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LANDAU QUANTIZATION AND EQUATORIAL STATES ON A SURFACE OF A NANOSPHERE

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The Landau quantization for the electron gas on a surface of sphere is considered. We show that in the regime of strong fields the lowest energy states are those with magnetic quantum numbers m of order of Φ/Φ_0, the number of magnetic flux quanta piercing the sphere. For the electron gas of low density (semiconducting situation), it leads to the formation of the electronic stripe on the equator of the sphere in high fields.

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The electronic properties of cylindrical and spherical nanosize objects attract much of the theoretical interest last years. This interest is mostly related to the physics of carbon macromolecules and, particularly, to the transport properties of carbon nanotubes [1]. One meets the spherical nanosize objects in the studies of the nonlinear optical response in composite materials [2], of simple metal clusters [3] and of the photonic crystals on the base of synthetic opals [4]. In the most of these studies, the coating of the nano-sphere is characterized by an effective dielectric function [5]. It was noted however [6] that this approach should be revised if the coating of a sphere has a width of a few monolayers, which limit is allowed by modern technologies.

In the recent papers we considered the electron gas on a sphere. We showed that various correlation functions in such gas exhibit maxima when the electrons are at the points-antipodes (north and south poles) [7]. The exact solution was found for a problem in the uniform magnetic field and the limits of weak and high fields were investigated [8]. In the high-field regime the formation of Landau levels was shown. The complexity of the special functions describing the exact solution, however, complicates the analysis of the physical picture in the high-field regime. In this paper we explore qualitative arguments, supported by the numerical calculations, to clarify the issue. We find that the minimum energy to the Hamiltonian is provided by the electronic states located near the equator of the sphere. For low densities of the gas one thus expects, that the high field pushes the electrons towards the equator and forms an electronic ring there.

We consider the electron gas moving within a thin layer on a surface of the sphere of radius r_0. We assume the Hamiltonian of the form $\mathcal{H} = -\nabla^2/2m_e + U(r)$, with m_e the electron mass. The chemical potential μ defines the total number of electrons N (with one projection of spin) and the areal density $\nu = N/4\pi r_0^2$. The confining quantum-well potential $U(r)$ restricts the radial motion within the thin layer $\delta r \ll r_0$. We are interested in the case $\delta r < \nu^{-1/2}$, when the first excited state of the radial motion lies above the chemical potential. Then one can ignore the radial component of the wave function and put $r = r_0$ in the remaining angular part of the Hamiltonian. In the absence of the magnetic field we have $\mathcal{H}_\Omega^{(0)} = -(2m_e r_0^2)^{-1}\Delta_\Omega$ with Δ_Ω being the angular part of the Laplacian. The solutions to this Hamiltonian are the spherical harmonics Y_{lm} and the spectrum is that of a free rotator model :

$$\Psi^{(0)}(\theta,\phi) = r_0^{-1} Y_{lm}(\theta,\phi), \quad E_l^{(0)} = (2m_e r_0^2)^{-1} l(l+1).$$

In the presence of the uniform magnetic field $\mathbf{B}$ directed towards a north pole of the sphere ($\theta = 0$) we choose the gauge of the vector potential as $\mathbf{A} = \frac{1}{2}(\mathbf{B}\times\mathbf{r})$. Then the angular part $\mathcal{H}_\Omega$ of the Hamiltonian

$$\mathcal{H} = \frac{1}{2m_e}(-i\nabla + e\mathbf{A})^2 + U(r)$$

acquires the form

$$\mathcal{H}_\Omega = (2m_e r_0^2)^{-1}[-\Delta_\Omega + 2ip\frac{\partial}{\partial\phi} + p^2\sin^2\theta]. \qquad (1)$$

The ruling parameter here is $p = \pi B r_0^2/\Phi_0$ with the magnetic flux quantum $\Phi_0 = 2\cdot 10^{-15}$ T·m^2. For a sphere of radius $r_0 = 100\,\text{nm}$ one has $p = 1$ at the field $B \simeq 600\,\text{Oe}$. The solutions of (1) are given by the oblate (angular) spheroidal functions and were analyzed in [8] to some detail.

