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THE QCD POMERON WITH OPTIMAL RENORMALIZATION

S.J.Brodsky, V.S.Fadin†, V.T.Kim□∇, L.N.Lipatov□, G.B.Pivovarov∆∇1)*

**Stanford Linear Accelerator Center,*
Stanford University, Stanford, CA 94309, USA

†Budker Institute of Nuclear Physics
630090 Novosibirsk, Russia

□St.Petersburg Nuclear Physics Institute
188350 Gatchina, Russia

∆Institute for Nuclear Research
117312 Moscow, Russia

∇International Institute of Theoretical and Applied Physics
Ames, IA 50011, USA

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It is shown that the next-to-leading order (NLO) corrections to the QCD Pomeron intercept obtained from the Balitsky – Fadin – Kuraev – Lipatov (BFKL) equation, when evaluated in non-Abelian physical renormalization schemes with Brodsky – Lepage – Mackenzie(BLM) optimal scale setting, do not exhibit the serious problems encountered in the $\overline{MS}$-scheme. A striking feature of the NLO BFKL Pomeron intercept in the BLM approach is that it yields an important approximate conformal invariance.

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The discovery of rapidly increasing structure functions in deep inelastic scattering at HERA at small-x is in agreement with the expectations of the QCD high-energy limit. The Balitsky – Fadin – Kuraev – Lipatov (BFKL) [1] resummation of energy logarithms is anticipated to be an important tool for exploring this limit. The highest eigenvalue, ω^{max}, of the leading order BFKL equation [1] is related to the intercept of the Pomeron which in turn governs the high-energy asymptotics of the cross sections: $\sigma \sim s^{\alpha_{I\!P}-1} = s^{\omega^{max}}$. The BFKL Pomeron intercept in LO turns out to be rather large: $\alpha_{I\!P} - 1 = \omega_L^{max} = 12\ln 2\,(\alpha_S/\pi) \simeq 0.55$ for $\alpha_S = 0.2$; hence, it is very important to know the next-to-leading order (NLO) corrections.

Recently the NLO corrections to the BFKL resummation of energy logarithms were calculated; see Refs. [2, 3] and references therein. The NLO corrections [2, 3] to the

1) gbpivo@ms2.inr.ac.ru

highest eigenvalue of the BFKL equation turn out to be negative and even larger than the LO contribution for $\alpha_S > 0.157$. However, one should stress that the NLO calculations, as any finite-order perturbative results, contain both renormalization scheme and renormalization scale ambiguities. The NLO BFKL calculations [2, 3] were performed by employing the modified minimal subtraction scheme ($\overline{MS}$) to regulate the ultraviolet divergences with arbitrary scale setting.

In this work we consider the NLO BFKL resummation of energy logarithms [2, 3] in physical renormalization schemes in order to study the renormalization scheme dependence. To resolve the renormalization scale ambiguity we utilize Brodsky-Lepage-Mackenzie (BLM) optimal scale setting [4]. We show that the reliability of QCD predictions for the intercept of the BFKL Pomeron at NLO when evaluated using BLM scale setting within non-Abelian physical schemes, such as the momentum space subtraction (MOM) scheme [5] or the Υ-scheme based on $\Upsilon \to ggg$ decay, is significantly improved compared to the $\overline{MS}$-scheme result [2, 3].

We begin with the representation of the $\overline{MS}$-result of NLO BFKL [2, 3] in physical renormalization schemes. The eigenvalue of the NLO BFKL equation at transferred momentum squared $t = 0$ in the $\overline{MS}$-scheme [2, 3] can be represented as the action of the NLO BFKL kernel (averaged over azimuthal angle) on the leading order eigenfunctions $(Q_2^2/Q_1^2)^{-1/2+i\nu}$ [2]:

$$\omega_{\overline{MS}}(Q_1^2,\nu) = \int d^2Q_2 \; K_{\overline{MS}}(\mathbf{Q}_1,\mathbf{Q}_2)\left(\frac{Q_2^2}{Q_1^2}\right)^{-\frac{1}{2}+i\nu} =$$

$$= N_C\chi_L(\nu)\frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi}\left[1 + r_{\overline{MS}}(\nu)\frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi}\right], \qquad (1)$$

where

$$\chi_L(\nu) = 2\psi(1) - \psi(1/2+i\nu) - \psi(1/2-i\nu)$$

is the function related with the leading order eigenvalue, $\psi = \Gamma'/\Gamma$ denotes the Euler ψ-function, the ν-variable is conformal weight parameter [6], N_C is the number of colors, and $Q_{1,2}$ are the virtualities of the reggeized gluons.

