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ISOTOPE SHIFTS IN FINITE NUCLEI AND PAIRING PROPERTIES OF NUCLEAR MATTER

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A uniform nuclear matter with s–wave pairing is studied within the local energy-density functional approach incorporating a few parameter sets extracted from the analysis of isotope shifts in finite nuclei. The dilute limit when the regime changes from weak to strong pairing is considered in detail, and, in strong coupling, the ground state properties of that system are found to be completely determined in leading order by the singlet scattering length a_{nn}. The density-dependent contact pairing interaction and energy cutoff adjusted to produce a realistic value of a_{nn} is shown to be a preferable choice between the deduced parameter sets.

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Recent studies [1 – 3] have shown that nuclear isotope shifts – the differential observables such as the odd-even mass differences and odd-even effects in charge radii along isotope chains – can be reasonably well reproduced within the local energy-density functional (LEDF) approach with an effective density-dependent contact pairing interaction. Most succesful description has been achieved with a phenomenological "gradient" force of the form [2]:

$$\mathcal{F}^{\xi} = C_0 f^{\xi}\left(x(\frac{\mathbf{r}_1+\mathbf{r}_2}{2})\right)\delta(\mathbf{r}_1-\mathbf{r}_2)\,, \quad f^{\xi}(x(\mathbf{r})) = f^{\xi}_{ex} + h^{\xi}x^{q}(\mathbf{r}) + f^{\xi}_{\nabla}r_0^2\left(\nabla x(\mathbf{r})\right)^2\,, \quad (1)$$

where x is the isoscalar dimensionless density, $x=(\rho_n+\rho_p)/2\rho_0$ with $\rho_{n(p)}$ the neutron (proton) density, $q=2/3$ [3]. The normalization factors are the density $2\rho_0$ and the inverse density of states C_0 at the Fermi surface in saturated nuclear matter: $C_0 = 2\epsilon_{0F}/3\rho_0 \equiv \equiv \hbar^2\pi^2/k_{0F}m$ with m the free nucleon mass and $\epsilon_{0F}=\hbar^2k_{0F}^2/2m$. Numerically, for the functional DF3 [4] used here, $C_0 = 308.2$ MeV·fm^3, $r_0 = 1.147$ fm, $2\rho_0 = 0.1582$ fm^{-3}, $k_{0F} = 1.328$ fm^{-1} and $\epsilon_{0F} = 36.57$ MeV. As shown in Ref. [2], the self-consistent LEDF calculations with the density-gradient term $\propto f^{\xi}_{\nabla}$ in pairing force provide desirable size of isotopic shifts and right order of odd-even staggering observed in finite nuclei. Different choices of the parameters of the pairing force (1) are possible. In particular, the following sets are deduced for the lead isotopes:

$$\begin{array}{llll}
f^{\xi}_{ex} = -0.56, & h^{\xi} = 0, & f^{\xi}_{\nabla} = 0 & (a)\\
f^{\xi}_{ex} = -1.20, & h^{\xi} = 0.56, & f^{\xi}_{\nabla} = 2.4 & (b)\\
f^{\xi}_{ex} = -1.60, & h^{\xi} = 1.10, & f^{\xi}_{\nabla} = 2.0 & (c)\\
f^{\xi}_{ex} = -1.79, & h^{\xi} = 1.36, & f^{\xi}_{\nabla} = 2.0 & (d)\\
f^{\xi}_{ex} = -2.00, & h^{\xi} = 1.62, & f^{\xi}_{\nabla} = 2.0 & (e)\\
f^{\xi}_{ex} = -2.40, & h^{\xi} = 2.16, & f^{\xi}_{\nabla} = 2.0 & (f)
\end{array} \quad (2)$$

These LEDF calculations are based on the general variational principle applied to the local functional with a fixed energy cutoff $\epsilon_c = 40$ MeV measured from ϵ_F, and on the coordinate-space technique which involves an integration in the complex energy plane of the Green's functions obtained by solving the Gor'kov equations exactly (see Refs. [3] for detailes).

