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FEMTOSECOND PARAMETRIC EXCITATION OF ELECTROMAGNETIC FIELD IN A CAVITY

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We discuss the method of parametric excitation of electromagnetic waves in a cavity (particularly from vacuum state) by creating of dense plasma layer which represents the "mobile" wall of the cavity. The mobile plasma layer may be produced by the irradiation of semiconductor film with femtosecond laser pulse. Parametrically excited radiation may be resolved from radiation of another origin by analysis of it's squeezing, specific angle distribution and by time resolved registration.

1. Introduction

We discuss the parametric excitation of electromagnetic waves in the cavity (in particular from the initial vacuum state) by use of power femtosecond laser pulses. The idea is to create dense electron-hole $(e-h)$ plasma in thin semiconductor film at a small time interval by irradiating it with power femtosecond laser pulses. The surface of such plasma may be considered as the mobile wall of the cavity. Both, this plasma layer and fixed metal mirror represent the cavity with one mobile wall (see Fig.1). The effect of parametric excitation of electromagnetic field due to the

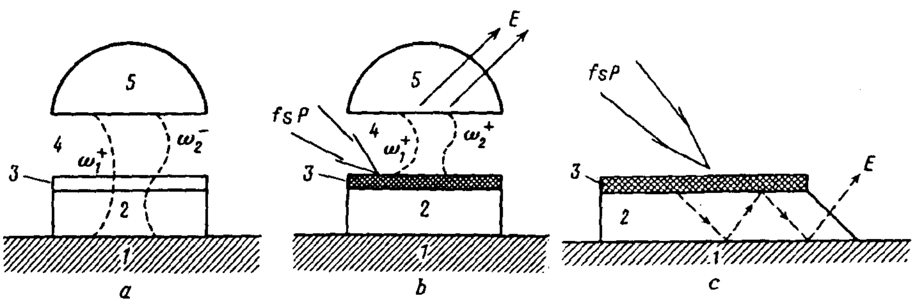

Fig.1. Emission of cavity modes due to the parametric excitation: *a*) before excitation, *b*) after excitation; *1* — metal layer; *2* — dielectric slab; *3* — semiconductor film; *4* — gap; *5* — dielectric prism; ω_1^-, ω_2^- (ω_1^+, ω_2^+) are vacuum cavity modes before (after) excitation; *fsP* is exciting femtosecond laser pulse; *E* is the light radiation due to the parametric excitation, c) another realization

motion of one of the cavity walls is essential for ultrashort time τ of displacement of the mobile wall. So using of femtosecond laser pulses in experiment proposed is justified. We consider the new regime of parametric excitation when τ is *comparable* with the time of propagating of light from one wall of the cavity to

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another: $\tau \sim \frac{L}{c}$, where L is the length of the cavity[2)]. This regime is opposite to the case $\tau \gg \frac{L}{c}$ considered before (see e.g. [2–4] and references herein).

In this case no stationary picture of distribution of electromagnetic energy in the cavity exists. It occurs that the value of excited energy $\overline{E}$ in the cavity depends on the *whole history* of the process, namely, the whole displacement of the mobile wall of the cavity.

In the present article we calculate in the framework of quantum electrodynamics in the cavity the parametric excitation of electromagnetic field in particular from vacuum state in case of instantaneous and also adiabatic slow excitation. Besides, the classic approach is considered. The angle distribution and the quantum statistical properties (squeezing etc.) of parametrically excited radiation are described.

2. Basic relations

We consider electromagnetic field in rectangular cavity with time-dependent length $L_z = L_z(t)$ but $L_x = \mathrm{Const}$ and $L_y = \mathrm{Const}$ and suppose that quality Q of the cavity is infinitely great. (We suppose also to be specific that in time $t < t_0$ and $t > t_0 + \tau$ the wall is at rest and that at $t_0 < t < t_0 + \tau$ the velocity $\dot{L}_z < 0$, i.e. size of the cavity is diminished. Note that phenomena considered below takes place also for more general dependence $L_z(t)$). This leads to simple boundary conditions for that cavity modes which have the vector $\mathbf{E}$ parallel to xy plane (see also [2]): $\mathbf{A}|_{z=L_z(t)} = 0$ where $\mathbf{A}$ is the vector-potential.

