A}) \ . \tag{4}$$

Here sums over L and R mean summation over all the left-handed and right-handed fermionic species respectively; e_L and e_R are charges of left and right fermions with respect to $U(1)$ field (say, hypercharge field in the Standard model).

Translation of Eq.(4) to the ^{3}He-A language gives

$$\frac{p_F^3}{2\pi^2}\left(\hat{\mathbf{l}}_0 \cdot (\mathbf{v}_s - \mathbf{v}_n)\right)\left(\delta\hat{\mathbf{l}} \cdot (\nabla \times \delta\hat{\mathbf{l}})\right) \ . \tag{5}$$

Here $\hat{\mathbf{l}}_0$ is the direction of the order parameter $\hat{\mathbf{l}}$ in the homogeneous ground state; and $\delta\hat{\mathbf{l}} = \hat{\mathbf{l}} - \hat{\mathbf{l}}_0$ is the deviation of the order parameter from its ground state direction.

Since for chiral fermions the chemical potential plays the part of the parameter b^0 in the fermionic Lagrangian, the connection between k^0 and b^0 is $k^0 = b^0/2\pi^2$ in ^{3}He-A. Though it agrees with the result obtained in relativistic system with nonzero chemical potential for chiral fermions [8], it does not coincide with Eq.(2) obtained in the massless limit $m/b^0 \to 0$.

The instability of the electromagnetic vacuum due to the 3+1 CS term has been discussed by Caroll, Field and Jackiw [12], Andrianov and Soldati [13], and Joyce and

Shaposhnikov [10]. In the case of the nonzero density of right electrons ($\mu_R \neq 0$) this instability leads to the conversion of the density of the right electrons to the hypermagnetic field. This effect was used in the scenario for nucleation of the primordial magnetic field [10]. In ^{3}He-A this phenomenon is represented by the well known helical instability of the counterflow, which is triggered by the term in Eq.(5) [14]. The conversion of the counterflow of the normal component (an analogue of μ_R in the Joyce–Shaposhnikov scenario) to the inhomogeneous $\hat{\mathbf{l}}$-field with $\nabla \times \hat{\mathbf{l}} \neq 0$ (an analogue of hypermagnetic field) due to this instability has been observed in rotating ^{3}He-A [15, 11].

Recently another type of the CS term has been found for both systems, ^{3}He-A and chiral relativistic fermions with nonzero μ or/and T. This is the mixed axial-gravitational CS term, which contains both the gauge field and the gravimagnetic field [2]:

$$\frac{1}{8\pi^2}\left(\sum_L \mu_L^2 e_L - \sum_R \mu_R^2 e_R\right) \mathbf{A}\cdot\mathbf{B_g} \ , \quad \mathbf{B_g} = \boldsymbol{\nabla}\times\mathbf{g} \ , \ \mathbf{g}\equiv g_{0i} \ . \tag{6}$$

Here g_{0i} is the element of the metric in the reference frame of the heat bath (in superfluids it is the element of the effective metric in the frame, in which the normal component is at rest). If the heat bath of chiral fermions is rotating in Minkowski space, the "gravimagnetic field" is expressed in terms of rotation velocity $\mathbf{\Omega}$:

$$\mathbf{B_g} = \boldsymbol{\nabla}\times\mathbf{g} = 2\frac{\mathbf{\Omega}}{c^2}. \tag{7}$$

Here c is the material parameter, which is the speed of light in relativistic system, and the initial slope in the energy spectrum of fermionic quasiparticles propagating in the plane transverse to the $\hat{\mathbf{l}}$-vector in ^{3}He-A [11]. The material parameters do not enter Eq.(6) explicitly: they enter only through the metric. That is why the same equation Eq.(6) can be applied to different fermionic systems, including those with varying speed of light. In relativistic system this equation describes the macroscopic parity violating effect: rotation of the heat bath (or of the black hole) produces the flux of the chiral fermions along the rotation axis [16].

Comparison of the Eqs.(6) and (4) suggests that the two CPT-odd terms can be unified if one uses the Larmor theorem and introduces the combined fields:

$$\mathbf{A}_{L(R)} = e_{L(R)}\mathbf{A} + \frac{1}{2}\mu_{L(R)}\mathbf{g} \ , \quad \mathbf{B}_{L(R)} = \boldsymbol{\nabla}\times\mathbf{A}_{L(R)} \ . \tag{8}$$

Then the general form of the CS CPT-odd term is

$$\frac{1}{4\pi^2}\left(\sum_L \mu_L \mathbf{A}_L\cdot\mathbf{B}_L - \sum_R \mu_R \mathbf{A}_R\cdot\mathbf{B}_R\right) \ . \tag{9}$$

Note that in the Standard Model the nullification of the CPT-odd term in Eq.(9) occurs if the "gyromagnetic" ratio e/μ is the same for all fermions. This happens because of the anomaly cancellation. For the CPT-odd term induced by the vacuum fermions, the anomaly cancellation was discussed in Refs.[7, 3]. In ^{3}He-A the "gyromagnetic ratio" is the same for two fermionic species, $e_L/\mu_L = e_R/\mu_R$, but the CS terms survive, since there is no anomaly cancellation in this system.

In ^{3}He-A there are also subtle points related to gauge invariance of the CS term, as discussed by Coleman and Glashow [17], and to the reference frame. They are determined by physical situations.

(i) The reference frame for superfluid velocity $\mathbf{v}_s$ is the heat bath frame – the frame of the normal component moving with velocity $\mathbf{v}_n$. At $T = 0$ this frame disappears: thermal fermions are frozen out. To avoid uncertainty in determination of the counterflow velocity $\mathbf{v}_s - \mathbf{v}_n$, and thus of the chemical potential of the chiral fermions, the limit $T \to 0$ must be taken after all other limits.

(ii) The leading terms in the low-energy effective action for the "electrodynamics" of ^{3}He-A are gauge invariant, because the main contributions to the effective action are induced by the low-energy fermions, which are "relativistic" and obey the gauge invariant Lagrangian. The Eq.(9) is an example of such gauge invariant term in the low-energy action. It is gauge invariant if the b^0 parameter (or μ_R) is constant, i.e. if the background counterflow and $\hat{\mathbf{l}}_0$ field are homogeneous. The inhomogeneous corrections, which correspond to the inhomogeneous b^0, violate the gauge invariance. This is natural, since these corrections are determined by the higher energy fermions, which do not obey the gauge invariance from the very beginning. This is in agreement with the conclusion made in Ref.[1], that for existence of the CS term the "weak condition" – the gauge invariance at zero 4-momentum – is required.

I thank Alex Vilenkin for discussions. This work was supported in part by the Russian Foundations for Fundamental Research grant 96-02-16072 and by European Science Foundation.

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