In the present paper, the empirical information gained from finite laboratory nuclei is used to study the ground state properties of uniform nuclear matter and the behavior of the energy gap Δ as a function of density $\rho = 2\rho_0 x$ (or the Fermi momentum $k_F = = (3\pi^2\rho/2)^{1/3} \equiv k_{0F}x^{1/3}$) in this system. The term $\propto f^{\xi}_{\nabla}$ vanishes in this case, thus only two parameters f^{ξ}_{ex} and h^{ξ} in (1) are relevant. The gap equation reads

$$\Delta(x) = -\int_{k\leq k_c} \frac{d\mathbf{k}}{(2\pi)^3} F^{\xi}(x) \frac{\Delta(x)}{\sqrt{(\epsilon_k - \epsilon_F(x))^2 + \Delta^2(x)}}, \tag{3}$$

where $k_c = \sqrt{2m(\epsilon_F + \epsilon_c)}/\hbar$, $F^{\xi}(x) = C_0 f^{\xi}(x)$ and $\epsilon_k = \hbar^2 k^2/2m$. The solution of Eq. (3) in the weak pairing approximation, $\Delta/\epsilon_F \ll 1$, is given in Ref. [5], eq. (25) (see also eqs. (10) and (22) in Refs. [3]). We find that this solution, to get a deeper insight into the physics, can be written as:

$$\Delta(k_F) = c\frac{\hbar^2 k_F^2}{2m} \exp\left[-\frac{\pi}{2}\cot\delta(k_F)\right], \tag{4}$$

where we have introduced the Fermi level phase shift $\delta(k_F)$ defined by

$$k_F \cot\delta(k_F) = -\frac{4k_{0F}}{\pi}\left(\frac{1}{f^{\xi}(k_F)} + \frac{k_c(k_F)}{2k_{0F}}\right) - \frac{k_F}{\pi}\ln\left(\frac{k_c(k_F) - k_F}{k_c(k_F) + k_F}\right), \tag{5}$$

with $k_c(k_F) = \sqrt{k_{0c}^2 + k_F^2}$; $k_{0c} = \sqrt{2m\epsilon_c}/\hbar$, and $c = 8e^{-2} \approx 1.083$. Eq. (5) corresponds to an exact solution of the nn scattering problem at the relative momentum $k = k_F$ with the states truncated by a momentum cutoff $k_c = k_c(k_F)$ for contact potential $C_0 f^{\xi}(k_F)\delta(\mathbf{r})$ (see, e.g., Ref. [6]).

Shown in Fig. 1 are the results for Δ in nuclear matter with the parameter sets of Eq. (2). The approximation (5) works well in the entire range of k_F in Fig. 1 for the set (a), but for the other sets this is true only at k_F greater than $\approx$1.2 fm^{-1} and also, for the sets (b), (c) and (d), at k_F less than 0.42, 0.14 and 0.042 fm^{-1}, respectively (in these regions, $\Delta/\epsilon_F \leq 0.1$). It should be stressed that ϵ_F entering the integrand of Eq. (3) can be expressed directly through density ρ by $\epsilon_F = \hbar^2 k_F^2(\rho)/2m$ with $k_F(\rho) = (3\pi^2\rho/2)^{1/3}$ only if the pairing is weak and the dependence of the Fermi energy ϵ_F (and the chemical potential μ) on Δ can be disregarded. Otherwise one should introduce the particle number condition

$$x = \frac{2}{\rho_0}\int_{k\leq k_c} \frac{d\mathbf{k}}{(2\pi)^3} n_k(x), \quad n_k(x) = \frac{1}{2}\left(1 - \frac{\epsilon_k - \epsilon_F(x)}{\sqrt{(\epsilon_k - \epsilon_F(x))^2 + \Delta^2(x)}}\right), \tag{6}$$

and to solve the system of the two equations (3) and (6) with respect to Δ and ϵ_F. The results shown in Fig. 1 correspond to such a solution.

