

## FLUCTUATIONS IN RANDOM $RL - C$ NETWORKS: NON-LINEAR $\sigma$ -MODEL DESCRIPTION

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Disordered  $RL - C$  networks are known to be an adequate model for describing fluctuations of electric fields in a random metal-dielectric composite. We show that under appropriate conditions the statistical properties of such a system can be studied in the framework of the Efetov's non-linear  $\sigma$ -model. This fact provides a direct link to the theory of Anderson localization.

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Optical properties of random metal-dielectric films (also known as cermets or semi-continuous metal films) attracted a lot of research interest recently, both theoretically and experimentally, see [1-4] and references therein. It was discovered that for metal concentrations close to the percolation threshold the absorption of microwave radiation in such materials fluctuates anomalously. In turn, these anomalous properties were traced back to high local field fluctuations detected in such compounds. A very insightful approach to the problem [1-4] is to represent the system as a large random network made of capacitors  $C$  and inductances  $L$ , the latter being in series with a weak resistor  $R$ . The network description naturally arises when discretizing the equations satisfied by the scalar potential of the electric field. The capacitors here are to model dielectric bridges, whereas isolated metallic granulas are indeed characterized by almost purely inductive response for frequencies  $\omega$  of radiation such that  $\omega_r \ll \omega \lesssim \omega_p$ , with  $\omega_p$  being the plasma frequency and  $\omega_r$  being the plasmon relaxation rate [1-4].

For frequencies close to  $\omega_0 = 1/(LC)^{1/2}$  an electromagnetic response of such a network is dominated by resonance effects as long as losses are small, i.e. the quality factor  $Q = (L/C)^{1/2}R^{-1}$  is large. The resonance frequencies can be determined as (generalized) eigenvalues of some linear lattice operator arising when solving the system of Kirchhoff equations  $\sum_j \sigma_{ij}(v_i - v_j) = 0$  for on-site potentials  $v_i \equiv v(\mathbf{r}_i)$  [4, 5]. Here  $\sigma_{ij}$  is the conductance between pair of nodes  $\mathbf{r}_i$  and  $\mathbf{r}_j$  if two nodes are connected by a direct bond and  $\sigma_{ij} = 0$  otherwise. In the simplest case, one can think of the network being connected to AC voltage by two external leads attached to lattice nodes with the coordinates  $\mathbf{r}_A$  and  $\mathbf{r}_B$ , the corresponding potentials being  $v_A = e^{-i\omega t}$  and  $v_B = 0$ , respectively. Omitting the common time-dependent factor it is easy to see that the amplitude of the potential  $v(\mathbf{r}_i)$  at an internal lattice node  $\mathbf{r}_i$  is given by:

$$v(\mathbf{r}_i) = \sum_j \left( \hat{D}^{-1} \right)_{ij} \sigma_{Aj}, \quad (1)$$

$$\hat{D}_{ij} = \left( \sigma_{Ai} + \sigma_{Bi} + \sum_{j \neq i} \sigma_{ij} \right) \delta_{ij} - (1 - \delta_{ij}) \sigma_{ij}.$$

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In a random  $RL - C$  network each nonzero conductance  $\sigma_{ij}$  at frequency  $f = \omega/2\pi$  is equal to either  $\sigma_0 = iC\omega$  or  $\sigma_1 = (R + iL\omega)^{-1}$ , with a specified probabilities (in what follows we concentrate on the case of equal probability for finding  $L$  and  $C$  bonds in the network). Then it is convenient to introduce "symmetric" variables  $h_{ij}$  such that  $h_{ij} = -1$  if  $\sigma_{ij} = \sigma_0$  and  $h_{ij} = 1$  if  $\sigma_{ij} = \sigma_1$ , so that  $\sigma_{ij} = e_{ij} \frac{1}{2} ([\sigma_0 + \sigma_1] + [\sigma_1 - \sigma_0] h_{ij})$ , with  $e_{ij} = 1$  for directly connected nodes and  $e_{ij} = 0$  otherwise. In terms of these variables we can write  $D = \hat{H} - \lambda \hat{W}$ , where