The calculations of Refs. [2, 3] allow us to decompose the NLO coefficient $r_{\overline{MS}}$ of Eq. (1) into β-dependent and conformal (β-independent) parts:

$$r_{\overline{MS}}(\nu) = r^{\beta}_{\overline{MS}}(\nu) + r^{conf}_{\overline{MS}}(\nu), \qquad (2)$$

where

$$r^{\beta}_{\overline{MS}}(\nu) = -\frac{\beta_0}{4}\left[\frac{1}{2}\chi_L(\nu) - \frac{5}{3}\right] \qquad (3)$$

and

$$r^{conf}_{\overline{MS}}(\nu) = -\frac{N_C}{4\chi_L(\nu)}\left[\frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3+\left(1+\frac{N_F}{N_C^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right) - \chi_L''(\nu) + \right.$$
$$\left. + \frac{\pi^2-4}{3}\chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu)\right] \qquad (4)$$

with

$$\varphi(\nu) = 2\int_0^1 dx\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right], \; \mathrm{Li}_2(x) = -\int_0^x dt\frac{\ln(1-t)}{t}. \qquad (5)$$

Here $\beta_0 = (11/3)N_C - (2/3)N_F$ is the leading coefficient of the QCD β-function, N_F is the number of flavors, $\zeta(n)$ stands for the Riemann zeta-function, $\mathrm{Li}_2(x)$ is the Euler dilogarithm (Spence-function). In Eq. (4) N_F denotes flavor number of the Abelian part of the $gg \to q\bar{q}$ process contribution. The Abelian part is not associated with the running of the coupling [7] and is consistent with the corresponding QED result for the $\gamma^*\gamma^* \to e^+e^-$ cross section [8].

The β-dependent NLO coefficient $r^{\beta}_{\overline{MS}}(\nu)$, which is related to the running of the coupling, receives contributions from the gluon reggeization diagrams, from the virtual part of the one-gluon emission, from the real two-gluon emission, and from the non-Abelian part [7] of the $gg \to q\bar{q}$ process.

The NLO BFKL Pomeron intercept then reads for $N_C = 3$ [2]:

$$\alpha^{\overline{MS}}_{I\!P} - 1 = \omega_{\overline{MS}}(Q^2, 0) = 12 \ln 2 \, \frac{\alpha_{\overline{MS}}(Q^2)}{\pi} \left[1 + r_{\overline{MS}}(0) \frac{\alpha_{\overline{MS}}(Q^2)}{\pi} \right] ; \tag{6}$$

$$r_{\overline{MS}}(0) \simeq -20.12 - 0.1020 N_F + 0.06692 \beta_0, \tag{7}$$

$$r_{\overline{MS}}(0)_{|N_F=4} \simeq -19.99.$$

One of the most popular physical schemes is MOM-scheme [5], based on renormalization of the triple-gluon vertex at some symmetric off-shell momentum. In order to eliminate the dependence on gauge choice and other theoretical conventions inherent to the MOM-scheme, one can consider renormalization schemes based on physical processes [4], e.g., V-scheme based on heavy quark potential or Υ-scheme based on $\Upsilon \to ggg$ decay [9].

