Pis'ma v ZhETF, vol.70, iss.9, pp.573 - 576

ON PROPAGATION OF SHORT PULSES IN STRONG DISPERSION MANAGED OPTICAL LINES

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Submitted 21 September 1999

We show that propagation of short pulses in the optical lines with strong dispersion management is described by an integrable Hamiltonian system. The leading nonlinear effect is formation of a collective dispersion which is a result of interaction of all pulses propogating along the line.

PACS: 42.65.Tg

One of the most important practical applications of Nonlinear Optics is the theory of propagation optical pulses in nonlinear optical fibers. Due to dispersion, pulses of small amplitude in such a system endure the chromatic spreading which limits the transmission capacity of a fiber. In 1973 Hasegava and Tappert [1] offered to use focusing nonlinearity for compensation of chromatic spreading. Competion of spreading and nonlinear focusing leads in a conservative fiber to formation of stationary pulses – optical solitons – which can be used as units of information. The theory of optical solitons and their interactions was developed by Zakharov and Shabat in 1971 [2]. At the moment optical solitons are proposed for using in dozens of large scale projected telecommunication systems.

Real optical lines are not conservative. For compensation of damping one should install a periodic array of amplifiers. Hasegava and Kodama [3] showed that these "traditional" soliton lines inherit the basic features of the conservative lines.

A more advanced (see [4]) proposal is to design a line that includes a periodic system of fiber legs with opposite signs of dispersion as well as a periodic array of amplifiers. Would such a line be linear, the designer could achieve the compensation of chromatic spreading. It makes such lines the most promising systems for ultrafast communication. However, to suppress the noise and provide a low-error transport of information one should use optical pulses of relatively high amplitude. As a result, in the process of long-distant pulse propagation the nonlinear effects are inevitably essential.

The theory of pulse propagation in the long periodic structures is a new and interesting chapter of the nonlinear physics. In many aspects periodic fibers differ from homogenous ones. In the last case the dispersion is a smooth function of frequency which can be approximated inside a narrow spectral band by a low-order polynomial. As a result, a pulse envelope can be described by a partical differential equation (PDE) of second or third orders, and a pulse interaction is local in time. In this situation, the optical solitons propagating without chromatic distortion are the basic objects. Due to locality of interaction the shape of an individual soliton does not depend on the presence of other pulses in the system. We will show in this article that in periodic systems it is not necessary true.

We study the lines where local nonlinearity and mean dispersion are much less than local dispersion. Such lines are characterized by time τ_0. This is the duration of pulse which broadens in factor two after passing a fiber leg with constant dispersion. If the pulse is short ($\tau \ll \tau_0$, τ is pulse duration), one can speak about strong dispersion management (SDM). In this letter we show that in this limit the line is described approximately by a completely integrable Hamiltonian system.

The leading nonlinear effect is the appearance of a collective average dispersion formed by a whole ensemble of pulses, propagating through the line. This dispersion is very nonlocal, thus the pulse envelopes cannot be described by any PDE in coordinate space. The pulses pass through each other without interaction, but the rate of chromatic spreading of an individual pulse depends on the presence of neighboring pulses.

The theory of short-pulses propagation. The basic model for description of dispersion managed fibers is the nonlinear Schrodinger equation with periodic coefficients

$$i\frac{\partial\Psi}{\partial x} + (d + \Phi'(x))\Psi_{tt} + R(x)|\Psi^2|\Psi = 0. \tag{1}$$

Here $\Psi(x,t)$ is the envelope of a wave pulse, $\Phi(x), R(x)$ are periodic functions of x with the same period 2π. As far as $< \Phi'(x) >= 0$, d is the average dispersion of the fiber. We assume

$$|R(x)| \ll |d + \Phi'(x)|. \tag{2}$$

In real transmission lines, this condition is usually satisfied.

In the assumption (2), equation (1) can be replaced with the approximate Gabitov–Turitzyn model (GT) [5–7]

$$i\frac{\partial\chi}{\partial x} = d\omega^2\chi_\omega - \int G(\Delta)\chi^*_{\omega_1}\chi_{\omega_2}\chi_{\omega_3}\delta_{\omega+\omega_1-\omega_2-\omega_3}\, d\omega_1 d\omega_2 d\omega_3. \tag{3}$$

Here

$$\chi_\omega = \Psi_\omega e^{i\Phi(x)\omega^2}, \quad \Delta = \omega^2 + \omega_1^2 - \omega_2^2 - \omega_3^2,$$
$$G(\Delta) = \frac{1}{2\pi}\int_0^{2\pi} R(x)e^{i\Phi(x)\Delta}dx = J(\Delta\tau_0^2). \tag{4}$$

Here τ_0 is a characteristic parameter of the line, $J(\xi)$ is a function of dimensionless variable $\xi \sim 1$, and $\Psi_\omega(x)$ is the Fourier transformation of $\Psi(x,t)$. In a general case,

$$J(-\xi) = J^*(\xi).$$

In a special case of constant nonlinearity (R =const) and piecewise constant dispersion $J(\xi) = \sin\xi/\xi$.

