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ROTATIONAL QUANTUM FRICTION IN SUPERFLUIDS: RADIATION FROM OBJECT ROTATING IN SUPERFLUID VACUUM

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We discuss the friction experienced by the body rotating in superfluid liquid at $T = 0$. The effect is analogous to the amplification of electromagnetic radiation and spontaneous emission by the body or black hole rotating in quantum vacuum, first discussed by Zel'dovich and Starobinsky. The friction is caused by the interaction of the part of the liquid, which is rigidly connected with the rotating body and thus represents the comoving detector, with the "Minkowski" superfluid vacuum outside the body. The emission process is the quantum tunneling of quasiparticles from the detector to the ergoregion, where the energy of quasiparticles is negative in the rotating frame. This quantum rotational friction caused by the emission of quasiparticles is estimated for phonons and rotons in superfluid ^{4}He and for Bogoliubov fermions in superfluid ^{3}He.

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Introduction. The body moving in the vacuum with linear acceleration a is believed to radiate the thermal spectrum with the Unruh temperature $T_U = \hbar a/2\pi c$ [1]. The comoving observer sees the vacuum as a thermal bath with $T = T_U$, so that the matter of the body gets heated to T_U (see references in [2]). Linear motion at constant proper acceleration (hyperbolic motion) leads to velocity arbitrarily close to the speed of light. On the other hand uniform circular motion features constant centripetal acceleration while being free of the above mentioned pathology (see the latest references in [3 - 5]). The latter motion is stationary in the rotating frame, which is thus a convenient frame for study of the radiation and thermalization effects for uniformly rotating body.

Zel'dovich [6] was the first who predicted that the rotating body (say, dielectric cylinder) amplifies those electromagnetic modes which satisfy the condition

$$\omega - L\Omega < 0 \ . \qquad (1)$$

Here ω is the frequency of the mode, L is its azimuthal quantum number, and Ω is the angular velocity of the rotating cylinder. This amplification of the incoming radiation is referred to as superradiance [7]. The other aspect of this phenomenon is that due to quantum effects, the cylinder rotating in quantum vacuum spontaneously emits the electromagnetic modes satisfying Eq.(1) [6]. The same occurs for any rotating body, including the rotating black hole [8], if the above condition is satisfied.

Distinct from the linearly accelerated body, the radiation by a rotating body does not look thermal. Also, the rotating observer does not see the Minkowski vacuum as a thermal bath. This means that the matter of the body, though excited by interaction with the quantum fluctuations of the Minkowski vacuum, does not necessarily acquire an intrinsic temperature depending only on the angular velocity of rotation. Moreover the vacuum of the rotating frame is not well defined because of the ergoregion, which exists at the distance $r_e = c/\Omega$ from the axis of rotation.

The problems related to the response of the quantum system in its ground state to rotation [3], such as radiation by the object rotating in vacuum [6, 9, 8, 7] and the vacuum instability caused by the existense of ergoregion [10], etc., can be simulated in superfluids, where the superfluid ground state plays the part of the quantum vacuum. We discuss the quantum friction due to spontaneous emission of phonons and rotons in superfluid ^{4}He and Bogoliubov fermions in superfluid ^{3}He.

Rotating frame. Let us consider a cylinder of radius R rotating with angular velocity Ω in the (infinite) superfluid liquid. In bosonic superfluids the quasiparticles are phonons and rotons; in fermi superfluids these are the Bogoliubov fermions. The phonons are "relativistic" quasiparticles: Their energy spectrum is $E(p) = cp + \mathbf{p} \cdot \mathbf{v}_s$, where c is a speed of sound and $\mathbf{v}_s$ is the superfluid velocity, the velocity of the superfluid vacuum; and this phonon dispersion is represented by the Lorentzian metric (the so-called acoustic metric [11, 12]):

$$g^{\mu\nu}p_\mu p_\nu = 0, \quad g^{00} = -1, \quad g^{0i} = v_s^i, \quad g^{ik} = c^2\delta^{ik} - v_s^i v_s^k. \tag{2}$$

When the body rotates, the energy of quasiparticles is not well determined in the laboratory frame due to the time dependence of the potential, caused by the rotation of the body. But it is determined in the rotating frame, where the potential is stationary. Hence it is simpler to work in the rotating frame. If the body is rotating surrounded by the stationary superfluid, i.e. $\mathbf{v}_s = 0$ in the laboratory frame, then in the rotating frame one has $\mathbf{v}_s = -\mathbf{\Omega} \times \mathbf{r}$. Substituting this $\mathbf{v}_s$ in Eq.(2) we get the interval $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, which determines the propagation of phonons in the rotating frame:

$$ds^2 = -(c^2 - \Omega^2 r^2)dt^2 - 2\Omega r^2 d\phi dt + dz^2 + r^2 d\phi^2 + dr^2 \ . \tag{3}$$