In the weak-field regime, $p \sim 1$, the jumps in the static magnetic susceptibility χ at half-integer p were demonstrated. The amplitude of these jumps is parametrically larger than the Pauli spin contribution and decreases with the increase of p. It was shown that the weak-field regime ends at $p^2 \sim p_c^2 = 2\sqrt{N}$. For $r_0 \sim 100\,\text{nm}$ (the case of opals) and in the metallic situation, e.g., at the densities $\nu \sim 10^{14}\,\text{cm}^{-2}$, we have $N \sim 10^5$ and $p_c \simeq 30$.

On the other hand, at the lower (semiconducting) densities, $\nu \sim 10^{10}\,\text{cm}^{-2}$, we have $N \sim 10$. Formally in this case $p_c \simeq 3$, i.e. the field is not small already at p of order of unity. The jumps in the susceptibility were predicted in [8] at the assumption $\sqrt{N} \gg 1$ which is violated in the latter case of lower ν. At the same time the experimentally accessible fields of order of 6 T result in $p \sim 100$ for the spheres with $r_0 \sim 100\,\text{nm}$, i.e. we come into the strong field regime.

For strong fields, $p \to \infty$, one observes the eventual formation of the Landau levels (LL). The spherical geometry brings into the problem some peculiarities which were partly discussed in [8]. First is the incomplete restructuring of the spectrum into the LL scheme. This restructuring takes place only for levels with initial momentum l lower than p, and the field remains weak for the levels with $l > p^2$. Secondly, the field-induced two-well potential $p^2\sin^2\theta$ in (1) localizes the electron states with moderate magnetic quantum numbers m ($|m| \ll p$) near the poles $\theta = 0$ and $\theta = \pi$. As a result, the correlations within one hemisphere only survive. Specifically, if an electron was initially in the northern hemisphere, then the probability to find it in the southern hemisphere is exponentially small.

At the same time, the spherical geometry produces yet another effect which is to be discussed here. The effective potential in the strong-field regime can be written as

$$U_{eff}(\theta) = \frac{p\omega_c}{4}\left(\frac{m/p}{\sin\theta} + \sin\theta\right)^2 \qquad (2)$$

with the cyclotron frequency $\omega_c = eB/m_e$. At small negative m we have two minima of $U_{eff}(\theta)$ near $\theta_0 = \arcsin(\sqrt{|m|/p})$ and $\theta_0 = \pi - \arcsin(\sqrt{|m|/p})$ where we expand

$$U_{eff}(\theta_0 + \theta) \simeq (\omega_c p \cos^2\theta_0)\,\theta^2. \qquad (3)$$

Rescaling here $\theta \to x/\sqrt{2p|\cos\theta_0|}$ we arrive at the quantum oscillator problem of the form

$$\mathcal{H} \simeq \frac{\omega_c|\cos\theta_0|}{2}\left(-d^2/dx^2 + x^2\right),$$

i.e. the well-known Landau quantization. As long as $|\cos\theta_0| \sim 1$, the wave-functions are extended at the scale $|\theta - \theta_0| \sim p^{-1/2}$. The possibility of quantum tunneling between θ_0 and $\pi - \theta_0$ produces the exponentially small splitting between the states centered at these points [8].

Thus we see that the energy levels are labeled by two quantum numbers, magnetic number m and LL number n, with approximate double degeneracy for given m, n.

This simple picture becomes inadequate, when $|m| \simeq p$ and $\theta_0 \simeq \pi/2$. In this case the harmonic potential in (3) weakens, which makes necessary the consideration of the fourth-order terms in the expansion. We have in this case

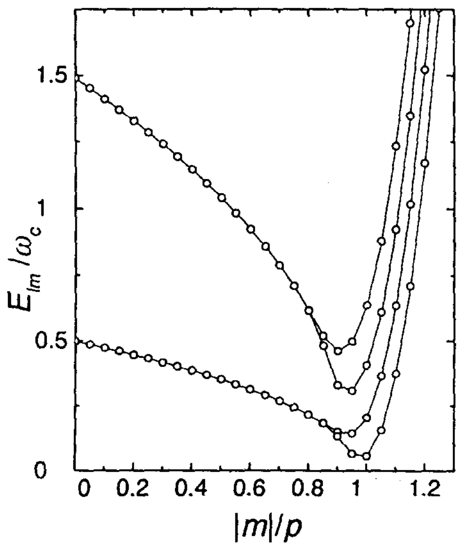
$$U_{eff}(\theta + \frac{\pi}{2}) \simeq \frac{\omega_c p}{4}\theta^4, \quad |m| = p. \tag{4}$$

