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SUBATTOSECOND ($\leq 10^{-18}$s) TIME MEASUREMENTS USING COHERENT x-RAY TRANSITION RADIATION

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The properties of the coherent x-ray transition radiation (CXTR) produced by intense, short electron beams microbunched in various devices, with periods corresponding to x-ray wavelengths $\sim 0.02-1$ nm and with modulation indices $\sim 0.001-1$, are investigated theoretically. The results of numerical calculations show that, just as in the case of incoherent and coherent transition radiation in the optical and microwave regions, CXTR can be used to study the microbunching processes and may thus find application for beam diagnostics in future x-ray FELs and advanced accelerator technology.

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Coherent transition radiation (CTR) is now used [1] for subpicosecond particle density distribution measurements. These measurements make use of the fact that the CTR spectrum is the Fourier transform of the longitudinal charge distribution and that, when the radiation wavelength λ is longer than the longitudinal σ_z and transverse σ_r sizes of the bunch, the radiation intensity is proportional to the square of the number N_b of electrons in the bunch. However, when x-ray FELs with self-amplified spontaneous emission (SASE) come online in the near future [2, 3], the femtosecond bunches will have subattosecond structures due to the particle grouping or microbunching (MB), with periods equal to the resonant radiation wavelength of the FEL.

The use of TR for the investigation of MB in the microwave region has been proposed [4] and now implemented [5, 6]. Just as the first works [7 – 9] on the application of optical TR for high-energy particle beam diagnostics were, in the words of [9], "initiated after the publication of the paper [10]," the present work follows [4 – 6], adopting the methods developed in them. Using a realistic model for MB, in this work we calculate the spectral and angular distributions of the CXTR and the dependence of the total CXTR photon number on the bunch and MB parameters. Numerical results given for the expected high-density, short bunches show the possibility of CXTR applications.

Suppose that after MB the particle distribution in a axially symmetric Gaussian bunch has the following form:

$$f(r,z) = \frac{N_b}{A}\left[\frac{\exp\left(-\frac{r^2}{2\sigma_r^2}\right)}{2\pi\sigma_r^2}\right]\left[\frac{\exp\left(-\frac{z^2}{2\sigma_z^2}\right)}{(2\pi)^{1/2}\sigma_z}[1+b_1\cos(k_r z)]\right], \quad (1)$$

where b_1 is the modulation index of the first harmonic of the MB (we neglect the much smaller higher harmonics), which takes place with a wavelength λ_r, $k_r = 2\pi/\lambda_r$, and

$$A = 1 + 2b_1\exp\left(-\frac{k_r^2\sigma_z^2}{2}\right) \quad (2)$$

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is a factor determined by the normalization of the distribution function (1) and is practically equal to 1 in the case under discussion ($k_r\sigma_z \gg 1$) .

Using (1) and expressions for the spectral angular distributions of the incoherent XTR intensity and its relation with CXTR [11], one can obtain the following expression for the number N_{CXTR} of CXTR photons produced by a bunch at an interface between vacuum and a substance with plasma frequency ω_p:

$$\frac{d^2N_{CXTR}}{d\omega d\theta} = \frac{2\alpha N_b^2 b_1^2}{\pi A^2}\frac{\omega_p^4}{\omega^5}\frac{\theta^3}{(\gamma^{-2}+\theta^2)(\gamma^{-2}+\theta^2+\omega_p^2/\omega^2)} \times$$
$$\times \exp\left(-(\frac{\omega}{v}\sigma_r\theta)^2\right)\exp\left(-(\frac{\omega}{v}-k_r)^2\sigma_z^2\right), \qquad (3)$$

where ω and θ are the CXTR photon emission frequency and polar angle, and v and $\gamma = E/mc^2$ are the particle velocity and Lorentz factor of the bunch, respectively.

As expected, the angular spectral distribution of the CXTR is mainly proportional to N_b^2 and b_1^2, decreases with increasing ω, and exceeds that of the incoherent XTR of N_b particles for $N_b b_1^2 \exp(-(\omega_r\sigma_r\theta/v)^2) \gg 1$ when $\omega \simeq \omega_r$. It is also seen that CXTR is a diffraction-limited radiation, since it vanishes with increasing σ_r when $\omega\sigma_r\theta/v$ becomes much larger than 1.