$$\hat{W}_{ij} = (Z + e_{Ai} + e_{Bi})\delta_{ij} - (1 - \delta_{ij})e_{ij}, \quad (2)$$

$$\hat{H}_{ij} = \left( \bar{h}_{Ai} + \bar{h}_{Bi} + \sum_{k \neq i} \bar{h}_{ik} \right) \delta_{ij} - (1 - \delta_{ij})\bar{h}_{ij}, \quad (3)$$

with  $Z = \sum_j e_{ij}$  standing for the coordination number of the lattice and  $\bar{h}_{ij} = h_{ij}e_{ij}$ . The frequency-dependent parameter  $\lambda$  is defined as

$$\lambda = \frac{\sigma_0 + \sigma_1}{\sigma_0 - \sigma_1} \approx \left( \frac{\omega}{\omega_0} - 1 \right) - \frac{i}{2Q} \equiv \text{Re}\lambda - i\frac{\Gamma}{2}, \quad (4)$$

where we have made use of  $\omega \approx \omega_0$  and  $Q \gg 1$ .

We see, that statistics of the scalar potential  $v(\mathbf{r})$  (and hence of the electric field  $\mathcal{E}_{ij}$  proportional to the voltage difference  $v(\mathbf{r}_i) - v(\mathbf{r}_j)$  on the bond  $ij$ ) is determined basically by properties of the operator  $\hat{H}$ . The operators of such a type acting on a lattice were suggested to be called Kirchhof Hamiltonians (KH) in [1]. Off-diagonal entries of such a Hamiltonian assume random values  $\pm 1$  for directly connected nodes. This property makes KH to be, in a sense, similar to a tight-binding Hamiltonian describing the motion of a quantum particle on a disordered lattice with an off-diagonal disorder. The latter model is a paradigmatic one in the theory of Anderson localization. That kind of analogy first discussed in [1] led the authors to relating the anomalous fluctuations of electric fields to localized properties of the corresponding eigenfunctions. Further numerical and experimental work confirmed the quantitative validity of the suggested picture.

At the same time, the question to which extent one can push forward the analogy between the Anderson model and the KH is far from being trivial. Indeed, the KH has a specific feature: the diagonal entries  $H_{ii}$  are strongly correlated with the off-diagonal ones  $H_{i \neq j}$ . It is known that correlations of various kinds can substantially modify the localisation behaviour, see e.g.[6]. Therefore, it is highly desirable to find an adequate approach allowing to shed more light on the question of equivalence between the models.

The main goal of the paper is to show that the equivalence indeed exists and the unifying concept is provided by the so-called Efetov's supermatrix non-linear  $\sigma$ -model (ENSM)[7]. The latter model is known to be the most powerful tool in understanding the fluctuation phenomena in disordered conductors last decade, see e.g. [8]. As a matter of fact, we derive ENSM from a version of the KH and thus provide a regular analytical background for the quantitative description of statistical properties of the semicontinuous films.

To derive ENSM from a microscopic random Hamiltonian one has to exploit some large parameter which physically controls the strength of the disorder. The experience of dealing with the usual tight binding models suggests that a role of such a parameter can be played, e.g. by a large radius of connectivity  $b$  (i.e. the large coordination number

$Z \sim b^d$ ][9, 10]. Formally, we consider a  $d$ -dimensional lattice of a linear size  $L$  with unit lattice spacing and a connectivity radius  $b$ . To facilitate bookkeeping of terms of the different order it is convenient to redefine  $e_{ij} \rightarrow e_{ij} Z^{-1/2}$  where  $e_{ij} = 1$  for  $|\mathbf{r}_i - \mathbf{r}_j| \leq b$  and  $e_{ij} = 0$  otherwise. The radius of connectivity  $b$  is chosen to satisfy  $1 \ll b \ll L$ . Both inequalities are important:  $b \gg 1$  allows one to map the problem to ENSM, whereas  $b \ll L$  is necessary to ensure the adequate description of effects of the Anderson localization. Indeed, as is shown recently [5] a full-connectivity  $LC$ -network with  $b = L$  can be mapped on the zero-dimensional version of the ENSM, which precludes the localization effects to be taken into account.