The azimuthal motion of the quasiparticles in the rotating frame can be quantized in terms of the angular momentum L, while the radial motion can be treated in the quasiclassical approximation. Then the energy spectrum of the phonons in the rotating frame is

$$E = c\sqrt{\frac{L^2}{r^2} + p_z^2 + p_r^2} - \Omega L \ . \tag{4}$$

Ergoregion in superfluids. The radius $r_e = c/\Omega$, where $g_{00} = 0$, marks the position of the ergoplane. In the ergoregion, i.e. at $r > r_e = c/\Omega$, the energy of quasiparticle in Eq.(4) can become negative for any rotation velocity and $\Omega L > 0$. We assume that the angular velocity of rotation Ω is small enough, so that the linear velocity on the surface of the cylinder ΩR is less than $v_L = c$ (the Landau velocity for nucleation of phonons). Thus phonons cannot be nucleated at the surface of cylinder. However in the ergoplane the velocity $v_s = \Omega r$ in the rotating frame reaches c, so that quasiparticle can be created in the ergoregion $r > r_e$.

The process of creation is, however, determined by the dynamics, i.e. by the interaction with the rotating body; there is no radiation in the absence of the body. If $\Omega R \ll v_L = c$ one has $r_e \gg R$, i.e. the ergoregion is situated far from the cylinder; thus the interaction of the phonons state in the ergoregion with the rotating body is small. This results in a small emission rate and thus in a small value of quantum friction, as will be discussed below.

Let us now consider other excitations: rotons and Bogoliubov fermions. Their spectra in the rotating frame are

$$E(p) = \Delta + \frac{(p - p_0)^2}{2m_0} - \Omega L \ , \tag{5}$$

$$E(p) = \sqrt{\Delta^2 + v_F^2(p - p_0)^2} - \Omega L \ . \tag{6}$$

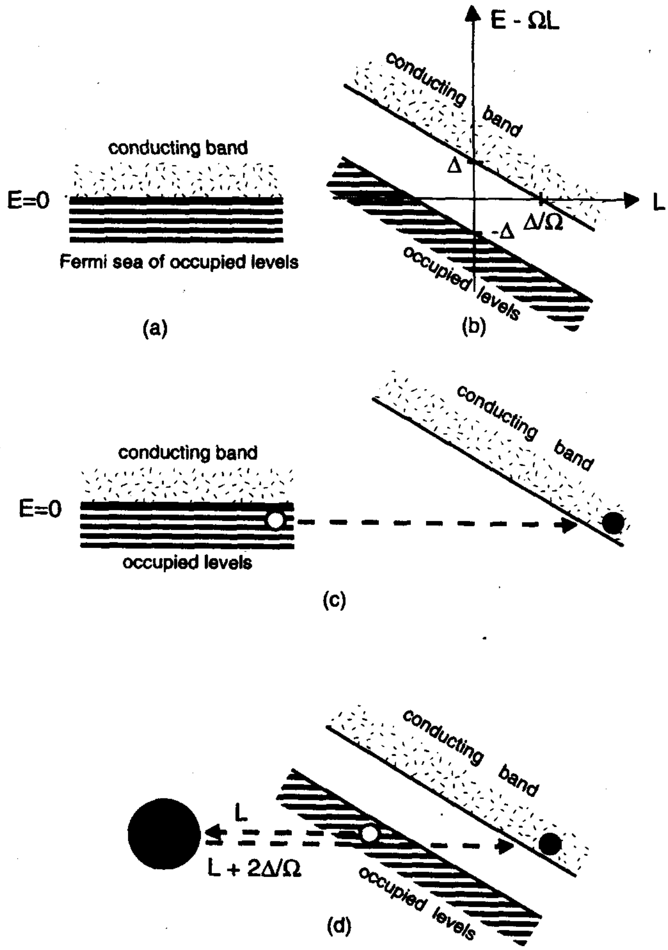
Here p_0 marks the roton minimum in superfluid ^{4}He and the Fermi momentum in Fermi liquid, while Δ is either a roton gap or the gap in superfluid ^{3}He-B. The Landau critical velocity for the emission of these quasiparticles is $v_L = \min(E(p)/p) \sim \Delta/p_0$. In ^{4}He the Landau velocity for emission of rotons is smaller than that for the emission of phonons, $v_L = c$. That is why the ergoplane for rotons, $r_e = v_L/\Omega$, is closer to the cylinder. However, for the rotating body the emission of the rotons is exponentially suppressed due to the big value of the allowed angular momentum for emitted rotons: the Zel'dovich condition Eq.(1) for roton spectrum is satified only for $L > \Delta/\Omega \gg 1$ (see Fig. b).