The angular and spectral distributions of CXTR photons obtained by the integration of (6) can be written approximately in the forms

$$\frac{dN_{CXTR}}{d\theta} \simeq \frac{\sqrt{\pi}v}{\sigma_z}\frac{dN_{CXTR}}{d\omega d\theta}\,|_{\omega=\omega_r}, \qquad (4)$$

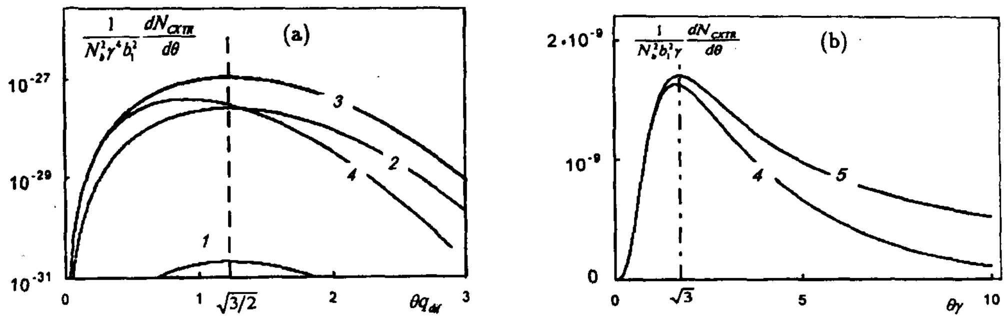
$$\frac{dN_{CXTR}}{d\omega} = \frac{N_b^2 b_1^2\alpha}{\pi A^2\omega}\exp\left[-\left(\frac{\omega}{v}-k_r\right)^2\sigma_z^2\right]\left\{\begin{array}{ll}\tau^4/(1+\omega_\gamma^2)^2, & \text{if} \quad \tau \ll 1,\\ (1+2\omega_\gamma^2)\ln\gamma, & \text{if} \quad \tau \gg 1, \xi \gg 1.\end{array}\right. \qquad (5)$$

The total number of CXTR photons per bunch is obtained by integrating expression (4) over θ or expression (5) over ω and is equal to

$$N_{CXTR} = \frac{\sqrt{\pi}v}{\sigma_z}\frac{dN_{CXTR}}{d\omega}\,|_{\omega=\omega_r}\,. \qquad (6)$$

In Eqs.(4) − (6) $E_1(x)$ is the exponential integral function, and the following notation is used: $\omega_\gamma = \omega/(\omega_p\gamma)$ and $\xi = \omega_p\sigma_r/v$ are dimensionless frequencies, and $\tau = \gamma/q_{dif}$, where $q_{dif} = \omega_r\sigma_r/v$ is the dimensionless transverse size of the bunch, which determines the diffraction limit. Particular cases of expressions (4) and (5) for various relationships among the values of τ, ξ, and γ will be analyzed in [12].

In the discussion above it was assumed that the particles in the bunches are monochromatic. The expected high-energy bunches for SASE FELs will have uncorrelated and correlated energy spreads equal to $\Delta E/E = \Delta\gamma/\gamma = 0.0002$ and 0.001, respectively [3]. It is clear that such energy spreads will have negligible influence on the XTR and CXTR angular distributions. However, they can result in substantial broadening of the narrow CXTR spectral distributions. Indeed, since the resonance wavelength in FELs is equal to $\lambda_r = l(1+K^2)/2\gamma^2$, where l and K are the undulator period and parameter, then particles with different energies will have different resonance frequencies, and the CXTR photons they produce will have wider spectral distributions. Assuming the particles have a Gaussian energy distribution, after convolution with the above expressions



