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UNIVERSALITY IN EFFECTIVE STRINGS

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We demonstrate that, due to the finite thickness of domain walls, and the consequent ambiguity in defining their locations, the effective string description obtained by integrating out bulk degrees of freedom contains ambiguities in the coefficients of the various geometric terms. The only term with unambiguous coefficient is the zeroth order Nambu – Goto term. We argue that fermionic ghost fields which implement gauge-fixing act to balance these ambiguities. The renormalized string tension, obtained after integrating out both bulk and world-sheet degrees of freedom, can be defined in a scheme independent manner; and we compute the explicit finite expressions, to one-loop, for the case of compact quantum electrodynamics and φ^4 theory.

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A long standing problem in the physics of interfaces in three dimensional systems is to describe the interface dynamics as a theory of fluctuating surfaces analogous to an effective Euclidean string theory [1 – 3]. The interface surface can be interpreted as the world-sheet of an effective string in three-dimensions, while the two phases on either side of the interface represent the vacuum expectation values of a fundamental field. For example in the Ising model, the field is the spin operator, and the interface is the set of links about which the spin changes sign. An effective string action of this surface has a typical geometric expansion which begins with the Nambu – Goto term (the area), next the extrinsic curvature and then higher order curvature corrections. It is usually argued that the higher order corrections are irrelevant as far as the low energy dynamics is concerned, and perturbation theory consisting in keeping only the area term and extrinsic curvature is valid. However, we will demonstrate that such an ansatz is in fact ambiguous from the outset. In particular, we will demonstrate that coefficients of the higher order curvatures are completely arbitrary, and depend upon the precise prescription implemented in defining the position of the interface. Such an ambiguity is a direct consequence of the finite thickness of the interface region. In spite of this ambiguity an effective string description is possible if one includes in the action fermionic degrees of freedom which reinstate the scheme independent nature of the fundamental action.

In this work we concentrate on interfaces which occur in 3D compact quantum electrodynamics (QED) [4]. The analysis can be easily applied to any other field theory in 3D which has a soliton solution to the classical equations of motion. In compact QED, monopole instantons cause the electric fields between two charged particles to form a flux tube, the potential between electric charges grows linearly with the distance between them, and the charges are confined [4]. This picture of confinement is, however, a purely classical one. In the full quantum theory the string of electric flux, along with the magnetic fields in the bulk, are not rigid but rather fluctuate.

Compact QED can be regarded as the low-energy effective theory for the Georgi – Glashow model where the $SU(2)$ symmetry is spontaneously broken to $U(1)$. The monopoles appearing in the broken gauge theory are the classical solutions of the original Higgs model which have finite Euclidean action [5]. The collection of monopoles behave as a gas of charged particles interacting through a Coulomb force. Since the charges are of order $\sqrt{4\pi}/g$, where g is the gauge coupling, the monopole configurations can be treated using a semi-classical approximation in the limit of weak gauge coupling. The system then reduces to the classical thermodynamics of a Coulomb gas and the partition function is a Sine-Gordon (SG) theory [4]:

$$Z = \int [d\varphi] \exp\left\{ -\frac{g^2}{32\pi^2} \int d^3x \, \left[(\partial\varphi)^2 - 2m^2 :\cos\varphi:\right] \right\}. \tag{1}$$

The monopole density is the coefficient of the cosine interaction, $\zeta = g^2m^2/32\pi^2$. The Grand canonical partition function for the monopoles is recovered by expanding in powers of ζ and performing the functional integral over φ. In the presence of the monopole solution the photon becomes massive and Wilson loop correlators obey an area law [4]. The relevant order parameter in the original gauge theory (compact QED) is the vacuum expectation value of the Wilson loop. There is a natural mapping between this correlator and the following correlation function in the SG model [4, 6]:

$$W(C) \equiv \left\langle \exp\left(\frac{i}{2} \oint_C dA \right) \right\rangle_{\text{QED}} = \left\langle \exp\left(\frac{g^2}{16\pi} \int_S \star d\varphi \right) \right\rangle_{\text{SG}}. \tag{2}$$

