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COMMENT ON THE PAPER "SUBWAVELENGTH DIAMETER OF LIGHT BEAMS IN ACTIVE MEDIA"

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In a recent paper [1] Kuznetsova considered the case of a cylindrical waveguide containing an amplifying medium and concluded that there is no frequency cutoff for such a case: for an arbitrary small radius a of a waveguide (however smaller than the cutoff radius a_{cut}) and an arbitrary small amplification δ (thus the complex permittivity of a medium is $\chi = \epsilon - i\delta$) light can propagate through such a waveguide with an amplification. This possibility, if it exists, of course, would be of an extremal importance for near-field optics and fiber communications.

But it is not the case, because an analysis given in [1] is incorrect. The derived conclusions are based on the cylindrical wave equation to describe the electric field $\mathbf{E}$ for the spherically symmetric TE mode (in this comment we follow the notation and concrete example given in [1]; the consideration is indeed similar for all waveguide modes):

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial}{\partial \rho}\Big(\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E)\Big) + \epsilon\frac{\omega^2}{c^2}E = 0 \tag{1}$$

whose solution for the boundary condition

$$E(\rho = a, z) = 0 \tag{2}$$

for the case of a real permittivity is well known: $E = AJ_0(q\rho)\exp(pz)$. Here J_0 is the Bessel function of zero order, $q = 3.83/a$ and p can be found from the relation:

$$p^2 = q^2 - \epsilon\frac{\omega^2}{c^2}. \tag{3}$$

Kuznetsova generalized eq. (3) for the case of a complex permittivity, thus the complex coefficient p can be found as a square root of the right side of (3), and arrived to the aforementioned conclusions that *propagating and amplifying* wave exists for such a waveguide. This conclusion is wrong and due to the incorrect selection of signs of the real and imaginary parts of the coefficient p, made on the nuclear basis that "it is evident that the direction of growth of the wave is the same as the direction of propagation".

This mistake can be easily understood from the following example. For the case when $a << a_{cut}$ and δ is an arbitrary small, Kuznetsova found that amplification of the wave inside the waveguide is approximately equal to the q, i. e. the same as the damping for the case of small losses or an empty waveguide, and explained that this is due to the fact that "the wave propagates almost perpendicularly to the waveguide" (?) thus acquiring a necessary gain coefficient during this long way in an amplifying medium. Well, but does it mean that the damping for the case of the positive δ (losses) is also due to such a "perpendicular" propagation? And what to do when there are no losses at all but damping is the same?

Indeed, the correct selection of signs of the real and imaginary parts of the coefficient p is different from that given in [1]. (This is especially clear when one will directly substitute an expression $E = AJ(\rho)\exp(p'z + ip''z)$ into (1) and then use the theorem that Bessel functions with the order exceeding-1 has only real zeroes [2] to fulfill the boundary conditions (2). We will not do it here due to the lack of space.) The only one physically reasonable solution is:

$$(p')^2 = \frac{1}{2}(\gamma^2 + \sqrt{\gamma^4 + \delta^2\frac{\omega^4}{c^4}}), \quad p'' = \frac{\delta}{2p'}\frac{\omega^2}{c^2}$$

and the negative sign should be used when finding the p' from the square root. (Here $\gamma^2 = q^2 - \epsilon\frac{\omega^2}{c^2}$ is the square of the damping constant for an empty waveguide (3), which is positive when $a < a_{cut}$.)

This solution describes an evanescent nonpropagating wave decreasing exponentially as $\exp(-|p|'z)$ for the case of a waveguide with the radius smaller than the cutoff radius a_{cut}. More than, the rate of damping of this wave does not depend on the sign of δ and is larger for both amplifying and absorbing media in comparison with an empty waveguide. Indeed, it is not so surprising, because physically the cutoff phenomenon is nothing more than a *reflection* of the propagating wave from subwavelength apertures governed by the phase relations. Existence of an amplifying medium in the other side of an aperture can not change fundamentally the conditions of such a reflection. What it can do, that is to increase the reflectivity coefficient which can be higher than unity as it takes place, for example, for the case of total internal reflection of light from an amplifying medium [3]: some amount of energy can be added to the reflected light for such a case. Thus, unfortunately, bright subwavelength-size sources of light can not be produced by the small subwavelength-aperture waveguides filled with an amplifying medium, and other ways to solve this fundamental for the near field optics problem should be found.

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3. F. Schuller, G. Niehnuis, and M. Ducloy, Phys. Rev. **A43**, 443 (1991).