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PRECOCIOUS ASYMPTOPIA FOR CHARM FROM THE RUNNING BFKL

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The running BFKL equation gives rise to a series of moving poles in the complex j-plane. Corresponding eigenfunctions (color dipole cross sections) are the oscillating functions of the color dipole size r. The first nodes for all sub-leading solutions (color dipole cross sections) accumulate at $r_1 \sim 0.1$ fm. Therefore the processes dominated by the dipole sizes $r \sim r_1$ are free of sub-leading BFKL corrections. A practically important example – the leptoproduction of charm. In a wide range of Q^2 the calculated $F_2^{cc}(x, Q^2)$ is exhausted by the leading BFKL pole and gives a perfect description of the experimental data.

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The generalized Balitskii – Fadin – Kuraev – Lipatov (BFKL [1]) equation for the interaction cross section $\sigma(x,r)$ of the color dipole r with the target reads [2]

$$\frac{\partial\sigma(x,r)}{\partial\log(1/x)} = \mathcal{K}\otimes\sigma(x,r), \tag{1}$$

where x is the Bjorken variable. The kernel $\mathcal{K}$ is related to the flux of the Weizsäcker-Williams soft gluons $|\mathbf{E}(\rho_1) - \mathbf{E}(\rho_2)|^2$. The Asymptotic Freedom (AF) dictates that the chromoelectric fields $\mathbf{E}(\rho)$ be calculated with the running QCD charge $g_S(r_m) = \sqrt{4\pi\alpha_S(r_m)}$ taken at shortest relevant distance $r_m = \min\{r,\rho\}$ and

$$\mathbf{E}(\rho) = g_S(r_m)\rho/\rho^2 \times (\text{screening factor}).$$

Within the infrared regularization scheme described in [2–5]

$$\mathbf{E}(\rho) = g_S(r_m)\frac{\rho}{\rho R_c}K_1(\rho/R_c)\,, \tag{2}$$

where $K_1(x)$ is the modified Bessel function. Our numerical results are for the Yukawa screening radius $R_c = 0.27$ fm. The anlysis of the lattice QCD data on the field strength correlators suggests similar R_c [6]. The so introduced running coupling may not exhaust all NLO effects but it correctly describes the crucial enhancement of long distance, and suppression of short distance, effects by AF.

Our findings on the running BFKL equation which are of prime importance for the problem under discussion are as follows [7, 8]. The spectrum of the running BFKL equation is a series of moving poles $\mathbb{P}_n$ in the complex j-plane with eigenfunctions

$$\sigma_n(x,r) = \sigma_n(r)\exp(\Delta_n\log(1/x)) \tag{3}$$

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being a solution of

$$\mathcal{K} \otimes \sigma_n = \Delta_n \sigma_n(r). \tag{4}$$

The leading eigen-function $\sigma_0(r)$ is node free. The sub-leading $\sigma_n(r)$ has n nodes. The intercepts Δ_n closely, to better than 10%, follow the law

$$\Delta_n = \frac{\Delta_0}{(n+1)} \tag{5}$$

suggested earlier by Lipatov [9] . The intercept of the leading pole trajectory, with our specific choice of the infrared regulator, $R_c = 0.27\,\text{fm}$, is $\Delta_0 \equiv \Delta_{\mathbf{I\!P}} = 0.4$. The sub-leading $\sigma_n(r)$ represented in terms of $\mathcal{E}(r) = \sigma_n(r)/r$, to a crude approximation is similar to Lipatov's quasi-classical eigenfunctions [9],

$$\mathcal{E}_n(r) \sim \cos[\phi(r)].$$

With $R_c = 0.27\,\text{fm}$ the node of $\sigma_1(r)$ is located at $r = r_1 \simeq 0.05 - 0.06\,\text{fm}$, for larger n the first node moves to a somewhat larger $r \sim 0.1\,\text{fm}$. Hence, $\sigma(x, r_1)$ is dominated by $\sigma_0(x, r_1)$. This observation explains the precocious asymptopia for the dipole cross section,

$$\sigma(x,r) \propto (1/x)^{\Delta_{\mathbf{I\!P}}},$$

at $r \sim 0.1\,\text{fm}$ derived previously from the numerical studies of the running BFKL equation [3 – 5]. Consequently, zooming at $\sigma(x, r_1)$ one can readily measure $\Delta_{\mathbf{I\!P}}$. The point we want to make here is that because $r_1 \sim 1/m_c$, the excitation of open charm provides the desired zooming. Indeed, we shall demonstrate that the effect of suppression of the sub-leading BFKL terms in $F_2^{cc}(x, Q^2)$ is remarkably strong[3)].