Here S is an arbitrary surface bounded by the contour C. The result can be shown to be independent of the choice of this surface. We are interested in the behaviour of (2) for large loops. The contour C will be assumed to lie at infinity in the $x_3 = 0$ plane. In this case, it is possible to reformulate the problem. The operator on the right hand side of (2) introduces a source for the SG field or, equivalently, one may assume that φ experiences a jump of magnitude 2π across the surface S. Since the potential is periodic in φ, this jump can be eliminated by shifting φ on one side of the surface by 2π. This shift renders the field continuous, however, it changes the boundary conditions as $x_3 \to +\infty$. Consequently, performing such a shift on φ reduces (2) to an evaluation of the path integral 1 with the boundary conditions $\varphi \to 0$ as $x_3 \to -\infty$ and $\varphi \to 2\pi$ as $x_3 \to +\infty$. These boundary conditions are precisely what is required for field configurations in the presence of a domain wall. The domain wall in this case describes a world sheet of a string of electric flux created by charges at infinity.

The first step in obtaining an effective string action is to obtain the classical solution satisfying the appropriate boundary conditions. The solution is the SG soliton:

$$\varphi_{cl}(x) = 4 \arctan \, e^{\,m x_3}, \tag{3}$$

which corresponds to a domain wall in the $x_3 = 0$ plane. The position of this domain wall is in fact ambiguous. Conventionally, its position is given by the surface on which $\varphi = \pi$ [6, 7]. However, other definitions are also possible, for example, the surface on which the energy density is maximal [8, 9]. These definitions, although agreeing at the classical level, do not agree once quantum corrections are included. Nevertheless, they will yield the same result within an order of m^{-1}. This uncertainty is due to the finite thickness of the domain wall and we will argue that this ambiguity translates into the non-universality of the world-sheet action.

Upon inserting (3) as a classical background field in the functional integral over φ in (1), the collective coordinate method can be used to separate the integral over fluctuations into an integral over the domain wall position, which we describe by a height function $f(x_1, x_2)$, and over the field fluctuations in the bulk:

$$Z = \int [d\varphi][df]\Delta_{\rm FP}[\varphi]\delta\left[K[f,\phi]\right] e^{-S_{\rm SG}[\varphi]}, \tag{4}$$

where $K[f,\phi] = \int dx_3 K(\mathbf{x}, x_3 - f)\varphi(x) - \pi$ and $\Delta_{\rm FP}[\varphi] = \delta K[f,\phi]/\delta f$ is the Faddeev-Popov determinant, $\mathbf{x}$ is (x_1, x_2) and $S_{\rm SG}$ is the same as in (1). A particular definition of the domain wall position corresponds to choosing a kernel K. There is no unique choice of this function. The definition of [6, 7] corresponds to $K = \delta(x_3 - f)$, while the standard collective coordinate method [8, 9], in which the fluctuations of the domain wall are associated with quasi-zero modes in the background of the classical solution (3), corresponds to $K = \varphi'_{cl}(x_3 - f)$.

Integrating over φ in (4) yields an effective action for the coordinate $f(\mathbf{x})$ of the domain wall. In the semi-classical approximation φ is replaced by its classical solution. The effective action is then given by,

$$S_{eff} = \sigma_0 \int \left(1 + \frac{1}{2}(\nabla f)^2\right) + \mathcal{O}\left((\nabla f)^2\right) \tag{5}$$

where the string tension is determined by the mass of the SG soliton [4, 10]:

$$\sigma_0 = \frac{g^2}{32\pi^2}\int \left((\partial_3\varphi_{cl})^2 + 2m^2\cos(\varphi_{cl})\right) dx_3 = \frac{g^2 m}{2\pi^2}.$$