All parameter sets (2) except (a) reproduce the neutron separation energies and the isotope shifts of charge radii $\langle r^2\rangle_{ch}$ of lead isotopes fairly well (see Ref. [3]). Shown also in Fig. 1 are the values of the $^1S_0$ pairing gap in nuclear matter obtained for the CD-Bonn potential without medium effects (using free single-particle spectrum $\epsilon_k = k^2/2m$) [7] and for the Gogny D1 force in the Hartree-Fock – Bogolyubov framework [8]. The agreement

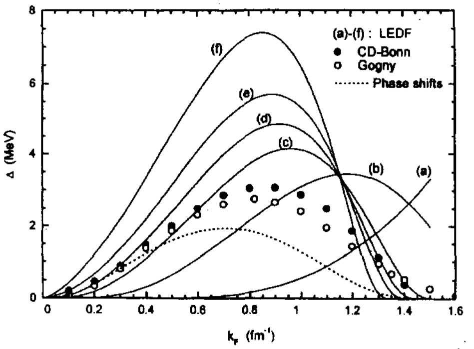

Fig.1. Pairing gap in nuclear matter as a function of the Fermi momentum. Curves (a)–(f) are calculated using Eqs. (3) and (6) with contact pairing force (1) and correspond, respectively, to the parameter sets (a)–(f) of Eq. (2). Full (open) circles are the solutions of the nonlocal gap equation with the CD-Bonn potential [7] (with the finite-range Gogny D1 force [8]). The dotted line shows Δ calculated using Eq. (4) with the free nn scattering phase shifts (see text)

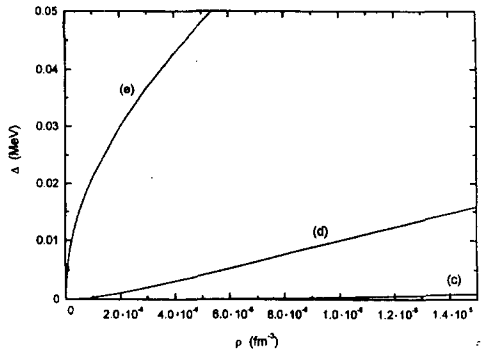

Fig.2. Pairing gap Δ at very low densities. Curves (c)–(e) correspond to the parameter sets (c)–(e) of Eq. (2), respectively

between the two latter calculations is relatively good while both deviate noticeably from our predictions. The curve for density-independent force, set (a), stands by itself with a positive derivative $d\Delta(x)/dx$ everywhere; no acceptable description of $\langle r^2\rangle_{ch}$ could be obtained in this case [3].

At very low densities (5) reduces to

$$k_F \cot\delta(k_F) \approx -\frac{1}{a_{nn}} + \frac{1}{2} r_{nn} k_F^2 - \frac{2k_F}{\pi}\left[\frac{k_F}{2k_{0c}} - \frac{2h^\xi}{(f_{ex}^\xi)^2}\left(\frac{k_F}{k_{0F}}\right)^{3q-1}\right], \tag{7}$$

where a_{nn} is the singlet scattering length,

$$a_{nn} = \frac{\pi}{2k_{0F}}\left(\frac{\sqrt{2m\epsilon_c}}{\hbar k_{0F}} + \frac{2}{f_{ex}^\xi}\right)^{-1} \equiv \frac{\pi}{4k_{0F}}\left(\frac{1}{f_{ex}^\xi} - \frac{1}{f_{cr}^\xi}\right)^{-1}, \tag{8}$$

and r_{nn} is the effective range, $r_{nn} = 4/\pi k_{0c}$. Here we have introduced the critical constant $f_{cr}^\xi = -2k_{0F}/k_{0c}$, the value of the vacuum strength f_{ex}^ξ at which the two-nucleon problem has a bound state solution at zero energy (in our case, $f_{cr}^\xi = -1.912$).

The first two terms in (7) would describe low-energy behavior of the nn s-wave phase shift through an expansion of $k\cot\delta$ in powers of the relative momentum $k = k_F$ if the interaction were density-independent – i.e., if the coupling strength and momentum cutoff were fixed by $f^\xi = f_{ex}^\xi$ and $k_c = k_{0c}$, respectively. It follows that, with a density-dependent effective force, such an expansion contains additional terms which are, for the parametrization used here, of the same order as the effective range term. This simply demonstrates that, for reproducing the pairing gap, the effective interaction even at very low densities need not necessarily coincide with the bare NN interaction as was discussed by Migdal many years ago [9].

At very low densities, at $k_F \to 0$, to leading order from (5) we obtain

$$\Delta = c\epsilon_F \exp\left(\frac{\pi}{2k_F a_{nn}}\right), \quad a_{nn} < 0 . \tag{9}$$

This expression agrees with the results of Ref. [10] based on a general analysis of the gap equation at low densities when $k_F|a_{nn}| \ll 1$. But we should stress that (9) is valid only in the weak-coupling regime corresponding to negative a_{nn}. In the opposite case, the gap in the dilute limit has to be found in a different way.

