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QUASI-LONG-RANGE ORDER IN THE RANDOM ANISOTROPY HEISENBERG MODEL

D.E.Feldman[1)]

Landau Institute for Theoretical Physics RAS
142432 Chernogolovka, Moscow region, Russia

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The random field and random anisotropy N-vector models are studied with the functional renormalization group in $4-\epsilon$ dimensions. The random anisotropy Heisenberg ($N=3$) model has a phase with the infinite correlation radius at low temperatures and weak disorder. The correlation function of the magnetization obeys a power law $\langle \mathbf{m}(\mathbf{r}_1)\mathbf{m}(\mathbf{r}_2)\rangle \sim |\mathbf{r}_1-\mathbf{r}_2|^{-0.62\epsilon}$. The magnetic susceptibility diverges at low fields as $\chi \sim H^{-1+0.15\epsilon}$. In the random field N-vector model the correlation radius is finite at the arbitrarily weak disorder for any $N>3$.

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The effect of impurities on the order in condensed matter is interesting since the disorder is almost inevitably present in any system. If the disorder is weak the short range order is the same as in the pure system. However, the large distance behavior can be strongly modified by the arbitrarily weak disorder. This happens in the systems of continuous symmetry in presence of the random symmetry breaking field [1]. The first experimental example of this kind is the amorphous magnet [2]. During the last decade a lot of other related objects were found. These are liquid crystals in the porous media [3], nematic elastomers [4], He-3 in aerogel [5] and vortex phases of impure superconductors [6]. The nature of the low-temperature phases of these systems is still unclear. The only reliable statement is that a long range order is absent [1, 7, 8]. However, other details of the large distance behavior are poorly understood.

The neutron scattering [9] reveals sharp Bragg peaks in impure superconductors at low temperatures and weak external magnetic fields. Since the vortices can not form a regular lattice [7] it is tempting to assume that there is a quasi-long-range order (QLRO), that is the correlation radius is infinite and correlation functions depend on the distance slow. Recent theoretical [10] and numerical [11] studies of the random field XY model, which is the simplest model of the vortex system in the impure superconductor [6], support this picture. The theoretical advances [10] are afforded by two new technical approaches: the functional renormalization group [12] and the replica variational method [13]. These methods are free from drawbacks of the standard renormalization group and give reasonable results. The variational method regards a possibility of spontaneous replica symmetry breaking and treats the fluctuations approximately. On the other hand the functional renormalization group provides a subtle analysis of the fluctuations about the replica symmetrical ground state. Surprisingly, the methods suggest close and sometimes even the same results.

Both techniques were originally suggested for the random manifolds [12, 13] and then allowed to obtain information about some other disordered systems with the abelian symmetry [10,14–16]. It is less known about the non-abelian systems. The simplest of them

1) e-mail: feldman@itp.ac.ru

are the random field (RF) [1] and random anisotropy (RA) [2] Heisenberg models. The latter was introduced as a model of the amorphous magnet [2]. In spite of a long discussion the question about QLRO in these models is still open. There is an experimental evidence in favor of no QLRO [17]. On the other hand recent numerical simulations [18] support the possibility of QLRO in these systems. The only theoretical approach, developed up to now, is based on the spherical approximation [19] and predicts absence of QLRO at $N \gg 1$ magnetization components. However, there is no reason for this approximation to be valid at $N \sim 1$.

In this letter we study the RF $O(N > 3)$ and RA $O(N > 2)$ models in $4-\epsilon$ dimensions with the functional renormalization group. The large distance behaviors of the systems are found to be quite different. While in the RF $O(N)$ model with $N > 3$ the correlation radius is always finite, the RA Heisenberg ($N = 3$) model has a phase with QLRO. In this phase the correlation function of the magnetization obeys a power law and the magnetic susceptibility diverges at low fields.