To demonstrate the mapping it is instructive to address the simplest nontrivial correlation function of the potentials:  $C_i(\Omega, \Gamma) = \langle v_{\omega_1}^*(\mathbf{r}_i) v_{\omega_2}(\mathbf{r}_i) \rangle$  where we introduce the frequency difference  $\Omega \propto (\omega_1 - \omega_2)/2\omega_0 \ll 1$  and the brackets stand for the disorder averaging<sup>2)</sup>.

Our starting expression is:

$$C_i(\Omega, \Gamma) = \left\langle \sum_{k_1, k_2} e_{Ak_1} e_{Ak_2} (\tilde{h}_{Ak_1} - \lambda_1^*) (\tilde{h}_{Ak_2} - \lambda_2) \times \right. \\ \left. \times \left[ \frac{1}{H - \lambda_1 W} \right]_{ik_1}^* \left[ \frac{1}{H - \lambda_2 W} \right]_{ik_2} \right\rangle. \quad (5)$$

To perform the disorder average we follow the standard procedure and represent the matrix element of the resolvent  $(H - \lambda W)^{-1}$  in terms of the Gaussian integral:

$$\left[ \frac{1}{H - (\text{Re}\lambda \pm i\Gamma/2)W} \right]_{ik} = \pm i \int \left[ \prod_{l=1}^N d\Psi_l(\pm) \right] s_l^*(\pm) s_k(\pm) \times \\ \times \exp\left\{ \pm \frac{i}{2} \sum_{m,n} \Psi_m^\dagger(\pm) [W_{mn}(\text{Re}\lambda \pm i\Gamma/2) - H_{mn}] \Psi_n(\pm) \right\} \quad (6)$$

over 4-component supervectors  $\Psi_l(\pm)$ ,

$$\Psi_l(\pm) = \begin{pmatrix} \mathbf{S}_l(\pm) \\ \boldsymbol{\eta}_l(\pm) \end{pmatrix}, \quad \mathbf{S}_l(\pm) = \begin{pmatrix} s_l(\pm) \\ s_l^*(\pm) \end{pmatrix}, \quad (7) \\ \boldsymbol{\eta}_l(\pm) = \begin{pmatrix} \chi_l(\pm) \\ \chi_l^*(\pm) \end{pmatrix}, \quad d\Psi_l = \frac{ds_l ds_l^*}{2\pi} d\chi_l^* d\chi_l,$$

with components  $s_l(+), s_l(-)$ ;  $l = 1, 2, \dots, N$  being complex commuting variables and  $\chi_l(+), \chi_l(-)$  forming the corresponding Grassmannian parts of the supervectors  $\Psi_l(\pm)$ .

To facilitate the presentation, it is appropriate to anticipate few facts whose validity can be verified by the same method as presented below. First of all, after averaging the double sum in the expression Eq.(5) is dominated in the limit  $1 \ll b \ll L$  by the diagonal terms with indices  $k_1 = k_2$ . Another fact which is useful to exploit from the very beginning is that all the resonance frequencies are concentrated in an interval of the order of  $\delta\omega/\omega_0 \sim Z^{-1/2}$  around  $\omega = \omega_0$ , so that the typical spacing  $\Delta$  between the

<sup>2)</sup> Actually, the lowest correlation function of the electric fields  $\mathcal{E}_{ij}(\omega) = v_\omega(\mathbf{r}_i) - v_\omega(\mathbf{r}_j)$  amounts to the local correlations of the scalar potential since, in fact, in the present model  $\langle v^*(\mathbf{r}_i) v(\mathbf{r}_i) \rangle \gg \langle v^*(\mathbf{r}_i) v(\mathbf{r}_j) \rangle$  for any  $i \neq j$ .

neighbouring resonances is of the order of  $\Delta \sim \omega_0/(NZ^{1/2})$ , with  $N \sim L^d$  being the total number of resonance frequencies. We anticipate nontrivial correlations occurring on the frequency scale  $\omega_1 - \omega_2 \sim \Delta$  [5]. For this reason we scale  $\text{Re}\lambda_{1,2} = (r \pm \Omega/2N)/Z^{1/2}$ , considering both  $r$  and  $\Omega$  to be of the order of unity. By the same reasoning we consider the losses to be small enough to ensure that  $\gamma = \Gamma(NZ^{1/2})$  is of the order of unity. Physically this requirement means that a typical resonance width  $\omega_0\Gamma$  is considered to be comparable with the typical resonance spacing  $\Delta$ .