At $f^{\xi}_{ex} > f^{\xi}_{cr}$, from (9) and (4) it follows that at low densities the pairing gap is exponentially small and eventually $\Delta(k_F \to 0) = 0$. Such a weak pairing regime with Cooper pairs forming in a spin singlet $l = 0$ state exists up to the critical point at which the attraction becomes strong enough to change the sign of the scattering length. Then the strong pairing regime sets in, and Δ should be determined directly from the combined solution of Eqs. (3) and (6). In the dilute systems, ϵ_F plays the role of the chemical potential μ. It is defined by $\mu = \epsilon_F(k_F) + U(k_F)$ with $U(k_F)$ the Hartree – Fock mean field at the Fermi surface which is negligible for the fermion gas. At the critical point μ becomes negative and a bound state of a single pair of nucleons with the binding energy $\epsilon_b = 2\mu$ $(= -\hbar^2/ma^2_{nn})$ becomes possible [11, 12]. This can be easily seen from the gap equation (3) written in the form

$$\left(\frac{k^2}{m} - 2\mu\right)\phi_k = -\mathrm{sgn}(\epsilon_k - \mu)\sqrt{1-\phi_k^2}\int_{k'\leq k_c}\frac{d\mathbf{k}'}{(2\pi)^3}F^{\xi}\phi_{k'} , \tag{10}$$

where we have introduced the functions $\phi_k = \Delta/\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}$ and replaced ϵ_F by μ. In the strong coupling regime $\mu < 0$, and in the dilute limit, when $\phi_k^2 << 1$, Eq. (10) reduces to the Schrödinger equation for a single bound pair where 2μ plays the role of the eigenvalue. Δ at low densities in this regime can be found from (6). In the leading order we get

$$\Delta = \frac{\hbar^2}{m}\left(\frac{2\pi\rho}{a_{nn}}\right)^{1/2}, \quad a_{nn} > 0 . \tag{11}$$

It follows that, in the dilute case, the energy needed to break a condensed pair goes smoothly from 2Δ to $2\mu = \epsilon_b$ as a function of the coupling strength when the regime changes from weak to strong pairing. But as seen from (9) and (11), the behavior of Δ at low densities is such that the derivative $d\Delta/d\rho$ at $\rho \to 0$ as a function of f^{ξ}_{ex} shows up a discontinuity from 0 to ∞. This is illustrated in Fig. 2 where we have plotted $\Delta(\rho)$ at very low densities for the sets (c)–(e) of Eq. (2) embracing both regimes. We note also that the transition between the two regimes is formally reflected by the fact that the analitical expressions (9) and (11) give a pure imaginary gap at the critical point when the scattering length changes sign.

When the Fermi momentum k_F approaches from below the upper critical point, where the pairing gap closes, Δ becomes exponentially small [10]. In weak coupling, Δ is also exponentially small at low densities. It is noteworthy that, as follows from (4), the gap closes exactly at the points where the phase shift passes zero, i.e where the integral in the right-hand side of Eq. (10) vanishes. In fact, this is a model independent result valid for any given interaction (at these points the gap equation becomes equivalent to the Schrödinger equation which has a plane wave solution with the wave number $k = k_F$).

As an illustration, we show in Fig. 1 by the dotted line the values of $\Delta(k_F)$ obtained from (4) using the "experimental" nn phase shifts, without electromagnetic effects[1)]. It is seen that Δ obtained this way closely follows the solution of the gap equation with the CD-Bonn potential [7] at low densities. The nn phase shift passes zero at the relative momentum $k = 1.71$ fm^{-1}, and the gap should vanish at the corresponding Fermi momentum. Unfortunately, the solutions for Δ are given in Ref. [7] only in the region up to $k_F = 1.4$ fm^{-1}.