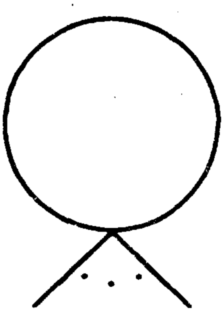
In S_{eff} all higher order corrections, coming from tree level diagrams, were ignored. However, if terms which would contribute to higher curvature corrections are ignored, it is possible to re-sum an infinite subset of the terms which were previously ignored. This can be achieved by considering a domain wall solution which is curved, unfortunately, such configurations are not solutions of the SG equations of motion. They can, on the other-hand, be considered as a constraint solution [11] – the solution of the equations of motion with a source term proportional to the argument of the delta function in (4). This can be included in the action by introducing a Lagrange multiplier field. For slowly varying f this equation can be solved by perturbation theory in the derivatives of f. If only the first derivatives of f are taken into account (i.e. ignoring higher curvature corrections), the solution can be constructed by the following simple arguments [2]. When one neglects higher derivatives this implies that f is a linear function of its arguments, which corresponds to the plane domain wall rotated through an angle θ with $\tan\theta = \sqrt{G}$ where $G \equiv 1 + (\nabla f)^2$. The classical solution in this case is obtained from (3) by rotation: $\varphi(x) = \varphi_{cl}((x_3 - f)/\sqrt{G})$. This solution is exact in the approximation of constant ∇f, since no source term is required to produce it. Consequently, the Lagrange multiplier appears only in the next order of the derivative expansion and is proportional to $\nabla^2 f$. The effective action with the above solution is equal to the area of the domain wall. The rotated wall area element contains the factor $1/\cos\theta = \sqrt{G}$, and the re-summed effective action is therefore,

$$S_{eff} = \sigma_0 \int d^2\mathbf{x}\, \sqrt{G(\mathbf{x})}. \tag{6}$$

Since G is the determinant of the induced metric on the string world sheet we have obtained the Nambu – Goto action.

The next term in the derivative expansion will be of order of the Lagrange multiplier squared, that is $(\nabla^2 f)^2$, and will depend on the choice of the constraint. This arbitrariness leads to an ambiguity in the coefficient of this term, which in the covariant description corresponds to the extrinsic curvature squared. It appears that the area term (6) is the only universal part of the effective action. This is not surprising, since the parameter of the derivative expansion is k/m, where k is a momentum of the excitation on the string world sheet. Higher derivative terms become important when $k \sim m$, i.e., when the wavelength of the excitation is of the same order of the thickness of the domain wall. Such excitations are of course indistinguishable from the fluctuations of the SG field in the bulk. As such, including such fluctuations in the effective string action leads to ambiguities. Of course, the computation of any physical quantity must be invariant under any choice of the constraint. It is the Faddeev – Popov determinant which cancels the ambiguities arising in the bosonic sector of the theory – the full effective action contains the fermionic ghosts coming from the determinant.

The universality of the Nambu – Goto term stems from the rotational invariance of the original model. This holds even when quantum corrections are included as long as the constraint respects rotational symmetry. Constraints which do not respect this symmetry would produce actions that are not reparametrization invariant. We have explicitly checked that when the kernel in (4) is chosen to be $K(\mathbf{x}, x_3 - f) = \varphi'_{cl}(x_3 - f)$, which is not rotationally invariant, the one-loop correction to the constant and to the $(\nabla f)^2$ terms in (6) disagree.



Tadpole diagrams to be subtracted in the normal ordering prescription

Let us now consider the quantum corrections. The string tension gains quantum corrections from fluctuations of the domain wall and of the field φ in the bulk. To study how these two corrections are correlated we calculate the one-loop corrections to the string tension. The parameter of the loop expansion, m/g^2, is small since m^2 is proportional to the monopole density. These corrections will be computed in the background field method starting from the classical solution (3). The one-loop corrections in 3D SG theory are linearly divergent, which leads to ambiguities in the definition of dimensional quantities like the string tension. However, the monopole gas representation implies the particular UV regularization based on the normal ordering prescription. To implement this prescription in the background field method, it is instructive to first study the one-loop corrections to the general classical solution. Expanding the quantum field as $\varphi = \varphi_{cl} + \eta$ and integrating out η, we obtain:

$$S_1^{bare} = \frac{1}{2}\ \mathrm{Tr}\ \ln\left(\frac{-\partial^2 + m^2\cos\varphi_{cl}}{-\partial^2 + m^2}\right).$$

