

MONOPOLE, HALF-QUANTUM VORTICES AND NEXUS IN CHIRAL SUPERFLUIDS AND SUPERCONDUCTORS

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Two exotic objects are still not identified experimentally in chiral superfluids and superconductors. These are the half-quantum vortex, which plays the part of the Alice string in relativistic theories [1] (A.S.Schwarz, Nucl. Phys. **B208**, 141 (1982)), and the hedgehog in the $\hat{\mathbf{l}}$ field, which is the counterpart of the Dirac magnetic monopole. These two objects of different dimensionality are topologically connected. They form the combined object which is called nexus [2] (John M.Cornwall, hep-th/9911125; Phys. Rev. **D59**, 125015 (1999); Phys. Rev. **D58**, 105028 (1998)) or center monopole [3] (N.N.Chernodub, M.I.Polikarpov, A.I.Veselov and M.A.Zubkov, Nucl. Phys. Proc. Suppl. **73**, 575 (1999)) in relativistic theories. Such combination will allow us to observe half-quantum vortices and monopoles in several realistic geometries.

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In relativistic quantum fields nexus is the monopole, in which N vortices of the group Z_N meet at a center (nexus) provided the total flux of vortices adds to zero (mod N) [2, 3, 4]. In a chiral superfluid with the order parameter of the $^3\text{He-A}$ type, the analog of the nexus is the hedgehog in the $\hat{\mathbf{l}}$ field, in which 4 vortices meet, each with the circulation quantum number $N = 1/2$. The total topological charge of the four vortices is $N = 2$ which is equivalent to $N = 0$ because the homotopy group, which describes the $^3\text{He-A}$ vortices, is $\pi_1 = Z_4$ [5], and thus $N = 0 \pmod{2}$. Each $N = 1/2$ vortex plays the part of the $1/4$ fraction of the "Dirac string" terminating on the hedgehog, while the hedgehog in the $\hat{\mathbf{l}}$ field plays the part of the Dirac magnetic monopole: The distribution of the vector potential of the electromagnetic field \mathbf{A} in the vicinity of the hedgehog in the electrically charged version of the $^3\text{He-A}$ (the chiral p -wave superconductor) is similar to that in the vicinity of magnetic monopole (see e.g. [6-8]).

The order parameter describing the vacuum manifold in chiral p -wave superfluid/superconductor ($^3\text{He-A}$ and also possibly the layered superconductor Sr_2RuO_4 [9]) is

$$A_{\alpha i} = \Delta \hat{d}_\alpha (\hat{e}_i^{(1)} + i\hat{e}_i^{(2)}). \quad (1)$$

Here $\hat{\mathbf{d}}$ is the unit vector of the spin-space anisotropy; $\hat{e}^{(1)}$ and $\hat{e}^{(2)}$ are unit mutually orthogonal vectors in the orbital space, they determine the superfluid velocity of the chiral condensate $\mathbf{v}_s = \frac{\hbar}{2m} \hat{e}_i^{(1)} \nabla \hat{e}_i^{(2)}$, where $2m$ is the mass of the Cooper pair; the orbital momentum vector is $\hat{\mathbf{l}} = \hat{e}^{(1)} \times \hat{e}^{(2)}$. The half-quantum vortex results from the identification of the points $\hat{\mathbf{d}}$, $\hat{e}^{(1)} + i\hat{e}^{(2)}$ and $-\hat{\mathbf{d}}$, $-(\hat{e}^{(1)} + i\hat{e}^{(2)})$, which correspond to the same order parameter Eq.(1). It is the combination of the π -vortex and π -disclination in the $\hat{\mathbf{d}}$ field:

$$\hat{\mathbf{d}} = \hat{\mathbf{x}} \cos \frac{\phi}{2} + \hat{\mathbf{y}} \sin \frac{\phi}{2}, \quad \hat{e}^{(1)} + i\hat{e}^{(2)} = e^{i\phi/2} (\hat{\mathbf{x}} + i\hat{\mathbf{y}}), \quad (2)$$