A finite renormalization due to the change of scheme can be accomplished by a transformation of the QCD coupling [5]:

$$\alpha_S \to \alpha_S \left[1 + T \frac{\alpha_S}{\pi} \right], \tag{8}$$

where T is some function of N_C, N_F, and for the MOM-scheme, of a gauge parameter ξ. Then the NLO BFKL eigenvalue in the MOM-scheme can be represented as follows

$$\begin{aligned} \omega_{MOM}(Q^2, \nu) &= N_C \chi_L(\nu) \frac{\alpha_{MOM}(Q^2)}{\pi} \left[1 + r_{MOM}(\nu) \, \frac{\alpha_{MOM}(Q^2)}{\pi} \right], \\ r_{MOM}(\nu) &= r_{\overline{MS}}(\nu) + T_{MOM}. \end{aligned} \tag{9}$$

The corresponding T-function for the transition from the $\overline{MS}$-scheme to the MOM-, V- and Υ- schemes can be found from Refs. [5, 4, 9] (Table 1).

Table 1

Scheme-transition function and the NLO BFKL coefficient in physical schemes

Scheme		$T = T^{conf} + T^{\beta}$	$r(0) = r^{conf}(0) + r^{\beta}(0)$	$r(0)$ ($N_F = 4$)
MOM	$\xi = 0$	$7.471 - 1.281\beta_0$	$-12.64 - 0.1020N_F - 1.214\beta_0$	-22.76
	$\xi = 1$	$8.247 - 1.281\beta_0$	$-11.87 - 0.1020N_F - 1.214\beta_0$	-21.99
	$\xi = 3$	$8.790 - 1.281\beta_0$	$-11.33 - 0.1020N_F - 1.214\beta_0$	-21.44
V		$2 - 0.4167\beta_0$	$-18.12 - 0.1020N_F - 0.3497\beta_0$	-21.44
Υ		$6.47 - 0.923\beta_0$	$-13.6 - 0.102N_F - 0.856\beta_0$	-21.7

One can see from Table 1 that in spite of a weak renormalization scheme dependence the problem of a large NLO BFKL coefficient remains. The large size of the perturbative corrections leads to significant renormalization scale ambiguity. The renormalization scale ambiguity problem can be resolved if one can optimize the choice of scales and renormalization schemes according to some sensible criteria. In the BLM optimal scale setting [4], the renormalization scales are chosen such that all vacuum polarization effects from the QCD β-function are resummed into the running couplings.

In the present case one can show that within the V-scheme (or the $\overline{MS}$-scheme) the BLM procedure does not change significantly the value of the NLO coefficient $r(\nu)$. This can be understood since the V-scheme, as well as $\overline{MS}$-scheme, are adjusted primarily to the case when in the leading order there are dominant QED (Abelian) type contributions, whereas in the BFKL case there are important leading order gluon-gluon (non-Abelian) interactions. Thus one can choose for the BFKL case the MOM-scheme [5] or the Υ-scheme.

Adopting BLM scale setting, the NLO BFKL eigenvalue in the MOM-scheme is

$$\omega_{BLM}^{MOM}(Q^2,\nu) = N_C \chi_L(\nu) \frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi} \left[1 + r_{BLM}^{MOM}(\nu) \frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi} \right], \quad (10)$$

$$r_{BLM}^{MOM}(\nu) = r_{MOM}^{conf}(\nu) . \quad (11)$$

The β-dependent part of the $r_{MOM}(\nu)$ defines the corresponding BLM optimal scale

$$Q_{BLM}^{MOM\,2}(\nu) = Q^2 \exp\left[-\frac{4 r_{MOM}^{\beta}(\nu)}{\beta_0} \right] = Q^2 \exp\left[\frac{1}{2}\chi_L(\nu) - \frac{5}{3} + 2\left(1 + \frac{2}{3} I\right) \right], \quad (12)$$

where $I = -2\int_0^1 dx \ln(x)/[x^2 - x + 1] \simeq 2.3439$. At $\nu = 0$ we have $Q_{BLM}^{MOM\,2}(0) = = Q^2 \left(4 \exp[2(1 + 2I/3) - 5/3]\right) \simeq Q^2\, 127$.

Fig.1, 2 and Table 2 give the results for the eigenvalue of the NLO BFKL kernel. We have used the QCD parameter $\Lambda = 0.1$ GeV which corresponds to $\alpha_S = = 4\pi/\left[\beta_0 \ln(Q^2/\Lambda^2)\right] \simeq 0.2$ at $Q^2 = 15$ GeV^2.