The inverse Fourier transform of (3) leads to the equation

$$i\frac{\partial\chi}{\partial x} + d\frac{\partial^2\chi}{\partial t^2} + \frac{1}{\tau_0^2}\int F(\frac{pq}{\tau_0^2})\,\chi^*(t+p+q)\,\chi(t+p)\,\chi(t+q)\,dpdq = 0. \tag{5}$$

If $s = pq/\tau_0^2 > 0$, then $F(s)$ is given by the expression.

$$F(s) = \frac{1}{2\pi}\int_0^\infty \left\{2J^*(z)K_0(\sqrt{2sz}) - \pi J(z)N_0(\sqrt{2sz})\right\}ds. \tag{6}$$

Here $K_0(q)$ is the Bessel function of imaginary argument, $N_0(q)$ – the Neumann's function. For negative s, $F(-s) = F^*(s)$. In the case of strong dispersion $s \ll 1$, one can use asymptotic expansions of the Bessel function at small values of argument.

Expansion in the powers of s leads to the result

$$F = F_0 + F_1 + F_2 + \cdots, \tag{7}$$

$$F_0 = \frac{2}{\pi}\int_0^\infty \left[\ln\frac{2}{|s|z} - C\right] ReJ(z)dz, \tag{8}$$

$$F_1 = \frac{2is}{\pi}\int_{-\infty}^\infty z\left[\ln\frac{2}{|s|z} + 1 - C\right] ImJ(z)dz, \tag{9}$$

$$F_2 = \frac{s^2}{4\pi}\int_0^\infty z^2\left[\ln\frac{2}{|s|z} + \frac{3}{2} - C\right] ReJ(z)dz. \tag{10}$$

Here C is the Euler constant. Let $\chi = Ae^{i\Phi}$. Plugging $F = F_0$ in (5) and performing the Fourier transform one obtains the system of equations for A and Φ

$$\frac{\partial A}{\partial x} = 0,$$

$$\frac{\partial \Phi}{\partial x} = -d\omega^2 + \hat{K}(A). \tag{11}$$

Here (see [8])

$$\hat{K}(A) = \frac{4}{\tau_0^2}\left[fA^2 + \int_{-\infty}^\infty \frac{A^2(\omega') - A^2(\omega)}{|\omega' - \omega|}d\omega'\right], \tag{12}$$

$$f = (2\ln\tau_0 + C)\,a + b,$$

$$a = \int_0^\infty J(z)dz, \quad b = \int_0^\infty \ln\frac{2}{z}\,J(z)dz. \tag{13}$$

In the order of magnitude

$$K(A) \simeq \frac{4}{\tau_0^2}\ln\frac{\tau_0}{\tau}A^2(\omega). \tag{14}$$

In the limit $\ln\tau_0/\tau \to \infty$ formula (14) is explicit. Eq. (11) mean that the system (4) in the limit $\tau_0/\tau \to 0$ is integrable.

The leading nonlinear effect is the formation of an additional collective dispersion $\hat{K}(A)$. Let the signal $\chi(t)$ be a superposition of a large number of identical pulses with random phases, separated by arbitrary intervals of time

$$\chi(t) = \sum_{n=1}^N \chi_0(t - \tau_n), \quad N \gg 1. \tag{15}$$

Then

$$\chi(\omega) = \chi_0(\omega)\sum_{n=1}^N e^{-i(\omega\tau_n - \phi_n)},$$

$$A^2(\omega) = N|A_0(\omega)|^2 = |\chi(\omega)|^2. \tag{16}$$

According to (11) all the pulses in the system endure the same chromatic spreading and remain identical. The value of spreading is defined by all pulses existing in the line.

This is a result of strong dispersional spreading of each pulse in the fibers of constant dispersion. Due to nonlinearity, this spreading cannot be completely compensated in the next fibers. As a result, each pulse generates long "tails", influencing the shapes of other pulses. In the developed model the pulse interaction is very nonlocal – they "feel each other", being separated by an arbitrarily long distance. This ultimate nonlocality is a weak point of the developed simple model.

Another weak point of the model is the plethora of solitonic solutions. To find such a solution, one can put

$$\Phi(\omega, x) = \lambda x + \Phi_0(\omega),$$

where $\Phi_0(\omega)$ is an arbitrary function of ω and λ is an arbitrary constant.

The amplitude $A(\omega)$ is an arbitrary positive solution of the equation

$$A(\omega)[\lambda + d\omega^2 - \hat{K}(A)] = 0. \tag{17}$$

One can arbitrarily separate the axis $-\infty < \omega < \infty$ into two sets $\Omega_1 \cup \Omega_2$ and put

$$\begin{aligned} A(\omega) = 0, \qquad & \omega \in \Omega_1, \\ \lambda + d\omega^2 - \hat{K}(A) = 0, \qquad & \omega \in \Omega_2. \end{aligned} \tag{18}$$

Here (18) is the Fredholm integral equation of the first type. The system (18) has an infinite number of solutions. To improve the model, one should take into account higher orders of expansion (7). This is outside the scope of this article.

We have shown that the propagation of short optical pulses with strong disperse managed nonlinear systems is quite different from the propagation in "traditional" lines with constant dispersion. Interaction of pulses is very nonlocal – even very distant pulses interact by the formation of average essential dispersion. In the leading order the system of pulses is described by an integrable Hamiltonian system. In the framework of this model, solitons do not have a universal form and their importance from the practical viewpoint is unclear. One can say that the nonlinearity plays in such systems a pure negative role. The program for the designer is to make the line "as linear as possible".

One of the authors (S.V.Manakov) is grateful to the Fellowship of the Italian Ministry of Foreign Affairs.

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