The expression for the energy in the cavity with time-dependent length $L_z(t)$ in case $\mathrm{div}\mathbf{A} = 0$ is:

$$E = \frac{L_x L_y}{4\pi} \sum_q \int_0^{L_z(t)} \left(\left|\dot{\mathrm{a}}_{\mathrm{q}}\right|^2 + \omega_q^2 \left|\mathrm{a}_{\mathrm{q}}\right|^2 + c^2 \left|\frac{\partial \mathrm{a}_{\mathrm{q}}}{\partial z}\right|^2 \right) dz - c^2 \frac{L_x L_y}{4\pi} \sum_q \frac{\partial \left|\mathrm{a}_{\mathrm{q}}\right|^2}{\partial z} \Big|_{z=0}^{z=L_z(t)} \tag{1}$$

where $\mathrm{a_q}$ are the 2D Fourier-transforms of $\mathbf{A}$ in xy variables and vector $\mathrm{q} = (k_x, k_y)$.

To get out the time-dependent boundary conditions it is natural to use variables $u = \xi(t) z$ where $\xi(t) = L_z^- / L_z(t)$. In that variables the wave equation for components $\mathrm{b_q}(u,t) = \mathrm{a_q}(z,t)$ of vector potential $\mathbf{A}$ on the condition $\mathrm{div}\mathbf{A} = 0$ takes the form:

$$\widehat{A}\widehat{B}\mathrm{b_q} = -\omega_q^2 \mathrm{b_q} \tag{2}$$

where

$$\widehat{A} = \frac{\partial}{\partial t} + [u\eta(t) + c\xi(t)]\frac{\partial}{\partial u}; \quad \widehat{B} = \frac{\partial}{\partial t} + [u\eta(t) - c\xi(t)]\frac{\partial}{\partial u} \quad \text{and} \quad \eta(t) = \frac{\dot{\xi}}{\xi}.$$

3. One-dimensional case $|\mathrm{q}| \ll k_z$

In the case $|\mathrm{q}| \ll k_z$ Eq.(2) may be transformed to the form $\widehat{A}\widehat{B}b = 0$ and so may be reduced to the first order equations in partial derivations $\widehat{A}b = 0$ and $\widehat{B}b = 0$. For modes with $\omega \ll 1/\tau$ one can find:

[2)] Different phenomena in QED in the cavity due to the *quasi-stationary* modulation of the wall (Lamb shift modulation etc.) see for instance in [1] and references herein.

$$b(u,t) = C^- \exp\left[ik_z u \mp i \int_{t_0}^{t} \omega(t')dt'\right], \quad t_0 < t < t_m,$$

$$b(u,t) = C^+ \exp\left[ik_z u \mp i \int_{t_m}^{t} \omega(t')dt'\right], \quad t_m < t < t_0 + \tau,$$

where $C^\pm$ are the constants $(C^+ \neq C^-)$; the value t_m is defined by the equation $\tilde{L}_z(t) = 0$ $(t_0 < t_m < t_0 + \tau)$; τ is the time of displacement of the mobile wall and $\omega(t) = ck_z\xi(t)$. The upper sign corresponds to the equation $\widehat{A}b = 0$ and the lower one to $\widehat{B}b = 0$.

Because of $\widehat{A}\widehat{B} = \widehat{B}\widehat{A}$ the linear combination of solutions of equations $\widehat{A}b = 0$ and $\widehat{B}b = 0$ is also the solution of Eq.(2). Taking into account boundary condition $A\,|_{z=L_z(t)} = 0$ one can write:

$$b = \left(\tfrac{4\pi}{L\omega^-}\right)^{\frac{1}{2}} \sum_{k_z} C^-_{k_z} \sin(k_z u) e^{i\omega^- t}, \quad t < t_0,$$
$$b = \left(\tfrac{4\pi}{L\omega^+}\right)^{\frac{1}{2}} \sum_{k_z} \left(C^+_{1k_z} \sin(k_z u) e^{i\omega^+ t} + C^+_{2k_z} \sin(k_z u) e^{-i\omega^+ t}\right), \quad t > t_0 + \tau, \tag{3}$$

where $\omega^- = ck_z$ and $\omega^+ = ck_z\frac{L^-}{L^+}$. The value L is the length of the cavity in z direction in coordinate (u,t) (in coordinate (u,t) we have $L = \mathrm{Const}$). By comparing two solutions (3) at $t = t_m = 0$ one can find:

$$C^+_{1k_z} = C^-_{k_z} \frac{1}{2}\left(\sqrt{\frac{\omega^+}{\omega^-}} + \sqrt{\frac{\omega^-}{\omega^+}}\right),$$
$$C^+_{2k_z} = C^-_{k_z} \frac{1}{2}\left(\sqrt{\frac{\omega^+}{\omega^-}} - \sqrt{\frac{\omega^-}{\omega^+}}\right).$$

If we substitute the solution (3) at time $t > t_0 + \tau$ into Eq.(1) on condition that $b(u,t) = a(z,t)$ we find:

$$E = \sum_{k_z} \left|C^-_{k_z}\right|^2 \omega^+ \gamma_k, \tag{4}$$

where:

$$\gamma_k = \frac{1}{4}\frac{(\omega^+)^2 + (\omega^-)^2}{\omega^+\omega^-}.$$

In opposite case $\omega \gg \frac{1}{\tau}$ one can find the exponentially small effect of excitation $\gamma_k \sim \frac{1}{2}(1 + e^{-2\pi\tau\omega^-})$.