Rescaling now $\theta \to xp^{-1/3}$ we arrive at the following Schrödinger equation:

$$\frac{\omega_c}{4p^{1/3}}\left(-d^2/dx^2 + x^4\right)\psi = E\psi.$$

The solution of the last equation apparently is not known [9]. For our purposes it suffices to note that the energy scales as $\omega_c/p^{1/3} \ll \omega_c$ and the wave-functions on the equator extend on the scale $|\theta - \pi/2| \sim p^{-1/3}$. In addition, we have no situation with two-well potential now and the energy levels are separated by the same scale $\omega_c/p^{1/3}$. The crossover between Eqs.(3) and (4) takes place at $1 - |m|/p \sim p^{-2/3}$.

With the further increase of $|m|$, at $|m| > p$, the minimum value of U_{eff} is found at $\theta = \pi/2$ and increases rapidly with $|m|$. In this case the energy levels $E_{lm} \sim \omega_c(m+p)^2/p$ and thus lie well above those with $|m| < p$.



The dependence of the four lowest energy levels E_{lm} on the magnetic number $m < 0$ in the strong-field regime, $p = \Phi/\Phi_0 = 100$. The calculated points are shown by circles, the lines are guide to the eye

We illustrate these qualitative results by the numerical calculations. We found the spectrum of Eq.(1) by diagonalizing $\mathcal{H}_\Omega$ in the basis of Legendre polynomials $P^m_{m+n}(\cos\theta)$

with $0 \le n \le 200$ [10]. The results are shown in the Figure. One verifies that the "equatorial" states with $|m| \simeq p \gg 1$ provide the minimum eigenvalues to the Hamiltonian. This dependence of the energy level scheme on m is probably of minor importance, if we consider the situation of a metal [8], when the chemical potential lies well above the bottom of the conduction band ($\mu \gg \omega_c$). In this case the number of electrons N on the sphere is expected to exceed the number of flux quanta p. Then several Landau levels are occupied (several lines in the Figure) and the electrons are distributed upon the whole sphere.

At the same time, the semiconducting coating of the sphere can lead to a different result. Indeed, if the cyclotron frequency ω_c is enough high, then the "polar" states with small negative m are poorly occupied. Meanwhile, the "equatorial" states with the energies $\sim \omega_c p^{-1/3}$ are occupied to a larger extent. This produces the effective ring on the equator of the sphere. The criterion for this phenomenon is $N < p$ or, equivalently, $B > \Phi_0 \nu$. Note that the latter inequalities correspond to the partially filled lowest Landau level in the usual planar geometry.

We see that in the spherical geometry of the electron gas the states with higher $|m|$ possess the lower energy. Our situation is thus opposite to the one discussed for the quantum Hall edge states [11]. Nevertheless both problems have a common ingredient, the linear-in-m spectrum for a given n (in our case in two domains, $|m| < p$ and $|m| > p$). Having effectively a case of one spatial dimension, we can consider the interaction effects as well. The problem however has a certain subtlety which is described below.

In certain cases one may hope to ignore the interaction between the states belonging to different "Landau levels" (different curves in the Figure). Considering now the lowest LL, one observes familiar branches of right- and left-going fermions, $|m| < p$ and $|m| > p$, with the negative and positive "Fermi velocities" $v_F = dE_{lm}/dm$, respectively. The absolute values of v_F for left and right movers are different. This point alone makes it difficult to pass to a bosonization description with one scalar field for both movers, and the notion of the chiral Luttinger liquid appears. A thorough consideration of the latter problem is beyond the scope of this study.

In conclusion, we considered the Landau quantization for the electron gas on a surface of sphere. The exact solution of this problem involves complicated functions, which are not very instructive for the analysis of states with large magnetic numbers m for the electron motion. We elucidate the role of the "equatorial" states with large m both analitically and numerically. These states are lower in energy, thus the electronic stripe on the equator can be realized for the semiconducting coating of the sphere in high magnetic fields.

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