The color dipole representation for the charm structure function (SF) reads [11]

$$F_2^{cc}(x,Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\langle\sigma\rangle = \frac{Q^2}{4\pi\alpha_{em}}\int_0^1 dz \int d^2\mathbf{r}|\Psi^{cc}(z,r)|^2\sigma(x,r), \tag{6}$$

Starting with the BFKL-Regge expansion

$$\sigma(x,r) = \sigma_0(r)(x_0/x)^{\Delta_0} + \sigma_1(r)(x_0/x)^{\Delta_1} + \sigma_2(r)(x_0/x)^{\Delta_2} + \ldots \tag{7}$$

with Δ_n determined in [7, 8] we arrive at the BFKL-Regge expansion for the charm SF

$$F_2^{cc}(x,Q^2) = \sum_n f_n^{cc}(Q^2)(x_0/x)^{\Delta_n}, \tag{8}$$

where the charm eigen-SF is as follows

$$f_n^{cc}(Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\langle\sigma_n\rangle. \tag{9}$$

In conjunction with the explicit form of the $c\bar{c}$ light-cone wave function, $\Psi^{cc}(z,r)$ [11],

$$|\Psi_T^{cc}(z,r)|^2 = \frac{8\alpha_{em}}{3(2\pi)^2}\left\{\left[z^2 + (1-z)^2\right]\varepsilon^2 K_1(\varepsilon r)^2 + m_c^2 K_0(\varepsilon r)^2\right\}, \tag{10}$$

3) The preliminary results have been reported at the DIS'98 Workshop [10].

where $K_{0,1}(x)$ are the modified Bessel functions, $\varepsilon^2 = z(1-z)Q^2 + m_c^2$, $m_c = 1.5\,\mathrm{GeV}$ is the c-quark mass and z is the light-cone fraction of photon's momentum carried by the quark of the $c\bar{c}$ pair, the eqs.(6), (9) show that the integral over r in (6) is dominated by

$$Q^{-2} \lesssim r^2 \lesssim m_c^{-2}.$$

Indeed, making use of the properties of modified Bessel functions, after z-integration one can write

$$f_n^{cc}(Q^2) \propto \int_{1/Q^2}^{1/m_c^2} \frac{dr^2}{r^2} \frac{\sigma_n(r)}{r^2}. \tag{11}$$

The dipole cross section $\sigma_n(r)$ in (11) is an oscillating function of r with the first node located inside the integration region. Then in a broad range of Q^2 one has strong cancellations in (11) for sub-leading poles which result in the leading pole dominance in charm production (Fig. 1). For large Q^2, far beyond the nodal region, the effect of cancellations disappears and

$$f_n^{cc}(Q^2) \propto \left[\alpha_S(Q^2)\right]^{-\gamma_n} \tag{12}$$

with $\gamma_n = 4/3\Delta_n$.

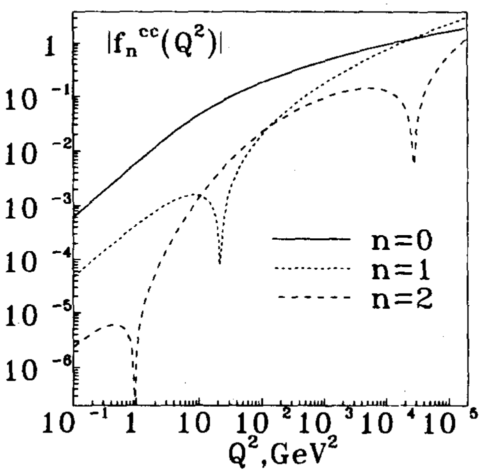

Fig.1. Modulus of the charm eigen-SF $|f_n^{cc}(Q^2)|$ for the BFKL poles with $n = 0, 1, 2$

For practical purposes it is convenient to represent $f_n^{cc}(Q^2)$ in an analytical form. The parameterization for the leading pole SF reads

$$f_0(Q^2) = a_0 \frac{R_0^2 Q^2}{1 + R_0^2 Q^2} \left[1 + c_0 \log(1 + r_0^2 Q^2)\right]^{\gamma_0}, \tag{13}$$

where $\gamma_0 = 4/3\Delta_0$.

For $n \geq 1$ the functions $f_n(Q^2)$ can be approximated by

$$f_n(Q^2) = a_n f_0(Q^2) \frac{1 + R_0^2 Q^2}{1 + R_n^2 Q^2} \prod_{i=1}^{n} \left(1 - \frac{z}{z_n^{(i)}}\right), \tag{14}$$

where

$$z = \left[1 + c_n \log(1 + r_n^2 Q^2)\right]^{\gamma_n} - 1 \tag{15}$$

and

$$\gamma_n = \gamma_0 \delta_n \tag{16}$$

with parameters listed in the Table.