To describe the large distance behavior at low temperatures we use the classical non-linear σ-model with the Hamiltonian

$$H = \int d^D x[J \sum_{\mu} \partial_\mu \mathbf{n}(\mathbf{x}) \partial_\mu \mathbf{n}(\mathbf{x}) + V_{imp}(\mathbf{x})], \tag{1}$$

where $\mathbf{n}(\mathbf{x})$ is the unit vector of the magnetization, $V_{imp}(\mathbf{x})$ the random potential. In the RF case it has the form

$$V_{imp} = -\sum_{\alpha} h_\alpha(\mathbf{x}) n_\alpha(\mathbf{x}); \quad \alpha = 1, ..., N, \tag{2}$$

where the random field $\mathbf{h}(\mathbf{x})$ has a Gaussian distribution and $\langle h_\alpha(\mathbf{x}) h_\beta(\mathbf{x}')\rangle = A^2 \delta(\mathbf{x}-\mathbf{x}')\delta_{\alpha\beta}$. In the RA case the random potential is given by the equation

$$V_{imp} = -\sum_{\alpha,\beta} \tau_{\alpha\beta}(\mathbf{x}) n_\alpha(\mathbf{x}) n_\beta(\mathbf{x}); \quad \alpha, \beta = 1, ..., N, \tag{3}$$

where $\tau_{\alpha\beta}(\mathbf{x})$ is a Gaussian random variable, $\langle \tau_{\alpha\beta}(\mathbf{x}) \tau_{\gamma\delta}(\mathbf{x}')\rangle = A^2 \delta_{\alpha\gamma}\delta_{\beta\delta}\delta(\mathbf{x}-\mathbf{x}')$. Random potential (3) corresponds to the same symmetry as a more conventional choice $V_{imp} = -(\mathbf{hn})^2$ but is more convenient for the further discussion.

The Imry – Ma argument [1, 8] suggests that in our problem the long range order is absent at any dimension $D < 4$. One can estimate the Larkin length, up to which there are strong ferromagnetic correlations, with the following qualitative renormalization group (RG) approach. Let one remove the fast modes and rewrite the Hamiltonian in terms of the block spins, corresponding to the scale $L = ba$, where a is the ultraviolet cut-off. Then let one make rescaling so that the Hamiltonian would restore its initial form with new constants $A(L), J(L)$. Dimensional analysis provides estimations

$$J(L) \sim b^{D-2} J(a); \quad A(L) \sim b^{D/2} A(a). \tag{4}$$

To estimate the typical angle ϕ between neighbor block spins, one notes that the effective field, acting on each spin, has two contributions: the exchange contribution and the random one. The exchange contribution of order $J(L)$ is oriented along the local average direction of the magnetization. The random contribution of order $A(L)$ may have any direction. This allows one to write at low temperatures that $\phi(L) \sim A(L)/J(L)$. The Larkin length corresponds to the condition $\phi(L) \sim 1$ and equals $L \sim (J/A)^{2/(4-D)}$ in agreement with the Imry – Ma argument [1]. If Eq.(4) were exact the Larkin length could be interpreted as the correlation radius. However, there are two sources of corrections to Eq.(4). Both of them are relevant already at the derivation of the RG equation for the pure system in $2+\epsilon$ dimensions [20]. The first source is the renormalization due to the interaction and the second one results from the rescaling of the magnetization which is necessary to ensure the fixed length condition $\mathbf{n}^2 = 1$. The leading corrections to Eq.(4) are proportional to $\phi^2 J$, $\phi^2 A$. Thus, the RG equation for the combination $(A(L)/J(L))^2$ is the following

$$\frac{d}{d\ln L}\left(\frac{A(L)}{J(L)}\right)^2 = \epsilon\left(\frac{A(L)}{J(L)}\right)^2 + c\left(\frac{A(L)}{J(L)}\right)^4, \quad \epsilon = 4 - D. \tag{5}$$

If the constant c in Eq.(5) is positive the Larkin length is the correlation radius indeed. But if $c < 0$ the RG equation has a fixed point, corresponding to the phase with the infinite correlation radius. As it is seen below, both situations are possible, depending on the system.