These results take place only when $\tau \sim L/c$ and in principle differ from that one found in [2–4] for the case $\tau \gg L/c$.

4. Three-dimensional case $|\mathrm{q}| \sim k_z$

If $|\mathrm{q}| \sim k_z$ one could not divide Eq.(2) into two first order differential equations. But for long wavelength modes with polarization parallel to xy plane in case $\delta L/L \ll 1$ one can use Eq.(4) but now $\omega_k^- = c\sqrt{|\mathrm{q}|^2 + k_z^2}$ and $\omega_k^+ = c\sqrt{|\mathrm{q}|^2 + (L^-/L^+)^2 k_z^2}$. In result one can find:

$$\overline{E} = \sum_{\mathrm{k}} \left|C_{\mathrm{k}}\right|^2 \omega_k^+ \gamma_k \sim V^+ \int d\Omega \int_{\omega_{\min}}^{\omega_{\max}} 2\gamma_k \sqrt{1 + \Delta\cos^2(\theta)}\, U_{0\omega} d\omega, \tag{5}$$

where $U_{0\omega} = \frac{|C_{\mathbf{k}}|^2}{2(2\pi c)^3}\omega^3$ is the initial spectral density of radiation; V^+ is the final volume of the cavity; $\omega = \omega_k^-$; $\Delta = \frac{L^{+^2}+L^{-^2}}{L^+L^-}$; $d\Omega$ is the differential of solid angle and θ is the angle between wave vector of the cavity mode and the direction of displacement of the mobile wall. The low limit ω_{min} of integration in Eq.(5) depends on the size of the cavity L^-: $\omega_{min} = \pi c/L^-$. The upper limit ω_{max} is different in two cases $\omega \ll 2\pi/\tau$ and $\omega \gg 2\pi/\tau$.

In the case $\omega \ll 2\pi/\tau$ ("instantaneous" approach) one have $\omega_{max} = 2\pi/\tau$ and the value γ_k equals to:

$$\gamma_k = \frac{1}{4}\frac{2 + \Delta\cos^2(\theta)}{\sqrt{1 + \Delta\cos^2(\theta)}}. \tag{6}$$

The quantity $\gamma_k(\Delta, \theta)$ represented in Fig.2 describes the angle distribution and effectiveness of excitation of eigenmodes of the cavity.

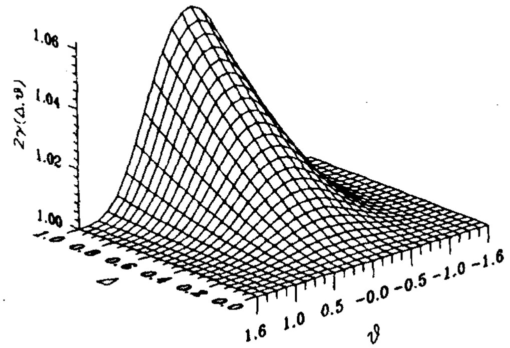

Fig.2. Relative spectral density of radiation $U_\omega/U_{0\omega} = 2\gamma_k$ as a function of Δ and observation angle θ in instantaneous approach

In the other case $\omega \gg 2\pi/\tau$ ("adiabatic" approach) one have $\omega_{\max} = \infty$ and the value γ_k equals to:

$$\gamma_k \sim \frac{1}{2}(1 + e^{-2\pi\tau\omega^-}). \tag{7}$$

5. Quantum consideration

1. Instantaneous approach $\omega \ll 2\pi/\tau$. Suppose that the initial state of the field is vacuum state. Quantum calculation for the model in which the frequency of each of the cavity modes is changed instantaneously from the initial value $\omega^- = \omega^-(L^-)$ to the final value $\omega^+ = \omega^+(L^+)$ (i.e. $\omega \ll 2\pi/\tau$ where ω is the frequency of the cavity mode) gives the expressions having the form of Eqs.(5),(6), but now $|C_{\mathbf{k}}|^2 = 3\hbar$. The total energy excited from the initial vacuum state is (in order to omit the energy of ground state we replace $\gamma_k \to \gamma_k - 1/2$):

$$\delta\overline{E} = V^+\frac{6\pi\hbar}{c^3\tau^4}I_1(\Delta) \tag{8}$$

where $I_1(\Delta) \sim \Delta^2/40$ at $\Delta \ll 1$.