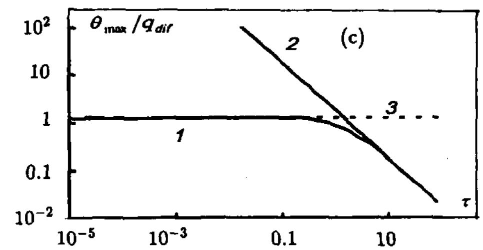


Fig.1. a,b) Angular distributions of CXTR photons per particle in a bunch at a copper–vacuum interface for $q_{dif} = 1.26 \cdot 10^5$, normalized to γ^4 (a) and to γ (b) for $\gamma = 25$, 200, $10^3 - 10^5$, 10^6, and $10^7 - 10^8$ or $\tau = 2.0 \cdot 10^{-5}, 1.6 \cdot 10^{-4}, 8.0 \cdot 10^{-4} - 8.0 \cdot 10^{-2}, 0.8$ and $8 - 80$ (curves *1*, *2*, *3*, *4* and *5*, respectively). c) Dependence on τ of the angle θ_{max} corresponding to the maximal CXTR and XTR intensity (curves *1* and *2*, respectively). Curve 3 is the direction $\theta = \sqrt{3}/\gamma/q_{dif}$

for the CXTR spectral distribution, one can obtain lengthy expressions (not given in this work) for the real CXTR spectral distribution.

Fig.1a and b show the angular distributions of CXTR photons produced at a vacuum–copper interface for $\lambda_r = 1$ Å at different values of γ in the case $\sigma_z = 2.5 \cdot 10^{-3}$ cm, $\sigma_r = 2 \cdot 10^{-3}$ cm. These bunch sizes correspond to the beam parameters for the TESLA x-ray FELs and the SLAC LCLS. As to the CXTR angular distributions, we can say that they have different behaviors, depending on the ratio $\tau = \gamma/q_{dif}$. If $\tau \ll 1$ (curves *1*, *2*, *3* in Fig.1) the maximum of the curves, which is determined by the transverse size of the bunch, occurs at approximately $\theta^{max}_{CXTR} = \theta_{dif} \simeq \sqrt{3/2}/q_{dif}$. This value is much smaller than the angle of maximal incoherent XTR, $\theta^{max}_{XTR} \simeq \sqrt{3}/\gamma$. In the case $\tau \gg 1$ the maximal radiation angle equals the maximal radiation angle of incoherent XTR (curve *5* in Fig.1b). If $\tau \sim 1$ (see curve *4* in Figs.1a,b), the maximal radiation angle decreases with increasing τ in the range from $\tau \simeq 0.32$ to $\tau \simeq 5$, and it remains smaller than both θ_{dif} and θ_{XTR}; this can be seen in Fig.1c, which shows the dependence on τ of the maximal angle. At $\tau \geq 5$ the maximal radiation angle approaches θ_{XTR}. The results of these numerical calculations show that by using angular discrimination one can separate the CXTR from the incoherent XTR, just as in the case of x-ray FELs [2,3], where the stimulated and spontaneous radiations can be separated by angular discrimination.

As to the CXTR spectral distributions shown in Fig.2 a,b, their width is of the order of expected SASE x-ray width, but they exhibit oscillations with periods equal to the resonant frequency (Fig.2b). By detecting these oscillations one can study the MB, at least at high values of N_b and b_1.

Fig.3 shows how the the normalized CXTR photon number depends on τ and γ (lower axis) for the DESY FEL bunch sizes. One can see that the behavior is different for $\tau \leq 1$ and $\tau \geq 1$ or for $\gamma \leq 10^6$ and $\gamma \geq 10^6$. In the latter case the photon number increases by