Rotating detector. Let us consider the system, which is rigidly connected to the rotating body and thus comprises the comoving detector. In superfluids the simplest model for such a detector consists of the layer near the surface of the cylinder, where the superfluid velocity follows the rotation of cylinder, i.e. $\mathbf{v}_s = \mathbf{\Omega} \times \mathbf{r}$ in the laboratory frame and thus $\mathbf{v}_s = 0$ in the rotating frame. This means that, as distinct from the superfluid outside the cylinder, in such a layer the quasiparticle spectrum has no $-\Omega L$ shift of the energy levels.

Since in the detector matter, i.e. in the surface layer, the vorticity in the laboratory frame is nonzero, $\nabla \times \mathbf{v}_s = 2\mathbf{\Omega} \neq 0$, this layer either contains vortices or is represented by the normal (nonsuperfluid) liquid, which is rigidly rotating with the body. Actually the whole rotating cylinder can be represented by the rotating normal liquid. The equilibrium state of the rotating normal liquid, viewed in the rotating frame, is the same as the equilibrium stationary normal liquid, viewed in the laboratory frame.

The rotating cylinder can also be represented by the cluster of the quantized vortices. The average superfluid velocity within the cluster is $< \mathbf{v}_s >= 0$ in the rotating frame. Thus within the cluster the superfluid is in the ground state in the rotating frame, while outside the cluster the superfluid is in the ground state in the laboratory frame. Such rigidly rotating clusters of vortices are experimentally investigated in superfluid ^{3}He (see e.g. [13]).

Thus we can discuss the complete system as consisting of two parts, each in its own ground state (see Figs. a – b for the case of Fermi liquid): (1) The matter of the detector



(a) Vacuum of the Fermi liquid within the rotating body at $r < R$. This vacuum is rotating together with the body and thus plays the role of the comoving detector. (b) "Minkowski" vacuum of superfluid outside the rotating body as viewed in rotating frame. In the ergoregion, i.e. at $r > r_e = v_L/\Omega$, where v_L is Landau critical velocity, the conducting band crosses the zero energy level. (c) Tunneling of particles from the vacuum of the detector matter to the "Minkowski" vacuum in the ergoregion produces radiation from rotating body and excitation of comoving detector. (d) Transition between the states in the "Minkowski" vacuum due to interaction with the rotating detector

in its ground state as seen in the rotating frame; (2) The superfluid outside the cylinder in its ground state (the "Minkowski" vacuum) in the laboratory frame. The radiation of fermions by the rotating cylinder is described by the rotating observer as a tunneling process (Fig. c): fermions tunnel from the occupied negative energy levels in the detector to the unoccupied negative energy state in the ergoregion. The same can be considered as the spontaneous nucleation of pairs: the particle is nucleated in the ergoregion and its partner hole is nucleated in the comoving detector. This process causes the radiation from the rotating body and also the excitation of the detector. From the point of view of the Minkowski (stationary) observer this is described as the excitation of the superfluid system by the time dependent perturbations.

Radiation of phonons to the ergoregion. For the Bose case the radiation of phonons can be also considered as the process in which the particle in the normal Bose liquid in the detector tunnels to the scattering state at the ergoplane, where also the energy is $E = 0$. In the quasiclassical approximation the tunneling probability is e^{-2S}, where at

$p_z = 0$:

$$S = \mathrm{Im} \int dr\ p_r = L \int_R^{r_e} dr \sqrt{\frac{1}{r^2} - \frac{1}{r_e^2}} \approx L \ln \frac{r_e}{R} \ . \quad (7)$$

Thus all the particles with $L > 0$ are radiated, but the radiation probability decreases at higher L. If the linear velocity at the surface is much less than the Landau critical velocity $\Omega R \ll c$, the probability of radiation of phonons with the energy (frequency) $\omega = \Omega L$ is

$$w \propto e^{-2S} = \left(\frac{R}{r_e}\right)^{2L} = \left(\frac{\Omega R}{c}\right)^{2L} = \left(\frac{\omega R}{cL}\right)^{2L} , \quad \Omega R \ll c \ . \quad (8)$$

If c is substituted by the speed of light, Eq.(8) is proportional to the superradiant amplification of the electromagnetic waves by rotating dielectric cylinder derived by Zel'dovich [7, 9].