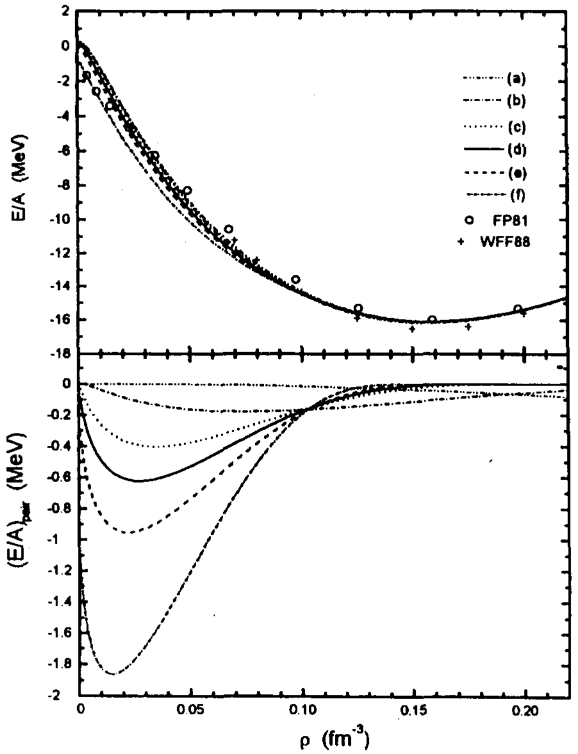

Fig.3. Energy per nucleon E/A (top) and pairing contribution to E/A (bottom) in symmetric nuclear matter. Curves (a)–(e) are calculated using Eq. (12) and correspond to the strength parameters (a)–(e) of Eq. (2), respectively. Open circles and crosses are the calculations of Ref. [13] and Ref. [14], respectively, for the UV14 plus TNI model

For symmetric nuclear matter, with our local functional, the energy per particle is

$$\frac{E}{A}(x) = \frac{2}{\rho_0 x}\int_{k\le k_c}\frac{d\mathbf{k}}{(2\pi)^3}\frac{\hbar^2k^2}{2m}n_k(x) + \frac{1}{3}\epsilon_{0F}a_+^v f_+^v(x)x + \frac{3\Delta^2(x)}{2f^\xi(x)x\epsilon_{0F}}\,, \qquad (12)$$

where $f_+^v(x) = (1 - h_{1+}^v x)/(1 + h_{2+}^v x)$. Numerically, $a_+^v = -6.422$, $h_{1+}^v = 0.163$ and $h_{2+}^v = 0.724$ [4]. The "particle-hole" term $\propto f_+^v$ vanishes in the dilute limit linearly in density. The chemical potential is

$$\mu(x) = \epsilon_F(x) + \frac{1}{3}\epsilon_{0F}a_+^v[f_+^{v\prime}(x)x^2 + 2f_+^v(x)x] + \frac{3f^{\xi\prime}(x)}{2f^{\xi 2}(x)}\frac{\Delta^2(x)}{\epsilon_{0F}}\,, \qquad (13)$$

where the prime denotes a derivative with respect to x, $\epsilon_F(x)$ and $\Delta(x)$ are determined from (3) and (6). The two last terms in (13), even in strong pairing regime, vanish in the dilute limit at least as x^q if $q < 1$ or linearly in x if $q \ge 1$. Thus, we see again that, in strong coupling, in the leading order $\mu = \epsilon_F = \epsilon_b/2 < 0$.

[1)] We thank Rupert Machleidt for providing us with these nn phase shifts.

The calculated energy per nucleon as a function of the isoscalar density ρ is shown in the upper panel in Fig. 3 together with the results of the nuclear matter calculations [13, 14] for the UV14 plus TNI model. It is seen that DF3 gives qualitatively reasonable description of the nuclear matter EOS and that pairing could contribute noticeable to the binding energy especially at lower densities. In the lower panel in Fig. 3 we have plotted the pairing energy per nucleon, $(E/A)_{pair}$, obtained by subtracting from (12) the corresponding value of E/A at $\Delta = 0$. The pairing contribution increases, as expected, when f^{ξ}_{ex} becomes gradually more attractive, with a shift to lower densities. For the sets (e) and (f) the attraction is strong, $f^{\xi}_{ex} < f^{\xi}_{cr}$. In these cases a nonvanishing binding energy in the dilute limit is solely due to Bose–Einstein condensation of the bound pairs, the spin-zero bosons, when all the three quantities, μ, E/A and $(E/A)_{pair}$, reach the same value $\epsilon_b/2 = \hbar^2/2ma^2_{nn}$ ($\epsilon_b = -0.0646$ and -1.616 MeV for the set (e) and (f), respectively). This is illustrated in Fig. 4 where we have plotted E/A and $(E/A)_{pair}$ as functions of ρ at very low densities.