With these facts in mind, we can easily average the products of the resolvents over  $\tilde{h}_{ij} = \pm 1/Z^{1/2}e_{ij}$  in the limit  $N \gg Z \gg 1$ . All further steps follow the method used in [5] (cf. [10, 11]) adopted to the present model. We have:

$$\begin{aligned} C_i(\Omega, \gamma) &= \frac{1}{Z} \sum_k e_{Ak} \int \left[ \prod_l d\Phi_l \right] s_i^*(-) s_i(+)^* s_k^*(+) s_k(-) \times \\ &\times \exp \left\{ \frac{i}{4NZ} (\Omega + i\gamma) \sum_{m < n} e_{mn} (\Phi_m^\dagger - \Phi_n^\dagger) (\Phi_m - \Phi_n) \right\} \times \\ &\times \exp \left\{ -\frac{1}{16Z} \sum_{m,n} e_{mn} \mathcal{K}(\Phi_m, \Phi_n) \right\}. \end{aligned} \quad (8)$$

Here the integration goes over the 8-component supervectors  $\Phi_l^\dagger = (\Psi_l^\dagger(+), \Psi_l^\dagger(-))$  and

$$\mathcal{K}(\Phi_a, \Phi_b) = -4ir (\Phi_a^\dagger - \Phi_b^\dagger) \hat{\Lambda} (\Phi_a - \Phi_b) + [(\Phi_a^\dagger - \Phi_b^\dagger) \hat{\Lambda} (\Phi_a - \Phi_b)]^2 \quad (9)$$

with  $\hat{\Lambda} = \text{diag}(1, 1, 1, 1, -1, -1, -1, -1)$ . Only those terms are left in the exponent which will later on contribute to the final expressions in the discussed limit.

Next step is to use the following functional Hubbard - Stratonovich transformation (cf.[10, 11, 5]):

$$\begin{aligned} \exp \left\{ -\frac{1}{16Z} \sum_{m,n} e_{mn} \mathcal{K}(\Phi_m, \Phi_n) \right\} &= \int \mathcal{D}(g) \exp \left\{ \frac{i}{8} \sum_{m=1}^N g_m(\Phi_m) \right\} \times \\ &\times \exp \left\{ -\frac{Z}{16} \sum_{mn} [\hat{e}^{-1}]_{mn} \int d\Phi_a d\Phi_b g_m(\Phi_a) \mathcal{C}(\Phi_a, \Phi_b) g_n(\Phi_b) \right\}, \end{aligned}$$

where  $[\hat{e}^{-1}]$  is the matrix inverse to the matrix  $\hat{e} = [e_{ij}]|_{i,j=1,\dots,N}$  and the kernel  $\mathcal{C}(\Phi_a, \Phi_b)$  is, in a sense, the inverse of a (symmetric) kernel  $\mathcal{K}(\Phi_a, \Phi_b)$ :

$$\int d\Phi \mathcal{K}(\Phi_a, \Phi) \mathcal{C}(\Phi, \Phi_b) = \delta(\Phi_a, \Phi_b), \quad (10)$$

with  $\delta(\Phi_a, \Phi_b)$  playing the role of a  $\delta$ -functional kernel in a space spanned by the functions  $g(\Phi)$ .