To derive the RG equations in a systematic way we use the method, suggested by Polyakov [20] for the pure system. The same consideration as in the XY [10] and random manifold [12] models suggests that near a zero-temperature fixed point in $4-\epsilon$ dimensions there is an infinite set of relevant operators. After replica averaging the relevant part of the effective replica Hamiltonian can be represented in the following form

$$H_R = \int d^D x [\sum_a \frac{1}{2T}\sum_\mu \partial_\mu \mathbf{n}_a \partial_\mu \mathbf{n}_a - \sum_{ab}\frac{R(\mathbf{n}_a\mathbf{n}_b)}{T^2}], \tag{6}$$

where a, b are replica indices, $R(z)$ is some function, T the temperature. In the RA case the function $R(z)$ is even due to the symmetry with respect to changing the sign of the magnetization.

The one-loop RG equations for the N-component model in $4-\epsilon$ dimensions are obtained by a straightforward combination of the methods of Refs. [12] and [20]. The RG equations become simpler after substitution for the argument of the function $R(z)$: $z = \cos\phi$. In terms of this new variable one has to find even periodic solutions $R(\phi)$. The period is 2π in the RF case and π in the RA case. In a zero-temperature fixed point the one-loop equations are

$$d\ln T/d\ln L = -(D-2) - 2(N-2)R''(0) + O(R^2, T); \tag{7}$$

$$\begin{aligned} 0 = dR(\phi)/d\ln L = \epsilon R(\phi) + (R''(\phi))^2 - 2R''(\phi)R''(0) - \\ -(N-2)[4R(\phi)R''(0) + 2\mathrm{ctg}\phi R'(\phi)R''(0) - (R'(\phi)/\sin\phi)^2] + O(R^3, T). \end{aligned} \tag{8}$$

The two-spin correlation function is given by the expression $\langle \mathbf{n}(\mathbf{x})\mathbf{n}(\mathbf{x}')\rangle \sim |\mathbf{x}-\mathbf{x}'|^{-\eta}$, where

$$\eta = -2(N-1)R''(0). \tag{9}$$

The same equations (7)–(9) were derived by a different and more cumbersome method in Ref. [21]. In that paper the critical behavior in $4+|\epsilon|$ dimensions was studied by considering analytical fixed point solutions $R(\phi)$. In the Heisenberg model, analytical solutions are absent and they are unphysical for $N \neq 3$ [21]. In this letter we search for non-analytical $R(\phi)$. In the non-analytical fixed point $R^{\rm IV}(\phi = 0) = \infty$ and hence the Taylor expansion over ϕ is absent [12]. Still a power expansion over $|\phi|$ exists, similar to the RF XY model [10].

Let us consider the RA model at $N = 3$. We solve Eq.(8) numerically. Since coefficients of Eq.(8) are large as $\phi \to 0$, it is convenient to use the expansion of $R(\phi)$ over $|\phi|$ at small ϕ. At larger ϕ the equation is integrated by the Runge – Kutta method. The solutions to be found have zero derivatives at $\phi = 0, \pi/2$. At $N = 3$ the solution with largest $|R''(0)|$, which corresponds to $\eta = 0.62\epsilon$ (9), has two zeroes in the interval $[0, \pi]$. There are also solutions with 4 and more zeroes. They all correspond to $\eta < 0.5\epsilon$. We shall see below that these solutions are unstable.

To test the stability of the solution with two zeroes we use an approximate method. The instability to the constant shift of the function $R(\phi)$ has no interest for us, since the constant shifts do not change the correlators [12]. To study the stability to the other perturbations it is convenient to rewrite Eq.(8), substituting $\omega(R''(\phi))^2$ for $(R''(\phi))^2$. The case of interest is $\omega = 1$, but at $\omega = 0$ the equation can be solved exactly. The solution at $\omega = 1$ can be found with the perturbation theory over ω. The exact solution at $\omega = 0$ is $R_{\omega=0}(\phi) = \epsilon(\cos 2\phi/24 + 1/120)$. The perturbative expansion provides the following asymptotic series for η: $\eta = \epsilon(0.67 - 0.08\omega + 0.14\omega^2 - \ldots)$. The resulting estimation $\eta = \epsilon(0.67 \pm 0.08)$ agrees with the numerical result well. This allows us to expect that the stability analysis of the solution $R_{\omega=0}$ of the equation with $\omega = 0$ provides information about the stability of the solution of Eq.(8). A simple calculation shows that $R_{\omega=0}$ is stable in the linear approximation. Thus, there is a stable zero-temperature fixed point of the RG equations with the critical exponent of the correlation function