The expansion of S_1 in the powers of φ_{cl} yields the usual Feynman diagrams while the normal ordering prescription consists in throwing out bubble diagrams (Figure). Thus, to one-loop, the normal ordering is implemented by adding the following counter-term to the effective action:

$$\begin{aligned} S_1 &= \frac{1}{2}\,\mathrm{Tr}\ \ln\left(\frac{-\partial^2 + m^2\cos\varphi_{cl}}{-\partial^2 + m^2}\right) - \\ &- \frac{m^2}{2}\int d^3x(\cos\varphi_{cl} - 1)\int \frac{d^3p}{(2\pi)^3}\frac{1}{p^2 + m^2}. \end{aligned} \tag{7}$$

This expression is free of all UV divergences. Notice that since the expansion of the renormalized effective action S_1 begins with φ_{cl}^4 terms, the mass of the photon does not acquire quantum corrections at one loop.

The spectrum of the operator describing the excitations of the soliton are well known. It consists of plane waves in the x_1 and x_2 directions; while the x_3 spectrum is gapped possessing a zero mode and then a set of continuum states corresponding to the scattering of plane waves off the potential given by the solitonic solution. Inserting the analytic form of the spectrum into (7) we find that the one-loop correction to the string tension is,

$$\begin{aligned} \Delta\sigma_1 &= \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\ln(p^2 + m^2)\nu(p_3) + \\ &+ \frac{1}{2}\int \frac{d^2p}{(2\pi)^2}\ln(\mathbf{p}^2) + 2m\int \frac{d^3p}{(2\pi)^3}\frac{1}{p^2 + m^2} = -\frac{m^2}{4\pi}. \end{aligned} \tag{8}$$

Here $\nu(p_3)$ is the difference of the density of scattering states in the presence and absence of the kink, and can be obtained using the exact eigenmodes of the linearized SG equation [12]:

$$\nu(p_3) = \int dx_3\ (\psi_{p_3}^*(x_3)\psi_{p_3}(x_3) - 1) = -\ \frac{2m}{p_3^2 + m^2}.$$

The first term in (8) corresponds to the trace over continuum states; the second term corresponds to the quasi-zero mode contribution; and the last term is the counter-term prescribed by normal ordering. The sum of these three terms is UV finite, although each term diverges individually. This leaves room for some ambiguity in the final finite result depending on what regularization scheme is implemented. However, if the sum of the terms is placed under one single integral, so that the function being integrated over has finite UV behaviour, the answer must be regularization independent. The difficulty in implementing such a scheme is that the quasi-zero mode is a 2D integral, while all others are 3D. It is possible to integrate out the p_3 component of the 3D integrals first, which then leaves a single two dimensional integral to perform. This is the scheme that was used in (8).

Thus far, we have shown that after including one-loop effects the string tension in the SG model is,

$$\sigma = \frac{g^2 m}{2\pi^2}\left(1 - \frac{\pi}{2}\,\frac{m}{g^2}\right).$$

We performed analogous calculation for φ^4 theory with the potential $\lambda(\varphi^2 - m^2/\lambda)^2/2$, the result is,

$$\sigma = \frac{4}{3}\frac{m^3}{\lambda}\left(1 - \frac{9}{32\pi}(4 - \ln 3)\frac{\lambda}{m}\right).$$

As mentioned earlier, different regularization schemes will lead to different final answers. The authors of [13] performed similar calculations on the φ^4 theory using the zeta-function regularization. That computation, however, did not include the integral over the quasi-zero branch of the theory, and therefore corresponds to obtaining the one-loop correction to the effective string tension rather than the one-loop renormalized string tension. Excluding that branch still leads to finite results in their regularization scheme since it is insensitive to power like divergences. Unfortunately, it yields an answer incompatible with the ansatz of first integrating out p_3 and then performing the finite integrals. This is to be expected, since the quasi-zero branch should and must contribute to the renormalized string tension.