2. Adiabatic approach $\omega \gg 2\pi/\tau$. In the case $\omega \gg 2\pi/\tau$ the electromagnetic field in the cavity is described by Schrödinger equation for harmonic oscillator with slowly changing frequency. It leads to relations (5) and (7) obtained in classical approach with $|C_{\mathbf{k}}|^2 = 3\hbar$. After integrating one have:

$$\delta\overline{E} = V^{+}\frac{6\hbar}{(2\pi c)^3}\frac{\omega_{\min}^3}{\tau}e^{-2\pi\tau\omega_{\min}}I_2(\Delta)$$

where $I_1(\Delta) \sim I_2(\Delta) \sim \Delta^2/40$ at $\Delta \ll 1$.

The value $\delta\overline{E}$ in both cases $\omega \ll 2\pi/\tau$ and $\omega \gg 2\pi/\tau$ obviously equals to the work of *nonstationary correction* to the (stationary) Casimir force during the time of displacement of the cavity wall (see e.g. [5]).

6. Specific properties of field's state after parametric excitation

The state of the field produced by the parametric excitation from the initial vacuum state is different from the electromagnetic field of another origin (which may appear in experiment) because of only even quantum levels are populated and specific distribution of wave packet in phase space takes place in the former case. These states are squeezed in the phase space in the direction of canonical variable P:

$$\langle\Delta Q^2\rangle_k = \frac{\hbar}{\omega_k}\left(\gamma_k + \sqrt{\gamma_k^2 - 1/4}\right),$$
$$\langle\Delta P^2\rangle_k = \hbar\omega_k\left(\gamma_k - \sqrt{\gamma_k^2 - 1/4}\right).$$

In result the uncertainty relation has the form: $\sqrt{\langle\Delta P^2\rangle_k\langle\Delta Q^2\rangle_k} = \hbar/2$ (last expression for oscillator with parametrically driven frequency was found in [6]; see also [7]). The states originated from the parametric excitation have some similarity to *Schrödinger cat* states, but the corresponding population of quantum levels is not a *Poisson* one.

7. Conclusion

Let now consider the possible experimental manifestation of the analyzed phenomena. By use of power femtosecond laser pulses one can produce rather dense $(e-h)$ plasma in short time interval τ in thin semiconductor layer of width l_s (see Fig.1) The initial length of such resonator is: $l_s + l_i$, where l_i is the thickness of the gap (insulator film) and the final length is l_i. Then we have $\tau \sim l_s/v$ where v is the velocity of electrons in plasma and $\Delta = \frac{(l_s+l_i)^2 - l_i^2}{l_i^2} \sim 2l_s/l_i \ll 1$. The velocity $v \sim 10^8$cm/sec is attainable for appropriate laser frequency ($v \sim \left(\frac{2(E_g - \hbar\omega)}{m^*}\right)^{\frac{1}{2}}$, where E_g is the energy gap for superconductor). If the value l_s is smaller than the extinction length for the excitation pulse then the plasma born simultaneously on the whole film. In the last case the parametric excitation of the electromagnetic field takes place due to the *change* of the *transmittance* of the semiconductor film (details well be published elsewhere). In the latter case the characteristic time τ is defined mainly by the form of the laser pulse.

The energy of the field after the parametric pumping (see Eq.(8)) is strongly depends on τ:

$$\delta\overline{E} \approx V^{+}\frac{3\hbar\pi^2}{5c^3\tau^4}\left(\frac{l_s}{l_i}\right)^2.$$

For example for $l_s \sim 10^3$Å, $l_i \sim 10^5$Å and $\tau \sim 10^{-14}$sec (it is attainable pulse time), $\lambda_{min} \sim 3 \cdot 10^{-4}$cm. For $l_i \sim 10^5$Å we find: $\overline{E}/V^{+} \sim 1.6 \cdot 10^6$eV/cm^3 and $\overline{N}/V^{+} \sim 5 \cdot 10^6$photons/cm^3. Note that the estimates given above are really upper estimations (for infinite quality Q of the cavity). Taking into account the dielectric susceptibility ε of the medium in the cavity one can obtain additional multiplier $\varepsilon^{5/2}$ in the estimate for the excited energy.

It occurs that the angle distribution of the parametrically excited radiation (which describes by the dependence $\gamma = \gamma(\theta)$ where θ is the angle between the direction of radiation and z-axis; see Fig.2) is different from that one of radiation of another origins appearing in experiment. Because of small time of pumping, parametrically excited light appears in time interval τ which is shorter than one required for appearing of radiation of another origins. These specific properties as well as quantum statistical properties described above (population of even levels and squeezing) may be used to distinguish parametrically excited field from other electromagnetic radiation in experiment (see e.g. [8, 9]).

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