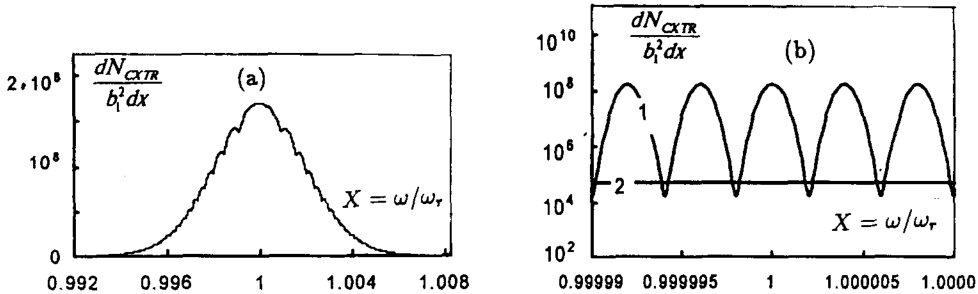


Fig.2. a) The envelope of the spectral distribution of the number of CXTR photons for a bunch at a copper–vacuum interface in a relatively wide frequency region around $\omega = \omega_r$. b) The spectral distribution in a narrow, central frequency region, showing the CXTR oscillations (curve *2*) and smooth behavior of the XTR spectrum for $\gamma = 5 \cdot 10^4$, $N_b = 6 \cdot 10^9$, and $\lambda_r = 1$ Å and with an energy spread $\Delta E/E = 10^{-3}$

a logarithmic law. For the parameters [2, 3] of the TESLA x-ray FEL and SLAC LCLS the values of N_{CXTR}/b_1^2 are equal to $2.5 \cdot 10^5$ and $3.1 \cdot 10^4$, respectively.

As we have said, measurement of the MB parameters is essential for SASE FELs, IFELs, and advanced methods of particle acceleration. In all these cases it is necessary to measure the particle grouping period and the modulation index at various distances from the undulator entrance. Measurement of the CXTR spectrum is of special interest and presents some specific difficulties, similar to those which will be encountered in SASE FELs [2,3]. The microbunched beam will be accompanied by undulator or SASE x-ray photons having much higher intensities and almost the same spectral distribution as the CXTR, but they can be separated by absorption in the relatively thick CXTR radiator foil at low intensities or by magnetic deflection of the beam. As is planned for the SASE FELs [2,3], the CXTR spectral measurements can be carried out after a large attenuation to avoid pile-up processes. From the experimentally measured CXTR spectrum and its maxima one can determine the grouping period. Measurement of the modulation index b_1 can be carried out by measuring the dependence of the CXTR yield on the number of particles in the bunches N_b or on the beam current. By measuring this dependence with the help of a simple ionization chamber and performing some calculations and fine spectral measurements, one can determine b_1 above certain values (depending on γ, N_b, etc).

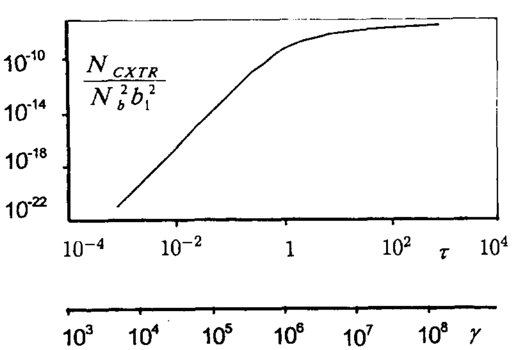


Fig.3. Normalized total number of CXTR photons versus τ or γ (the lower x axis) for the TESLA x-ray FEL beam parameters, $E = 25\,\mathrm{GeV}$, $\sigma_r = 20\,\mu\mathrm{m}$, $\sigma_z = 25\,\mu\mathrm{m}$, and $N_b = 6 \cdot 10^9$

Many simplifying assumptions have been made in the above calculations. For instance, we adopted a model of the particle distribution which took into account only the first harmonic of the MB. This assumption is justified by the expectation that the modulation indices b_2, b_3, etc. of the second and higher harmonics are much smaller, if no special measures are taken to enhance the higher harmonics. The correctness of this assumption is also supported by spectral measurements [5,6]. Another assumption, that MB does not affect the transverse particle distribution, is again justified by the experimental data [5]. The factor [13] taking the angular divergence of the beam into account may be neglected, because the primary beams of SASE FELs have very small emittances.

In conclusion, we note that although at present there are no very short bunches with bunch lengths $\sim 10^{-17} - 10^{-18}$ s, the fact that short laser pulses with lengths less than or of the order of the wavelength have been obtained recently leads one to hope that soon, when the difficulties connected with the Coulomb repulsion have been overcome, it will be possible to obtain charged particle beams with lengths less than 1 nm. Then the proposed method could be used for measuring the lengths of those beams as well as for observing and monitoring microbunching.

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