In the preceding, all modes were including in computing the determinant. However, it is possible to integrate out only the scattering states, i.e. to omit the second term in (8), and obtain an effective action for the quasi-zero branch (as in [13]). This branch contains the modes responsible for shifting the surface in the x_3 direction. Omitting the second term in 8 is equivalent to keeping f fixed and integrating out only bulk modes. The one-loop correction with only bulk modes included is badly UV divergent and is not regularization independent. In the naive cut-off regularization the effective string tension for the f fields will be $\sim \Lambda^2 \ln(\Lambda^2/m^2)$ as can be easily checked from (8). Of course, hard modes (with $k \gg m$) of the field f cancel this divergence, rendering the physical string tension finite. The implication is that the one-loop renormalization of the string tension is known precisely, even though all the interaction terms are not known.

To summarize we have argued that the accuracy of the macroscopic description of the confining string in compact QED is limited by the fact that the string is not infinitely thin. As a consequence the higher-derivative terms in the world-sheet action are not universal. Formally, it is possible to obtain an effective string theory action by integrating out φ in (4) exactly. However, the world-sheet action will be scheme-dependent and must be supplemented by the fermionic ghosts coming from the constraints. In addition, the action contains rather peculiar divergences and the finite physical quantities obtained from such an effective action appear only after delicate cancellations between these divergences and the contribution of hard modes of the string coordinates.

It is worth mentioning that the derivation of the effective domain wall action by collective coordinate method in one dimension lower would lead to essentially the same conclusions. In 2D theory, which can be thought of as a high-temperature reduction of 3D one [14], domain walls correspond to soliton paths. Solitons in 2D SG theory are known to be described by a local field theory, the Thirring model [15] and soliton operators [16] look very much like the dimensional reduction of the Wilson loops (2) [14]. Where the only difference is a local factor rendering the solitons fermionic. Fermion propogators have a well defined sum-over-path representation where the action is the supersymmetrized length of the world-line. Nevertheless, at weak SG coupling, solitons cannot be described by the world-line theory, since the four-fermion interaction in the Thirring model, which corresponds to a contact interaction in the sum-over-path picture, becomes infinitely strong.

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1. D.Jasnow, Rep. Prog. Phys. **47**, 1059 (1984).
2. R.K.P.Zia, Nucl. Phys. **B251[FS13]**, 676 (1985).
3. G.Münster, Nucl. Phys. **B340**, 559 (1990); G.Münster, hep-th/9802006.
4. A.M.Polyakov, Nucl. Phys. **B120**, 429 (1977).
5. G. 't Hooft, Nucl. Phys. **B79**, 276 (1974); A.M.Polyakov, JETP Lett. **20**, 194 (1974) [Pis'ma ZhETF **20**, 430 (1974)].
6. A.M.Polyakov, Nucl. Phys. **B486**, 23 (1997).
7. T.H.R.Skyrme, Proc. Roy. Soc. **A262**, 237 (1961); Nucl. Phys. **31**, 556 (1962); J.K.Perring and T.H.R.Skyrme, Nucl. Phys. **31**, 550 (1962).
8. J.L.Gervais and B.Sakita, Phys. Rev. **D11**, 2943 (1975); J.L.Gervais, A.Jevicki, and B.Sakita, Phys. Rev. **D12**, 1038 (1975).
9. J.L.Gervais, A.Jevicki, and B.Sakita, Phys. Rev. **D12**, 1038 (1975); E.Tomboulis, Phys. Rev. **D12**, 1678 (1975).
10. N.J.Snyderman, Nucl. Phys. **B218**, 381 (1983).
11. I.Affleck, Nucl. Phys. **B191**, 429 (1981).
12. J.Goldstone and R.Jackiw, Phys. Rev. **D11**, 1486 (1975).
13. R.V.Konoplich, Nucl. Phys. **B323**, 660 (1989).
14. N.O.Agasyan and K.Zarembo, Phys. Rev. **D57**, 2475 (1998).
15. S.Coleman, Phys. Rev. **D11**, 2088 (1975).
16. S.Mandelstam, Phys. Rev. **D11**, 3026 (1975).