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ON SELF-INDUCED TRANSPARENCY IN LASER-PLASMA INTERACTIONS

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We study fully relativistic non-linear one-dimensional equations describing steady-state solutions for an electromagnetic wave interacting with plasma in the self-induced transparency regime. In addition to the well-known solution that corresponds to the transmission of the electromagnetic wave into plasma, another steady-state solution is shown to exist in a certain range of the amplitude of the wave. The latter one corresponds to the full reflection of the incident wave. The co-existence of the two solutions indicates the possibility of a hysteresis-like behaviour in self-induced transparency.

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The fast progress of short-pulse laser technology in the last decade has made it possible to study laser-plasma interactions at relativistic intensities of the optical wave [1, 2]. Numerous interesting physical phenomena have been predicted and observed in this new regime. One of the basic effects of this kind is the possibility for a strong electromagnetic wave to propagate in overdense plasmas due to the relativistic increase of the electron mass [3 – 5] — the so-called self-induced transparency. It still remains a theoretical concept that has not been verified experimentally.

A considerable number of analytical and numerical results on self-induced transparency have been published by now (see [6 - 10] and references therein). In the present paper we would like to indicate that laser-plasma interactions in the self-induced transparency regime can exhibit even more complicated behaviour than one is used to think. Namely, we have found that instead of only one characteristic value of the amplitude of the optical wave, which defines the onset of self-induced transparency, there actually exist two such thresholds and a transition region between them where plasma may assume either transparent or opaque shape at the same intensity of the incident wave. Although our results have been obtained under steady-state assumptions, they can be of relevance for the propagation of a finite-lenght optical pulse through plasma, provided that its typical length scale is much larger than the relativistic plasma skin-depth.

Thus, we consider a strong electromagnetic wave interacting at normal incidence with a semi-infinite plasma layer with sharp boundary. In the present paper we are specifically interested in the steady-state field and density distribution, which is considered to be the result of an infinitely slow transition from zero amplitude of the incident wave to its current non-zero value. In one-dimensional (1D) approximation, all parameters of the problem depend only on the coordinate z and time t. To be specific, we assume the plasma to occupy the half-space $z > 0$; the electromagnetic wave propagates along z-axis from $-\infty$, its amplitude is constant in time. The ions are assumed immobile, and for the sake of simplicity we consider a step-like ion density distribution: $n_i = n_0\Theta(z)$, where $\Theta(z)$ is the Heavyside step-function.

Following Chen and Sudan [11], the governing set of equations describing the propagation of an electromagnetic wave through "cold electron fluid" can be presented as (note the usage of relativistic units $\hbar = c = 1$ throughout the paper):

$$\left(\Delta - \frac{\partial^2}{\partial t^2}\right)\mathbf{a} = \nabla\frac{\partial\phi}{\partial t} + \frac{\omega_p^2 n}{n_0\gamma}\left(\mathbf{a} - \nabla\psi\right) \quad , \tag{1}$$

$$\partial\psi/\partial t = -\phi - \gamma + 1 \quad , \tag{2}$$

with $n = n_i - n_0\omega_p^{-2}\,\Delta\phi$ and $\gamma^2 = 1 + (\mathbf{a} - \nabla\psi)^2$. Here n and γ denote the local density and Lorentz-factor of the moving "electron fluid", and $\mathbf{a}$ and ϕ are the dimensionless vector and scalar potentials of the electromagnetic field, respectively. The Coulomb gauge $\nabla\cdot\mathbf{a} = 0$ is used. The plasma frequency ω_p is defined in the usual way as $\omega_p^2 = 4\pi e^2 n_0/m$.