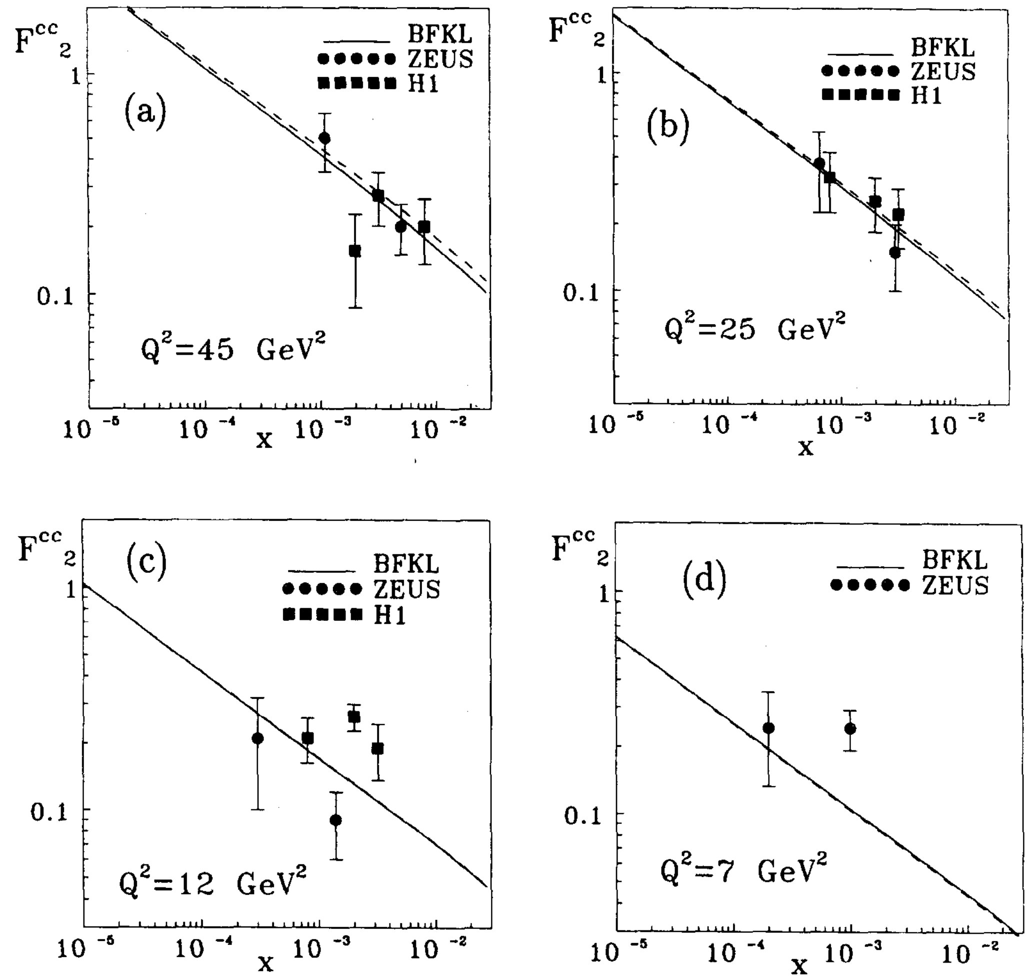

Fig.2. The predicted charm structure function $F_2^{cc}(x, Q^2)$ (solid line) vs. H1 and ZEUS data. The contribution of the leading pole with $\Delta_{\mathbb{P}} = 0.4$ is shown by dashed line. Fig.a corresponds to $Q^2 = 45\,\mathrm{GeV}^2$, Fig.b – $Q^2 = 25\,\mathrm{GeV}^2$, Fig.c – $Q^2 = 12\,\mathrm{GeV}^2$ and Fig.d – $Q^2 = 7\,\mathrm{GeV}^2$

n	a_n	c_n	r_n^2, GeV^{-2}	R_n^2, GeV^{-2}	$z_n^{(1)}$	$z_n^{(2)}$	δ_n
0	0.0214	0.2619	0.3239	0.2846			
1	0.0782	0.0352	0.0793	0.2958	0.2499		1.9249
2	0.0044	0.0362	0.0884	0.2896	0.0175	3.447	1.7985

In Fig.2 our predictions for the charm structure function are compared with data from H1 [12] and ZEUS [13]. We correct for threshold effects by the rescaling [14] $x \to$ $\to x(1 + 4m_c^2/Q^2)$. From both Fig.1 and Fig.2 it follows that the charm production in a wide range of the photon virtualities, $Q^2 \lesssim 10^2\,\mathrm{GeV}^2$, provides the unique opportunity of getting hold of elusive BFKL asymptotics and measuring $\Delta_{\mathbf{P}}$ already at currently available x.

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