The hedgehog in the orbital momentum field, $\hat{\mathbf{l}} = \hat{\mathbf{r}}$, produces the superfluid velocity field (or the vector potential in the corresponding superconductor):

$$\mathbf{v}_s = \frac{e}{mc} \mathbf{A}, \quad \mathbf{A} = \sum_a \mathbf{A}^a, \quad (3)$$

where \mathbf{A}^a is the vector potential for the Dirac monopole with the a -th Dirac string with the topological charge N_a (number of circulation quanta). Choosing the spherical coordinate system (r, θ, ϕ) in such a way that the string a occupies the lower half-axis $z < 0$, the vector potential \mathbf{A}^a of such string can be written as [8]:

$$\mathbf{A}^a = \frac{\hbar c}{4e r} N_a \hat{\phi} \frac{1 - \cos \theta}{\sin \theta}. \quad (4)$$

The superfluid vorticity and the corresponding magnetic field in superconductor are

$$\nabla \times \mathbf{v}_s = -\frac{\hbar}{4m r^3} \sum_a N_a \mathbf{r} + \frac{\hbar}{2m} \sum_a N_a \int_0^R dr \delta(\mathbf{r} - \mathbf{r}_a(r)), \quad (5)$$

$$\mathbf{B} = -\frac{\hbar c}{4e r^3} \sum_a N_a \mathbf{r} + \frac{\hbar c}{2e} \sum_a N_a \int_0^R dr \delta(\mathbf{r} - \mathbf{r}_a(r)), \quad \sum_a N_a = -2. \quad (6)$$

Here $\mathbf{r}_a(r)$ is the position of the a -th line, assuming that the lines are emanating radially from the monopole, i.e. the coordinate along the line is the radial coordinate. The regular part of the magnetic field corresponds to the monopole with the magnetic charge $g = \hbar c/2e$, the magnetic flux $4\pi g$ of the monopole is supplied by the Abrikosov vortices. The lowest energy of the monopole occurs when all the vortices emanating from the monopole have the lowest circulation number: this means that there must be four vortices with $N_1 = N_2 = N_3 = N_4 = -1/2$.

The half-quantum vortices are accompanied by the spin disclinations. Assuming that the $\hat{\mathbf{d}}$ -field is confined in the plane the disclinations can be characterized by the winding numbers ν_a which have values $\pm 1/2$ in half-quantum vortices. The corresponding spin-superfluid velocity \mathbf{v}_{sp} is

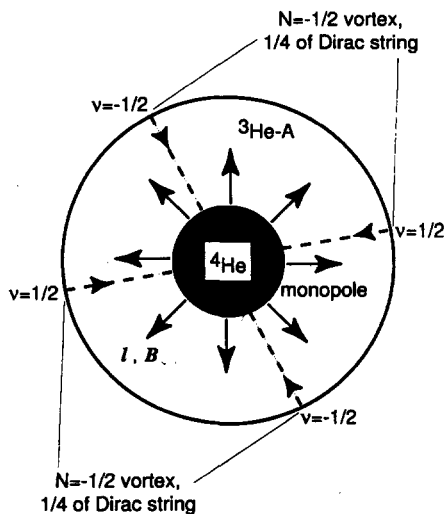
$$\mathbf{v}_{sp} = \frac{e}{mc} \sum_{a=1}^4 \nu_a \mathbf{A}^a, \quad \sum_{a=1}^4 \nu_a = 0, \quad (7)$$

where the last condition means the absence of the monopole in the spin sector of the order parameter. Thus we have $\nu_1 = \nu_2 = -\nu_3 = -\nu_4 = 1/2$.

The spin-orbit coupling can be neglected if the size of the bubble is less than spin-orbit length (about $10 \mu\text{m}$ in $^3\text{He-A}$). Assuming that the superfluid velocity is everywhere perpendicular to $\hat{\mathbf{l}}$ and has a form $\mathbf{v}_s = \tilde{\mathbf{v}}_s(\theta, \phi)/r$, the energy of the nexus in the spherical bubble of radius R is

$$\begin{aligned} E &= \int_0^R r^2 dr \int d\Omega \left(\frac{1}{2} \rho_s \mathbf{v}_s^2 + \frac{1}{2} \rho_{sp} \mathbf{v}_{sp}^2 \right) = R \int d\Omega \left(\frac{1}{2} \rho_s \tilde{\mathbf{v}}_s^2 + \frac{1}{2} \rho_{sp} \tilde{\mathbf{v}}_{sp}^2 \right) = \\ &= \frac{1}{2} R \int d\Omega \left((\rho_s + \rho_{sp}) \left[(\bar{\mathbf{A}}^1 + \bar{\mathbf{A}}^2)^2 + (\bar{\mathbf{A}}^3 + \bar{\mathbf{A}}^4)^2 \right] + \right. \\ &\quad \left. + 2(\rho_s - \rho_{sp}) (\bar{\mathbf{A}}^1 + \bar{\mathbf{A}}^2) (\bar{\mathbf{A}}^3 + \bar{\mathbf{A}}^4) \right), \quad \bar{\mathbf{A}}^a(\theta, \phi) = \frac{mcr}{e} \mathbf{A}^a. \end{aligned} \quad (8)$$