In 1D approximation, the gauge condition reads $\partial a_z/\partial z = 0$, which implies that the longitudinal component of the vector potential is irrelevant. The first of the above equations splits then into two:

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2} - \frac{\omega_p^2 n}{n_0\gamma}\right)\mathbf{a}_\perp = 0 \tag{3}$$

and

$$\frac{\partial\psi}{\partial z} = \frac{n_0\gamma}{\omega_p^2 n}\,\frac{\partial^2\phi}{\partial t\partial z} \quad . \tag{4}$$

As has already been said, we are searching for possible steady-state solutions when the plasma density does not depend on time: $\partial n/\partial t = 0$. It immediately follows from the above equations that the amplitude of the optical field $a_\perp^2$ may not depend on t, either. One must conclude that a steady-state solution of this type is only possible for circular polarization of the incident wave when the direction of the optical field varies in time but its amplitude not. In the following we always assume this to be the case.

As a result, inside the plasma layer ($z \geq 0$) the above set of governing equations reduces to just one non-linear equation for the transverse component of the vector potential:

$$\left(\frac{\partial^2}{\partial z^2} + \omega_0^2\right)\mathbf{a}_\perp = \frac{\mathbf{a}_\perp}{(1 + a_\perp^2)^{1/2}}\left(\omega_p^2 + \frac{\partial^2(1 + a_\perp^2)^{1/2}}{\partial z^2}\right) \quad , \tag{5}$$

where all time-dependence is assumed to be given by the factor $\exp(i\omega_0 t)$ with ω_0 being the frequency of the incident optical wave.

If a solution $\mathbf{a}_\perp(z)$ is known, the corresponding particle density distribution can be found as

$$n(z) = n_i + \frac{m}{4\pi e^2}\frac{\partial^2(1 + a_\perp^2)^{1/2}}{\partial z^2} \quad . \tag{6}$$

Any physically meaningful electron density distribution must conserve the total charge, which means

$$\int_{-\infty}^{\infty}(n(z) - n_i)\,dz = 0 \quad .$$

Substituting here Eq.(6), one sees that the boundary of the electron cloud in the steady state must be shifted with respect to the edge of the ion density profile by some distance

z_0, where z_0 satisfies the following equation (under the assumption that the z-derivative of the amplitude of the optical field vanishes at $+\infty$):

$$z_0 = -\frac{1}{\omega_p^2} \left. \frac{\partial(1+a_\perp^2)^{1/2}}{\partial z} \right|_{z=z_0} . \tag{7}$$

All our considerations imply that the plasma electrons always remain inside the plasma layer (that is, on the right side of $z = 0$), which is quite natural because the light pressure of the incoming wave is applied from the left. An additional consistency condition is then $z_0 \geq 0$, which means, in turn,

$$\left. \frac{\partial a_\perp^2}{\partial z} \right|_{z=z_0} \leq 0 . \tag{8}$$

Note that $z = 0$ is no longer a physical boundary for the transverse elecromagnetic wave; the actual boundary is $z = z_0$.

For further analysis, it is convenient to write the circularly polarized field $\mathbf{a}_\perp$ in terms of its amplitude $T(z)$ and phase $\theta(z)$ such that $a_x = T(z)\cos(\theta(z) + \omega_0 t)$, $a_y = T(z)\sin(\theta(z) + \omega_0 t)$. Eq.(5) splits then into two non-linear scalar equations. This set of equations can be easily shown to have two constants of motion, C_1 and C_2, which fully determine possible steady-state distributions of electromagnetic field and the particle density inside plasma. In terms of the constants of motion, the equation for the field amplitude can be conveniently written as

$$(\partial T/\partial z)^2 = (1+T^2)\left[C_2 - C_1^2 T^{-2} - \omega_0^2 T^2 + 2\omega_p^2 (1+T^2)^{1/2}\right] . \tag{9}$$

It looks now similar to the equation of motion of an anharmonic oscillator, where z plays the role of "time". One sees that, in general, there are two turning points T_{min} and T_{max}, where $\partial T/\partial z$ vanishes, so that the amplitude of the optical field T oscillates between these limits. From a physical point of view, such oscillations correspond to the interference pattern of two counter-propagating plane waves. They must be of relevance for a problem with two boundaries, when part of the energy can be reflected from the right boundary and interfere with the wave coming from the left. In the present paper we restrict our consideration to monotonic solutions, which is the most natural choice for a semi-infinite layer.