With the help of these relations one easily brings each term of the sum in Eq.(8) to the form:

$$\int \mathcal{D}(g) \mathcal{F}_{+,-}[g_i] \mathcal{F}_{-,+}[g_k] \exp[\mathcal{L}\{g\} + \delta\mathcal{L}_1\{g\}], \quad (11)$$

where

$$\mathcal{F}_{\pm, \mp}[g] = \frac{\int d\Phi s^*(\pm)s(\mp) \exp\left\{\frac{i}{8}g(\Phi)\right\}}{\int d\Phi \exp\left\{\frac{i}{8}g(\Phi)\right\}}, \quad (12)$$

$$\begin{aligned} \mathcal{L}\{g\} &= \sum_m \ln \int d\Phi \exp\left\{\frac{i}{8}g_m(\Phi)\right\} - \\ &- \frac{Z}{16} \sum_{m,n} [e^{-1}]_{mn} \int d\Phi_a d\Phi_b g_m(\Phi_a) \mathcal{C}(\Phi_a, \Phi_b) g_n(\Phi_b), \end{aligned} \quad (13)$$

$$\begin{aligned} \delta\mathcal{L}_1\{g\} &= i \frac{(\Omega + i\gamma)}{4NZ} \sum_{m < n} e_{mn} \times \\ &\times \frac{\int d\Phi_m d\Phi_n \exp\left\{\frac{i}{8}[g_m(\Phi_m) + g_n(\Phi_n)]\right\} (\Phi_m^\dagger - \Phi_n^\dagger) (\Phi_m - \Phi_n)}{\int d\Phi_m d\Phi_n \exp\left\{\frac{i}{8}[g_m(\Phi_m) + g_n(\Phi_n)]\right\}} \end{aligned} \quad (14)$$

and we restricted ourselves by the leading order term in  $\delta\mathcal{L}_1\{g\}$  which is only a small correction to  $\mathcal{L}\{g\}$ .

Next step is to evaluate the functional integral over  $g(\Phi)$  by the saddle-point method, justified by two large parameters:  $Z$  and  $N$ . The saddle-point configuration  $g_m^{(s)}(\Phi)$  can be found by requiring the vanishing variation of the "action"  $\mathcal{L}\{g\}$  and satisfies the following system of equations:

$$Z \sum_n [e^{-1}]_{mn} g_n^{(s)}(\Phi_a) = i \frac{\int d\Phi_b \mathcal{K}(\Phi_a, \Phi_b) \exp\left\{\frac{i}{8}g_m^{(s)}(\Phi_b)\right\}}{\int d\Phi_b \exp\left\{\frac{i}{8}g_m^{(s)}(\Phi_b)\right\}}. \quad (15)$$

When deriving Eq.(15) we have used Eq.(10).

Given the form of the kernel Eq.(9) and exploiting  $Z \sum_n [e^{-1}]_{mn} = 1$  one can find a space-independent solution  $g_n^{(s)}(\Phi) \equiv g^{(s)}(\Phi)$  to equation Eq.(15):

$$g^{(s)}(\Phi_a) = 4(r - G_1)(\Phi_a^\dagger \hat{\Lambda} \Phi_a) + 4iG_2(\Phi_a^\dagger \Phi_a) + i(\Phi_a^\dagger \hat{\Lambda} \Phi_a)^2 \quad (16)$$

provided the real coefficients  $G_1, G_2$  are solutions of the system of two conjugate equations:

$$G_2 \pm iG_1 = \int_0^\infty du \exp\left\{\pm \frac{i}{2}u(r - G_1 \pm iG_2) - \frac{u^2}{8}\right\}. \quad (17)$$

Hints to verifying such a solution can be found in [5].

For further analysis it is very important that a solution to equations Eq.(17) exists for arbitrary  $-\infty < r < \infty$  such that  $G_2(r) > 0$ . Actually, the mean density of resonances is merely given by  $\rho(r) = \frac{1}{\pi}G_2(r)$  [5].