In the simplest case, which occurs in the ideal Fermi gas approximation when the Fermi liquid corrections are neglected, one has $\rho_s = \rho_{sp}$ [10]. In this case the $1/2$ -vortices with positive spin current circulation ν do not interact with $1/2$ -vortices with negative ν . The energy minimum occurs when the orientations of two positive- ν vortices are opposite, so that these two $\frac{1}{4}$ fractions of the Dirac strings form one line along the diameter (see Figure). The same happens for the other fractions with negative ν . The mutual orientations of the two diameters is arbitrary in this limit. However, in real $^3\text{He-A}$ one has $\rho_{sp} < \rho_s$ [10]. If ρ_{sp} is slightly smaller than ρ_s , the positive- ν and negative- ν strings repel each other, so that the equilibrium angle between them is $\pi/2$. In the extreme case $\rho_{sp} \ll \rho_s$, the ends of four half-quantum vortices form vertices of a regular tetrahedron.



Arrows outward show distribution of the orbital momentum $\hat{\mathbf{l}}$ field and simultaneously the distribution of superfluid vorticity $\nabla \times \mathbf{v}_s$ in superfluid $^3\text{He-A}$ or of magnetic field \mathbf{B} in a chiral superconductor. The arrows inward show the direction of the vorticity or magnetic flux concentrated in 4 half-quantum vortices (dashed lines). The charge $\nu = \pm 1/2$ is the number of the circulation quanta of the spin current velocity \mathbf{v}_{sp} . The stability of the monopole in the center of the droplet is supported by the cluster of the ^4He liquid, which provides the radial boundary condition for the $\hat{\mathbf{l}}$ -vector. The cluster forms the core of the monopole.

Such monopole can be experimentally realized in the mixed $^4\text{He}/^3\text{He}$ droplets obtained via the nozzle beam expansion of the He gases [11]. The ^4He component of the mixture forms the cluster in a central region of the droplet [11]. If the size of the cluster is comparable with the size of the droplet, the radial distribution of the $\hat{\mathbf{l}}$ vector is stabilized by the boundary conditions on the surface of the droplet and on the boundary of the cluster (see Figure). The ^4He cluster plays the part of the core of the nucleus. The half-quantum vortices emanating from the nucleus are well defined if the radius of the droplet exceeds the coherence length $\xi \sim 200 - 500 \text{ \AA}$.

In a p -wave superconductor such monopole will be formed in a thin spherical layer. In Sr_2RuO_4 superconductor the spin-orbit coupling between the spin vector $\hat{\mathbf{d}}$ and crystal lattice seems to align the $\hat{\mathbf{d}}$ vector along $\hat{\mathbf{l}}$ [12]. In this case the half quantum vortices are energetically unfavourable, and instead of 4 half-quantum vortices one would have 2 singly quantized vortices in the spherical shell.

Monopole of this kind can be formed also in the so called ferromagnetic Bose condensate in optical traps. Such condensate is described by vector or spinor chiral order parameter [13].

There are interesting properties of the system related to the fermionic spectrum of such objects. In particular, the number of fermion zero modes on $N = 1/2$ vortex under

discussion is twice less than that on the vortex with $N = 1$. This is because such $N = 1/2$ vortex can be represented as the $N = 1$ vortex in one spin component with no vortices in another spin component. Thus, according to [14], in the core of the $N = 1/2$ vortex there is one fermionic level (per 2D layer) with exactly zero energy. Since the zero-energy level can be either filled or empty, there is an entropy $(1/2) \ln 2$ per layer related to the vortex. The factor $(1/2)$ appears because the particle excitation coincides with the antiparticle (hole) excitation in superconductors, i.e. the quasiparticle is a Majorana fermion, see also [15]. Such fractional entropy also arises in the Kondo problem [16]. According to [17], the $N = 1$ vortex has spin $S = 1/4$ per layer, this implies the spin $S = 1/8$ per layer for $N = 1/2$ vortex. Similarly the anomalous fractional charge of the $N = 1/2$ vortex is $1/2$ of that discussed for the $N = 1$ vortex [18].

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