Under the additional condition that the amplitude of the wave is monotonic at $+\infty$, we still have two two opportunities:

A. $T(z) = \text{const}$, and $\partial T/\partial z = 0$ everywhere inside the plasma. This, in turn, implies $\partial\theta/\partial z = -k_0 = \text{const}$, where

$$k_0^2 = \omega_0^2 - \omega_p^2 (1+T^2)^{-1/2} ; \tag{10}$$

B. $\partial T/\partial z$ vanishes at some point T_∞ like $(\partial T/\partial z)^2 \sim (T - T_\infty)^\alpha$ with $\alpha \geq 2$. This is the case for $T_\infty = 0$ (which means no transmitted wave at $+\infty$), and $C_1 = 0$, $C_2 = -2\omega_p^2$. Eq.(9) reduces then to

$$(\partial T/\partial z)^2 = (1+T^2)\left[2\omega_p^2\left(\sqrt{1+T^2} - 1\right) - \omega_0^2 T^2\right] . \tag{11}$$

The first of the above options is just the well-known self-induced transparency regime of propagation [3]. The necessary condition of the existence of such a solution is $k_0^2 \geq 0$, that is, $\omega_0^2 \geq \omega_p^2(1+T^2)^{-1/2}$, which is in agreement with earlier results on self-induced transparency. One can also re-write the above expression as a condition on the amplitude of the wave:

$$T^2 \geq \left(\omega_p^4 - \omega_0^4\right)/\omega_0^4 \ . \tag{12}$$

The second option is more complicated. Eq.(11) is seen to have solutions under the condition $\omega_p^2 \geq \omega_0^2$ (one may note that in the limit of $\omega_p \gg \omega_0$ the corresponding solution has been found in [12]). For each choice of the ratio of ω_p/ω_0, the field T that satisfies Eq.(11) cannot exceed some limiting value defined by the r.h.s. of Eq.(11):

$$T^2 \leq 4\omega_p^2\left(\omega_p^2 - \omega_0^2\right)/\omega_0^4 \ . \tag{13}$$

From the above analysis, one can conclude that, surprisingly, the two solutions **A** and **B** may co-exist. Namely, if one switches to new dimensionless variables ν and τ such that

$$\nu = \frac{n_0}{n_{cr}}\ , \quad \tau = \left(\frac{T}{2}\right)^2 \frac{n_{cr}}{|n_0 - n_{cr}|}\ ,$$

there can be found four distinct regions in the (ν, τ) phase space (see Fig.1):

I. $\nu < 1$. This is the usual transparency regime in underdense plasma;

II. $\nu > 1$, $\tau < (\nu+1)/4$. Overdense plasma: no transmitted wave, full reflection of the incident energy;

III. $\nu > 1$, $\tau > \nu$. Only self-induced transparency solutions in overdense plasma;

IV. $\nu > 1$, $(\nu+1)/4 < \tau < \nu$. Both transparent and opaque solutions co-exist for the same initial plasma density and the same amplitude of the incident wave.

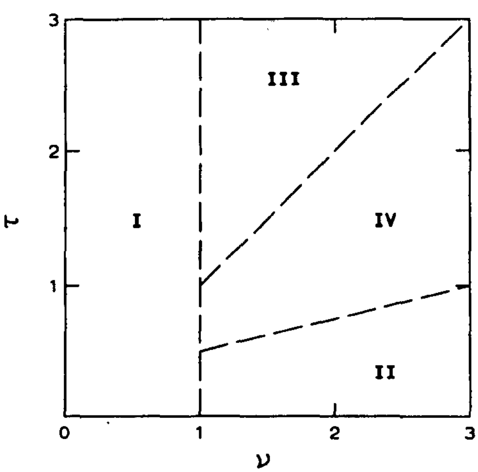

Fig.1. Four distinct regions in the (ν,τ) phase space: I — the usual transparency regime in underdense plasma, II — the full reflection of a low-intensity wave in overdense plasma, III — the self-induced transparency regime, IV — the transition region