One of the striking features of this analysis is that the NLO value for the intercept of the BFKL Pomeron, improved by the BLM procedure, has a very weak dependence on the gluon virtuality Q^2. The minor Q^2-dependence obtained leads to approximate scale and conformal invariance. Thus one may use conformal symmetry [6, 10] for the continuation of the present results to the case $t \neq 0$.

Table 2

The NLO BFKL Pomeron intercept in the BLM scale setting within non-Abelian physical schemes

Scheme		$r_{BLM}(0)$ ($N_F = 4$)	$\alpha_{IP}^{BLM} - 1 = \omega_{BLM}(Q^2,0)$		
			$Q^2 = 1$ GeV^2	$Q^2 = 15$ GeV^2	$Q^2 = 100$ GeV^2
MOM	$\xi = 0$	-13.05	0.134	0.155	0.157
	$\xi = 1$	-12.28	0.152	0.167	0.166
	$\xi = 3$	-11.74	0.165	0.175	0.173
Υ		-14.01	0.133	0.146	0.146

Note that the application of fast apparent convergence [11] and the principle of minimal sensitivity [12] to the NLO BFKL eigenvalue problem lead to difficulties with the conformal weight dependence, an essential ingredient of BFKL calculations [13].

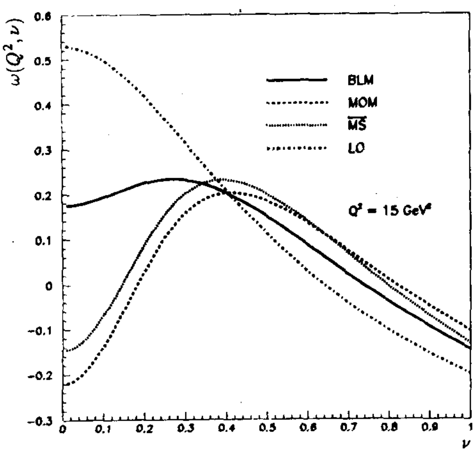

Fig.1. ν-dependence of the NLO BFKL eigenvalue: BLM (in MOM-scheme) - solid, MOM-scheme (Yennie gauge: $\xi = 3$) - dashed, $\overline{MS}$-scheme - dotted. LO BFKL ($\alpha_S = 0.2$) - dash-dotted

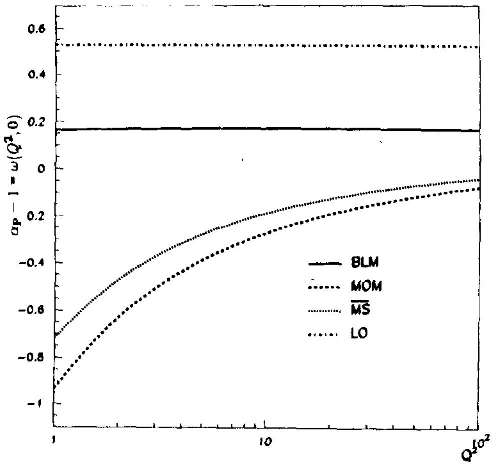

Fig.2. Q^2-dependence of the BFKL Pomeron intercept in the NLO. The notation is as in Fig.1

It is worth noting also that since the BFKL equation can be interpreted as the "quantization" of a renormalization group equation [10] the effective scale should depend on the BFKL eigenvalue ω, associated with the Lorentz spin, rather than on ν. This issue and other recent approaches to the NLO BFKL [14] will be discussed in more detail in the extended version of this work [13].

To conclude, we have shown that the NLO corrections to the BFKL equation for the QCD Pomeron become controllable and meaningful provided one uses physical renormalization scales and schemes relevant to non-Abelian gauge theory. BLM optimal scale setting sets the appropriate physical renormalization scale by absorbing the non-conformal β-dependent coefficients. The strong renormalization scale dependence of the NLO corrections to BFKL resummation then largely disappears. A striking feature of the NLO BFKL Pomeron intercept in the BLM approach is its very weak Q^2-dependence, which provides approximate conformal invariance.

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