$$\eta = 0.62\epsilon. \tag{10}$$

The critical exponent γ of the magnetic susceptibility $\chi(H) \sim H^{-\gamma}$ in the weak uniform field H is given by the equation

$$\gamma = 1 + (N-1)R''(0)/2 = 1 - 0.15\epsilon. \tag{11}$$

In 4 dimensions the one-loop RG equations for the RA Heisenberg model allow to obtain the exact large-distance asymptotics of the correlation function. It obeys the equation $\langle \mathbf{n}(\mathbf{x})\mathbf{n}(\mathbf{x}')\rangle \sim \ln^{-0.62}|\mathbf{x} - \mathbf{x}'|$. Numerical analysis of Eq.(8) shows that at $N > 10$ the QLRO is absent in the RA model. This agrees with the results of the spherical approximation $N = \infty$.

Let us demonstrate the absence of physically acceptable fixed points in the RF case at $N > 3$. We derive some inequality for critical exponents. Then we show that the inequality has no solutions. We use a rigorous inequality for the connected and disconnected correlation functions [22]

$$\langle \mathbf{n}_a(\mathbf{q})\mathbf{n}_a(-\mathbf{q})\rangle - \langle \mathbf{n}_a(\mathbf{q})\mathbf{n}_b(-\mathbf{q})\rangle \leq \text{const}\sqrt{\langle \mathbf{n}_a(\mathbf{q})\mathbf{n}_a(-\mathbf{q})\rangle}, \tag{12}$$

where $\mathbf{n}(\mathbf{q})$ is a Fourier-component of the magnetization, a, b are replica indices. The large distance behavior of the connected correlation function in a zero-temperature fixed point can be derived from the expression $\chi \sim \int \langle\langle \mathbf{n}(\mathbf{0})\mathbf{n}(\mathbf{x})\rangle\rangle d^D x$ and the critical exponent of the susceptibility (11). In a fixed point Eq.(12) provides an inequality for the critical exponents of the connected and disconnected correlation functions [22]. Both exponents can be expressed via $R''(0)$. This allows to obtain the following relation

$$4 - D \leq \frac{3 - N}{N - 1}\eta, \tag{13}$$

where η is given by Eq.(9). Since Eq.(13) is incompatible with the requirement $\eta > 0$ there are no accessible fixed points for $N > 3$. This suggests the strong coupling regime with a presumably finite correlation radius.

In the RA case a similar consideration uses the connected and disconnected correlation functions of the field $(n_x(\mathbf{r})n_y(\mathbf{r}))$ in presence of Gaussian disorder (3). At $N = 3$ the resulting condition for the critical exponent, $\eta \geq \epsilon/2$, rules out all but one fixed points of RG equation (8).

The question of the large distance behavior of the RF and RA Heisenberg models was discussed by Aharony and Pytte on the basis of an approximate equation of state [23]. They also obtained QLRO in the RA case. However, we believe that this is an occasional coincidence since the equation of state [23] is valid only in the first order in the strength of the disorder, while higher orders are crucial for critical properties [24]. In particular, the approach [23] incorrectly predicts the absence of QLRO in the RF XY model and its presence in the RA spherical model. It also provides incorrect critical exponents in the Heisenberg case.

The RA Heisenberg model is relevant for the amorphous magnets [2]. At the same time, for their large distance behavior the dipole interaction may be important [17]. Besides, a weak nonrandom anisotropy is inevitably present due to mechanical stresses.

In conclusion, we have found that the random anisotropy Heisenberg model has the infinite correlation radius and a power dependence of the correlation function of the magnetization on the distance at low temperatures and weak disorder in $4 - \epsilon$ dimensions. On the other hand, the correlation radius of the random field $O(N > 3)$ model is always finite.

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