The number of phonons with the frequency $\omega = \Omega L$ emitted per unit time can be estimated as $\dot{N} = We^{-2S}$, where W is the attempt frequency $\sim \hbar/ma^2$ multiplied by the number of localized modes $\sim RZ/a^2$, where Z is the height of the cylinder and a is the thickness of normal fluid layer of order of interatomic space. Since each phonon carries the angular momentum L, the cylinder rotating in superfluid vacuum (at $T = 0$) is loosing its angular momentum, which means the quantum rotational friction.

Radiation of rotons and Bogoliubov quasiparticles. The minimal L value of the radiated quasiparticles, which have the gap Δ, is determined by this gap: $L_{min} = \Delta/\Omega p_0 = v_L/\Omega$, where $v_L = \Delta/p_0$ is the Landau critical velocity. Since the tunneling rate exponentially decreases with L, only the lowest possible L must be considered. In this case the tunneling trajectory with $E = 0$ is determined by the equation $p = p_0$ both for rotons and Bogoliubov quaiparticles. For $p_z = 0$ the classical tunneling trajectory is thus given by $p_r = i\sqrt{|p_0^2 - L^2/r^2|}$. This gives for the tunneling exponent e^{-2S} the equation

$$S = \mathrm{Im} \int dr\ p_r = L \int_R^{r_e} dr \sqrt{\frac{1}{r^2} - \frac{1}{r_e^2}} \approx L \ln \frac{r_e}{R} \ . \quad (9)$$

Here the position of the ergoplane is $r_e = L/p_0 = v_L/\Omega$. Since the rotation velocity Ω is always much smaller than the gap, L is very big. That is why the radiation of rotons and Bogoliubov quasiparticles with the gap is exponentially suppressed.

Friction due to transitions in "Minkowski vacuum". Radiation can occur without excitation of the detector vacuum, via direct interaction of the particles in the Minkowski vacuum with the rotating body. In the rotating frame the states in the occupied band and in the conducting band have the same energy, if they have opposite momenta L. Then a transition between the two levels is energetically allowed and will occur if the Hamiltonian has a nonzero matrix element between the states L and $-L$. The necessary interaction is provided by any violation of the axial symmetry of the rotating body, e.g. by roughness on the surface (thus the interaction is localized ar $r \sim R$). A wire moving along the circular orbit is another practical example. In case of the rotating vortex cluster the axial symmetry is always violated.

In the quasiclassical approximation the process of radiation is as follows (see Fig. d). The particle from the occupied band in the ergoregion tunnels to the surface of the rotating body, where after interaction with the nonaxisymmetric disturbance it changes its angular momentum. After that it tunnels back to the ergoregion to the conducting band. In this process both a particle and a hole are produced in the Minkowski vacuum, as a result the tunneling exponent is twice larger than in Eqs.(8) and (9).

Discussion. The rotational friction experienced by the body rotating in superfluid vacuum at $T = 0$, is caused by the spontaneous quantum emission of the quasiparticles from the rotating object to the "Minkowski" vacuum in the ergoregion. The emission is not thermal and depends on the details of the interaction of the radiation with the rotating body. In the quasiclassical approximation it is mainly determined by the tunneling exponent, which can be approximately characterized by the effective temperature $T_{eff} \sim \hbar\Omega(2/\ln(v_L/\Omega R))$. The vacuum friction of the rotating body can be observed only if the effective temperature exceeds the temperature of the bulk superfluid, $T_{eff} > T$. For the body rotating with $\Omega = 10^3$ rad/s, T must be below 10^{-8} K. However, high rotation velocity can be obtained in the system of two like vortices, which rotate around their center of mass with $\Omega = \kappa/4\pi R^2$ (κ is the circulation around each vortex, R is the radius of the circular orbit).

The process discussed in the paper occurs only if there is an ergoplane in the rotating frame. For the superfluid confined within the external cylinder of radius R_{ext}, this process occurs at high enough rotation velocity, $r_e(\Omega) = v_L/\Omega < R_{ext}$, when the ergoplane is within the superfluid. On the instability of the ergoregion in quantum vacuum towards emission see e.g. Ref.[10].

If $r_e(\Omega) > R_{ext}$ and ergoregion is not present, then the interaction between the coaxial cylinders via the vacuum fluctuations becomes the main mechanism for dissipation. This causes the dynamic Casimir forces between the walls moving laterally (see Review [14]). As in [14] the nonideality of the cylinders is the necessary condition for quantum friction.

The case of the rotating body is not the only one in superfluids, where the ergoregion is important. The ergoregion also appears for the lineraly moving textures, where the speed of the order parameter texture exceeds the local "speed of light" [15].

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