The most important consequence of the existence of the solution  $g^{(s)}(\Phi_a)$  in the form Eq.(16) with  $G_2 \neq 0$  is actually the simultaneous existence of a whole *continuous manifold* of the saddle points parametrized as:

$$g_T(\Phi_a) = g^{(s)}(\hat{T}\Phi_a), \quad \hat{T}^\dagger \hat{\Lambda} \hat{T} = \hat{\Lambda}. \quad (18)$$

If it hadn't been for the condition  $G_2 \neq 0$  all these solutions would trivially coincide:  $g_T(\Phi) \equiv g^{(s)}(\Phi)$  for any  $\hat{T}$  defined as above. In the actual case presence of the combination  $\Phi_a^\dagger \Phi_a$  which is *not invariant* with respect to a transformation  $\Phi_a \rightarrow \hat{T}\Phi_a$  ensures the existence of the mentioned manifold. This fact is just a particular manifestation of the

phenomenon of spontaneous breakdown of symmetry. Different nontrivial solutions are actually parametrized by the supermatrices  $\hat{T}$  which are elements of a graded coset space  $UOSP(2, 2/4)/UOSP(2/2) \otimes UOSP(2/2)$ .

As a result, the functional integral over  $g_m(\Phi)$  is dominated by "Goldstone modes" slowly changing in space and parametrized as:  $g_m^{(G)}(\Phi) = g^{(s)}(\hat{T}_m \Phi)$ , with the matrices  $\hat{T}_m$  which depend on the lattice site index  $m = 1, \dots, N$ . Our next step is to determine the effective action for these modes that is  $\mathcal{L}\{g_m^{(G)}\} + \delta\mathcal{L}_1\{g_m^{(G)}\}$ .

First of all we notice that:

$$\int d\Phi \exp \left\{ \frac{i}{8} g_m^{(G)}(\Phi) \right\} = 1,$$

which allows one to perform the following manipulations:

$$\begin{aligned} & \int d\Phi_a d\Phi_b g_m^{(G)}(\Phi_a) \mathcal{C}(\Phi_a, \Phi_b) g_n^{(G)}(\Phi_b) = \\ & = i \int d\Phi g_n^{(G)}(\Phi) \exp \left\{ \frac{i}{8} g_m^{(G)}(\Phi) \right\} = i \int d\Phi g^{(s)}(\hat{T}_n \hat{T}_m^{-1} \Phi) \exp \left\{ \frac{i}{8} g^{(s)}(\Phi) \right\} = \\ & = -4G_2^2 \text{Str} \left( \hat{T}_n \hat{T}_m^{-1} \hat{\Lambda} \left( \hat{T}_n \hat{T}_m^{-1} \right)^{-1} \hat{\Lambda} \right) = \\ & = -4(\pi\rho)^2 \text{Str} \left( \hat{T}_m^{-1} \hat{\Lambda} \hat{T}_m \hat{T}_n^{-1} \hat{\Lambda} \hat{T}_n \right). \end{aligned} \quad (19)$$

Here we first exploited the saddle-point equation Eq.(15) together with Eq.(10) and then performed a change of variables:  $\hat{T}_m \Phi \rightarrow \Phi$  which does not effect the measure  $d\Phi$  because of the (pseudo) unitarity of the matrices  $\hat{T}$ . Then the integral can be readily evaluated with help of the explicit form Eq.(16) (see e.g. examples of similar calculations in [5, 11]) and brought to the final form by employing the cyclic permutation and the mentioned relation between  $G_2$  and the density of resonances  $\rho(r)$ . In the very same way we also find:

$$\begin{aligned} \delta\mathcal{L}_1\{g\} &= \frac{(\Omega + i\gamma)}{4NZ} G_2 \sum_{m < n} e_{mn} \text{Str} \left( \hat{T}_m^{-1} \hat{\Lambda} \hat{T}_m \hat{\Lambda} + \hat{T}_n^{-1} \hat{\Lambda} \hat{T}_n \hat{\Lambda} \right) = \\ &= \pi\rho(\Omega + i\gamma) \frac{1}{4N} \sum_m \text{Str} \left( \hat{T}_m^{-1} \hat{\Lambda} \hat{T}_m \hat{\Lambda} \right). \end{aligned} \quad (20)$$

The pre-exponential factors  $\mathcal{F}_{+,-}[g_i]$  are calculated analogously and are given by

$$\mathcal{F}_{+,-}[g_i] = \pi\rho \left[ \hat{T}_i^{-1} \hat{\Lambda} \hat{T}_i \right]^{2,5}; \quad \mathcal{F}_{-,+}[g_k] = \pi\rho \left[ \hat{T}_k^{-1} \hat{\Lambda} \hat{T}_k \right]^{6,1},$$

where the indices of supermatrix elements are inherited from the structure of the diadic product  $\Phi \otimes \Phi^\dagger$ .