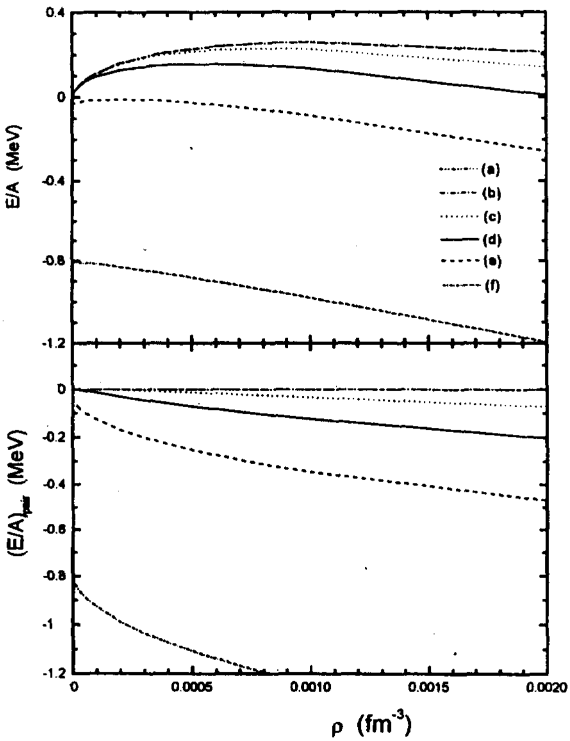

Fig.4. Energy per nucleon E/A (top) and pairing contribution to E/A (bottom) for symmetric nuclear matter at low densities. The notations are the same as in Fig.3

In conclusion, we have considered nuclear matter with s-wave pairing within the LEDF framework and demonstrated some results, including extrapolation to the dilute limit, with a few possible parameter sets of the pairing force deduced from experimental data for finite nuclei. At low densities, in the $T = 0$ case of symmetric $N = Z$ matter, the $^3S_1 - ^3D_1$ pairing leading to a Bose deuteron gas formation is more important [15] since the $n - p$ force is more attractive than in the $p - p$ or $n - n$ pairing channels. Thus, our approach, with the $^1S_0$ pairing only, would be more appropriate for an assymetric $N \neq Z$ case and for pure neutron systems. From this point of view the best choice for the LEDF calculations seems to be the pairing force with set (d) of Eq. (2) since it gives the singlet scattering length $a_{nn} \approx -17.2$ fm which corresponds to a virtual state at $\approx$ 140 keV known experimentally. As seen in Fig. 1, with this choice the behaviour of Δ at low densities agrees well with the calculations based on realistic NN forces. At higher densities, however, our predictions for Δ with the set (d) go much higher reaching a maximum of $\approx$ 4.84 MeV at $k_F \approx 0.92$ fm^{-1} while the calculations of Ref. [7] give a maximum of about 3 MeV at $k_F \approx 0.82$ fm^{-1}. With a bare NN interaction, assuming charge independence and $m_n = m_p$ in the free single-particle energies, the pairing gap would be, at given k_F, exactly the same both in symmetric nuclear matter and in neutron matter. As shown in Refs. [16, 17], if one includes medium effects in the effective pairing interaction, Δ in neutron matter would be substantially reduced to values of the order of 1 MeV at the most. Whether such a mechanism works in the same direction for symmetric nuclear matter is still an open question. With a smaller gap compared, for instance, to the one obtained with the Gogny D1 force which is also shown in Fig. 1, it would be difficult to explain the observed nuclear pairing properties. Calculation with the preferable set (d) of Eq. (2) leads to a larger pairing energy in nuclear matter than the Gogny force, but in finite nuclei there is a compensation due to the repulsive gradient term. The force (1) contains dependence on the isoscalar density only since we have analysed the existing data on separation energies and charge radii for finite nuclei with a relatively small assymetry $(N - Z)/A \leq 0.25$. An extrapolation to neutron matter with such a simple force would give a larger pairing gap than for nuclear matter. This suggests that some additional dependence on the isovector density $\rho_n - \rho_p$ should be present in the effective pairing interaction. This possibility is planned to be tested in our future work.

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