Between the purely reflecting and purely self-induced-transparent regimes, there exists thus a transition region IV where both solutions are equally possible, and the system has to choose which of the two steady-state solutions to realize. It is beyond the scope of the present paper to show how this choice is made (note that the boundary conditions are not sufficient to fix the situation). We just emphasize that the situation is not unique, and that systems that can assume several stationary states usually demonstrate hysteresis-like behaviour, when the current state of the system depends on its history. It still remains to be investigated whether this is the case for self-induced transparency. One must also note that in terms of the (ν, τ) phase-space diagram, an increase of the amplitude of the incident wave corresponds to the increase of τ along a line of constant ν. In realistic situations, there is thus no way to avoid the controversial region IV.

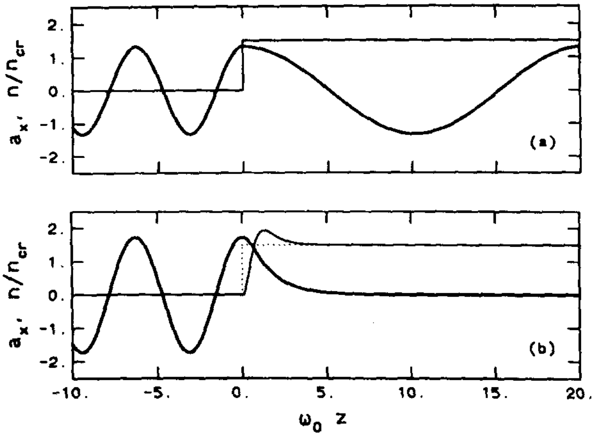

Fig.2. Dimensionless vector potential a_x (thick line) and the relative electron density n/n_{cr} (thin line) in the vicinity of the plasma boundary for two different regimes of propagation: (a) — self-induced transparency; (b) — full reflection. The input parameters are the same in both cases: $n_0 = 1.5\ n_{cr}$, $a_{in}^2 = 0.75$

As a matter of example, Fig.2 demonstrates the vector potential and the electron density distribution in the vicinity of the plasma boundary. The instant in time is chosen in such a way that the x-component of the vector potential at $z = 0$ is at its maximum. Fig.2a corresponds to the self-induced transparency regime of propagation, and Fig.2b — to the opaque solution. The amplitude of the incoming wave a_{in} and the plasma density n_0 are the same in both cases. Both parameters are properly chosen to encounter the region where the two solutions co-exist: $n_0/n_{cr} = 1.5$, $a_{in}^2 = 0.75$. The field at the left boundary of the plasma layer is found as a numerical solution of the (non-linear) boundary conditions. For our choice of parameters, the shift of the boundary of the electron cloud appears to vanish: $z_0 = 0$.

The most noticeable feature of the fully reflecting case is the presence of strong light pressure which influences the medium. The rearrangement of the plasma electrons under the action of the incident wave is clearly seen in Fig.2b. The electrons form a smooth maximum situated at about $z \simeq 1.3\omega_0^{-1}$ ($\sim 0.2\ \mu$m for an optical wavelength of 1 μm). This "deformation" of the electron cloud has a dramatic effect on the propagation of light: in Fig.2a about 72% of the incident energy is transmitted into plasma, while in Fig.2b the incoming light is fully reflected. Note that in our case self-induced transparency is already possible for $a_{in}^2 = 0.3125$. In fact, one sees that even at an irradiance that is 2.4 times higher, plasma may still "resist" to transmit the incoming light.

To conclude, we have considered interaction of a strong optical wave with overdense plasma in the self-induced transparency regime. It has been shown that in a certain range of the amplitude of the incident wave two possible steady-state solutions of the corresponding non-linear equations co-exist. One of them describes well-known self-induced transparency, while the other one corresponds to the full reflection of the incident wave. The co-existence of these two indicates that overdense plasmas under the action of a strong laser wave may exhibit hysteresis-like behaviour when the current state of the system depends on its history.

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