We see, that calculating the correlation function Eq.(8) in the limit  $L \gg b \gg 1$  amounts to evaluating the following integral over the set of supermatrices  $\hat{Q}_m = \hat{T}_m^{-1} \hat{\Lambda} \hat{T}_m$ :

$$C_i = \frac{(\pi\rho)^2}{Z} \sum_k e_{Ak} \int \left[ \prod_{l=1}^N d\hat{Q}_l \right] Q_i^{2,5} Q_k^{6,1} e^{\mathcal{L}(\hat{Q})}, \quad (21)$$

$$\mathcal{L}(\hat{Q}) = \frac{Z(\pi\rho)^2}{4} \sum_{m,n=1}^N [e^{-1}]_{mn} \text{Str} \hat{Q}_m \hat{Q}_n + \quad (22)$$

$$+i \frac{\pi\rho(\Omega + i\gamma)}{4N} \sum_{m=1}^N \text{Str} \left( \hat{Q}_m \hat{\Lambda} \right).$$

The action Eq.(22) is actually equivalent to the discretized version of the supermatrix nonlinear  $\sigma$ -model well-studied in the context of the Anderson localization [7, 8]. Indeed, following [9] notice that the condition  $b \gg 1$  ensures slow variation of the matrices  $\hat{Q}_m$  with index  $m$ , so that it is legitimate to pass from the lattice to continuum, when the action assumes the standard form:

$$\mathcal{L}(\hat{Q}) = \frac{\pi\rho}{8} \int d\mathbf{r} \text{Str} \left[ D (\nabla \hat{Q})^2 + 2i \frac{(\Omega + i\gamma)}{V} \hat{Q}(\mathbf{r}) \hat{\Lambda} \right], \quad (23)$$

where  $D = \pi\rho Z^{-1} \sum_{|\mathbf{r}| < b} \mathbf{r}^2$  plays the role of the effective diffusion constant and  $V = \int d\mathbf{r}$ . The large value of  $D \propto b^2$  ensures that a typical spatial scale  $\xi$  of variation of  $\hat{Q}(\mathbf{r})$  is large:  $\xi \propto b^2$ . Thus, for distances of the order of  $b$  the matrices  $\hat{Q}$  do not change and therefore:

$$C_i = (\pi\rho)^2 \int \mathcal{D}\hat{Q}(\mathbf{r}) Q^{2,5}(\mathbf{r}_i) Q^{6,1}(\mathbf{r}_A) e^{\mathcal{L}(\hat{Q})}. \quad (24)$$

In conclusion, we managed to express the correlation function of the scalar potentials in terms of ENSM. Similar reduction is possible also for other quantities of interest. As is well-known[7, 8], explicit evaluation of the integrals of the type Eq.(24) crucially depends on the parameter  $g = 2\pi\rho DL^{d-2}$ . For  $g \rightarrow \infty$  the integral is dominated by the constant configuration:  $\hat{Q}(\mathbf{r}) = \hat{Q}_0$  and the result for Eq.(24) is very simple:  $\bar{C}_i = 2i\pi\rho(\mathbf{r})/(\Omega + i\gamma)$ . This is the so-called "zero-dimensional" limit corresponding to the infinite-range connectivity model [5]. One can take into account weak localization effects finding  $1/g$  corrections in any dimension  $d$ , see e.g.[8]. For a quasi one-dimensional lattice one can calculate integrals exactly in the limit  $\Omega, \eta \rightarrow 0$  [9]. One should be able also to study singular parts of higher correlation functions. These questions are left for further investigations.

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